



CURRICULUM FET

MATHEMATICS

2018 REVISION MATERIAL



#MASTER THE BASICS

Booklet 2 – 2018:

- Financial Mathematics
- Probability
- Trigonometry
- Euclidean Geometry

It does not matter what percentage you got in the past, you can always improve if you practice!

Practice makes perfect! Collect the marks!

NOTE: PLEASE USE THIS BOOKLET TOGETHER WITH OTHER REVISION MATERIAL LIKE BOOKLET 1, PREVIOUS YEARS QUESTION PAPERS' REVISION MATERIAL (WITH QUESTION PAPERS FROM 2015 – 2017/18 SUPP.)

FROM 2018 WINTER SCHOOL ONWARDS!

RESOURCE MATERIAL

BOOKLET 2: 2018

<p>1. FINANCIAL MATHEMATICS</p> <ul style="list-style-type: none"> • Simple and compound interest • Logarithms in the context • Present value and future value • annuities (investments, sinking funds, loans and bond repayments) • Nominal and effective interest rates • Depreciation (reducing balance and straight line) 	<p>2. PROBABILITY</p> <ul style="list-style-type: none"> • Probability rules (identity, mutually exclusive events, independent events, complementary) • Venn-diagram, Tree-diagram, Contingency table • Counting principles
<p>3. TRIGONOMETRY</p> <ul style="list-style-type: none"> • Trigonometric definitions Special angles, Reduction formulae • Compound and double angle formulae • Identities and equations (general solution as well) • Solution of triangles and problems in 2D and 3D (sine-, cosine-, area rules) • Trigonometric Functions and transformations 	<p>4. EUCLIDEAN GEOMETRY</p> <ul style="list-style-type: none"> • Solving riders using properties of parallel lines, triangles and quadrilaterals • Circle geometry – prove theorems and apply with converses. • Ratio and proportion prove theorems and apply with converses. • Similarity – Prove and apply • Pythagoras Theorem – apply.

REMEMBER: PRACTICE MAKES PERFECT!!!! HARDWORK NEVER KILLS!!!!

SO, PRACTICE!!! PRACTICE!!!! PRACTICE!!!!

A. FINANCIAL MATHEMATICS

FINANCIAL MATHEMATICS

NOVEMBER 2008

QUESTION 7

- 7.1 R1 570 is invested at 12% p.a. compound interest. After how many years will the investment be worth R23 000? (4)
- 7.2 A farmer has just bought a new tractor for R800 000. He has decided to replace the tractor in 5 years' time, when its trade-in value will be R200 000. The replacement cost of the tractor is expected to increase by 8% per annum.
- 7.2.1 The farmer wants to replace his present tractor with a new one in 5 years' time. The farmer wants to pay cash for the new tractor, after trading in his present tractor for R200 000. How much will he need to pay? (3)

FEB/MARCH 2009

QUESTION 9

- 9.1 R2 000 was invested in a fund paying $i\%$ interest compounded monthly. After 18 months the value of the fund was R2 860,00. Calculate i , the interest rate. (4)

QUESTION 10

Rowan plans to buy a car for R125 000,00. He pays a deposit of 15% and takes out a bank loan for the balance. The bank charges 12,5% p.a. compounded monthly.

Calculate:

- 10.1 The value of the loan borrowed from the bank (1)
- 10.2 The monthly repayment on the car if the loan is repaid over 6 years (5)
- [6]

NOVEMBER 2009

QUESTION 9

- 9.1 A photocopier valued at R24 000 depreciates at a rate of 18% p.a. on the reducing-balance method. After how many years will its value be R15 000? (4)

NOVEMBER 2010**QUESTION 7**

- 7.1 At what annual percentage interest rate, compounded quarterly, should a lump sum be invested in order for it to double in 6 years? (5)

[14]**FEB/MARCH 2011****QUESTION 8**

- 8.1 R1 430,77 was invested in a fund paying i % p.a. compounded monthly. After 18 months the fund had a value of R1 711,41. Calculate i . (4)

NOVEMBER 2011**QUESTION 7**

- 7.1 How many years will it take for an article to depreciate to half its value according to the reducing-balance method at 7% per annum? (4)

- 7.2 Two friends each receive an amount of R6 000 to invest for a period of 5 years. They invest the money as follows:

- Radesh: 8,5% per annum simple interest. At the end of the 5 years, Radesh will receive a bonus of exactly 5% of the principal amount.
- Thandi: 8% per annum compounded quarterly.

Who will have the bigger investment after 5 years? Justify your answer with appropriate calculations. (6)

FEBRUARY/MARCH 2012**QUESTION 7**

- 7.1 Lerato wants to purchase a house that costs R850 000. She is required to pay a 12% deposit and she will borrow the balance from a bank. Calculate the amount that Lerato must borrow from the bank. (2)
- 7.2 The bank charges interest at 9% per annum, compounded monthly on the loan amount. Lerato works out that the loan will carry an effective interest rate of 9,6% per annum. Is her calculation correct or not? Justify your answer with appropriate calculations. (4)
- 7.3 Lerato takes out a loan from the bank for the balance of the purchase price and agrees to pay it back over 20 years. Her repayments start one month after her loan is granted. Determine her monthly instalment if interest is charged at 9% per annum compounded monthly. (4)

NOVEMBER 2012**QUESTION 7**

- 7.1 A business buys a machine that costs R120 000. The value of the machine depreciates at 9% per annum according to the diminishing-balance method.
- 7.1.1 Determine the scrap value of the machine at the end of 5 years. (3)
- 7.1.2 After five years the machine needs to be replaced. During this time, inflation remained constant at 7% per annum. Determine the cost of the new machine at the end of 5 years. (3)
- 7.1.3 The business estimates that it will need R90 000 by the end of five years. A sinking fund for R90 000, into which equal monthly instalments must be paid, is set up. Interest on this fund is 8,5% per annum, compounded monthly. The first payment will be made immediately and the last payment will be made at the end of the 5-year period.
- Calculate the value of the monthly payment into the sinking fund. (5)
- 7.2 Lorraine receives an amount of R900 000 upon her retirement. She invests this amount immediately at an interest rate of 10,5% per annum, compounded monthly.
- She needs an amount of R18 000 per month to maintain her current lifestyle. She plans to withdraw the first amount at the end of the first month.
- For how many months will she be able to live from her investment? (6)

[17]

MARCH 2013**QUESTION 7**

- 7.1 Raeesa invests R4 million into an account earning interest of 6% per annum, compounded annually. How much will her investment be worth at the end of 3 years? (3)

QUESTION 8

Jeffrey invests R700 per month into an account earning interest at a rate of 8% per annum, compounded monthly. His friend also invests R700 per month and earns interest compounded semi-annually (that is every six months) at $r\%$ per annum. Jeffrey and his friend's investments are worth the same at the end of 12 months. Calculate r . [3]

QUESTION 7

- 7.1 Diane invests a lump sum of R5 000 in a savings account for exactly 2 years. The investment earns interest at 10% p.a., compounded quarterly.
- 7.1.1 What is the quarterly interest rate for Diane's investment? (1)
- 7.1.2 Calculate the amount in Diane's savings account at the end of the 2 years. (3)
- 7.2 Motloi inherits R800 000. He invests all of his inheritance in a fund which earns interest at a rate of 14% p.a., compounded monthly. At the end of each month he withdraws R10 000 from the fund. His first withdrawal is exactly one month after his initial investment.
- 7.2.1 How many withdrawals of R10 000 will Motloi be able to make from this fund? (5)
- 7.2.2 Exactly four years after his initial investment Motloi decides to withdraw all the remaining money in his account and to use it as a deposit towards a house.
- (a) What is the value of Motloi's deposit, to the nearest rand? (4)
- (b) Motloi's deposit is exactly 30% of the purchase price of the house. What is the purchase price of the house, to the nearest rand? (1)
- [14]

B. PROBABILITY

1) Definition

- If an outcome set S consists of $n(S)$ equally likely outcomes and $n(E)$ of these correspond to an event E , then
- $P(E) = \frac{n(E)}{n(S)}$, where $P(E)$ denotes the probability of event E .
- $0 \leq P(E) \leq 1$
- If $E = \emptyset \Rightarrow P(\emptyset) = 0$
- If $E = S \Rightarrow P(E) = 1$

2) Addition rule:

- $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

3) Mutually exclusive (disjoint)

- $P(A \text{ and } B) = P(\emptyset) = 0$
- $P(A \text{ or } B) = P(A) + P(B)$

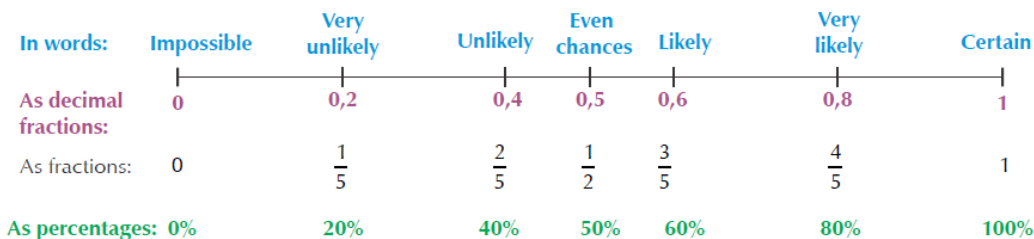
4) Independent (Multiplication law)

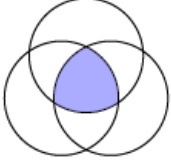
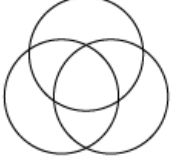
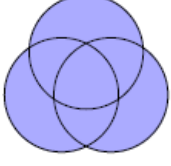
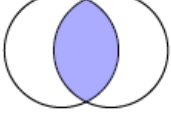
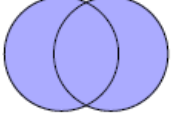
- Outcome of second event is not affected by the outcome of the first event.
- $P(A \text{ and } B) = P(A) \times P(B)$
- Probability that one event AND another happens

5) Complementary Rule

- Cannot occur at the same time.
- They are mutually exclusive
- $P(\text{not } A) = P(A') = 1 - P(A)$
- $P(\text{not winning}) = 1 - P(\text{winning})$

6) Probability scale



Words	Symbols	Venn diagram
"all"	$A \text{ and } B \text{ and } C / A \cap B \cap C$	
"none"		
"at least one"	$A \text{ or } B \text{ or } C / A \cup B \cup C$	
"both A and B"	$A \text{ and } B / A \cap B$	
"A or B"	$A \text{ or } B / A \cup B$	

DBE/EXEMPLAR 2014

QUESTION 11

11.1 Events A and B are mutually exclusive. It is given that:

- $P(B) = 2P(A)$ & $P(A \text{ or } B) = 0,57$

Calculate $P(B)$. (3)

11.2 Two identical bags are filled with balls. Bag A contains 3 pink and 2 yellow balls.

Bag B contains 5 pink and 4 yellow balls. It is equally likely that Bag A or Bag B is chosen. Each ball has an equal chance of being chosen from the bag. A bag is chosen at random and a ball is then chosen at random from the bag.

11.2.1 Represent the information by means of a tree diagram. Clearly indicate the probability associated with each branch of the tree diagram and write down all the outcomes. (4)

11.2.2 What is the probability that a yellow ball will be chosen from Bag A? (1)

11.2.3 What is the probability that a pink ball will be chosen? (3)

[11]

QUESTION 12

Consider the word M A T H S.

- 12.1 How many different 5-letter arrangements can be made using all the above letters? (2)
- 12.2 Determine the probability that the letters S and T will always be the first two letters of the arrangements in QUESTION 12.1. (3)
- [5]**

VRAAG 11

11.1 Gebeurtenisse A en B is onderling uitsluitend. Dit word gegee dat:

- $P(B) = 2P(A)$ & $P(A \text{ of } B) = 0,57$
Bereken $P(B)$. (3)

11.2 Twee identiese sakke word met balle gevul. Sak A bevat 3 pienk en 2 geel balle.

Sak B bevat 5 pienk en 4 geel balle. Dit is ewe waarskynlik dat Sak A of Sak B gekies word. Elke bal het 'n gelyke kans om uit die sak gekies te word. 'n Sak word willekeurig gekies en dan word 'n bal willekeurig uit die sak gekies.

11.2.1 Stel die inligting deur middel van 'n boomdiagram voor. Dui duidelik die waarskynlikheid aan wat deur elke tak van die boomdiagram verteenwoordig word en skryf al die uitkomst neer. (4)

11.2.2 Wat is die waarskynlikheid dat 'n geel bal uit Sak A gekies sal word? (1)

11.2.3 Wat is die waarskynlikheid dat 'n pienk bal gekies sal word? (3)

[11]

VRAAG 12

Beskou die woord M A T H S.

- 12.1 Hoeveel verskillende 5-letter-rangskikkings kan gevorm word as al die letters hierbo gebruik word? (2)
- 12.2 Bepaal die waarskynlikheid dat letter S en T altyd die eerste twee letters van die rangskikkings in VRAAG 12.1 sal wees. (3)
- [5]**

DBE/NOVEMBER 2014

QUESTION 11

A survey concerning their holiday preferences was done with 180 staff members.

The options they could choose from were to:

- Go to the coast
- Visit a game park
- Stay at home

The results were recorded in the table below:

	Coast	Game Park	Home	Total
Male	46	24	13	83
Female	52	38	7	97
Total	98	62	20	180

11.1 Determine the probability that a randomly selected staff member:

11.1.1 is male (1)

11.1.2 does not prefer visiting a game park (2)

11.2 Are the events 'being a male' and 'staying at home' independent events. Motivate your answer with relevant calculations. (4)

[7]

QUESTION 12

12.1 A password consists of five different letters of the English alphabet. Each letter may be used only once. How many passwords can be formed if:

12.1.1 All the letters of the alphabet can be used (2)

12.1.2 The password must start with a 'D' and end with an 'L' (2)

12.2 Seven cars of different manufacturers, of which 3 are silver, are to be parked in a straight line.

12.2.1 In how many different ways can ALL the cars be parked? (2)

12.2.2 If the three silver cars must be parked next to each other, determine in how many different ways the cars can be parked. (3)

[9]

DBE/NOVEMBER 2014 (AFRIKAANS)

VRAAG 11

'n Opname oor die vakansievoorkeure van 180 personeellede is gedoen. Die opsies waaruit hulle kon kies, was om:

- Kus toe te gaan
- 'n Wildreservaat te besoek
- Tuis te bly

Die resultate is in die tabel hieronder aangeteken:

	Kus	Wild- reservaat	Tuis	Totaal
Manlik	46	24	13	83
Vroulik	52	38	7	97
Totaal	98	62	20	180

11.1 Bepaal die waarskynlikheid dat 'n personeellid wat ewekansig gekies is:

11.1.1 Manlik sal wees (1)

11.1.2 Verkies om nie 'n wildreservaat te besoek nie (2)

11.2 Is die gebeure 'manlik' en 'tuis bly' onafhanklike gebeure? Motiveer jou antwoord met nodige bewerkings. (4)

[7]

VRAAG 12

12.1 'n Kodewoord bestaan uit vyf verskillende letters van die Engelse alfabet. Elke letter mag slegs een maal gebruik word. Hoeveel kodewoorde kan gevorm word indien:

12.1.1 Al die letters in die alfabet gebruik kan word (2)

12.1.2 Die kodewoord met 'n 'D' moet begin en met 'n 'L' moet eindig (2)

12.2 Sewe motors van verskillende vervaardigers, waarvan 3 silwer is, moet in 'n reguitlyn geparkeer word.

12.2.1 Op hoeveel verskillende maniere kan AL die motors geparkeer word? (2)

12.2.2 Indien die drie silwer motors langs mekaar geparkeer moet word, bepaal op hoeveel verskillende maniere die motors geparkeer kan word. (3)

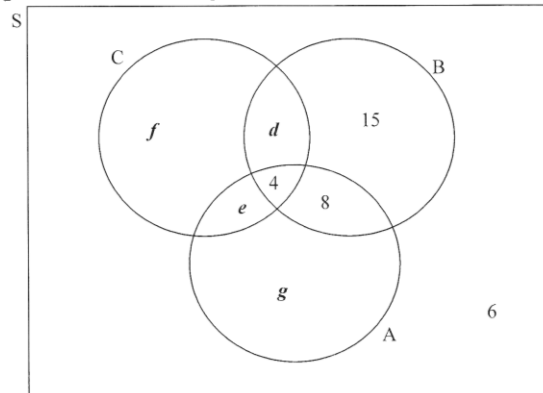
[9]

DBE/MARCH 2015**QUESTION 10**

10.1 Research was conducted about driving under the influence of alcohol. Information obtained from traffic authorities in 54 countries on the methods that are used to measure alcohol levels in a person, are summarised below:

- 4 countries use all three methods (A, B and C).
- 12 countries use the alcohol content of breath (A) and blood-alcohol concentration (B).
- 9 countries use blood-alcohol concentration (B) and certificates issued by doctors (C).
- 8 countries use the alcohol content of breath (A) and certificates issued by doctors (C).
- 21 countries use the alcohol content of breath (A).
- 32 countries use blood-alcohol concentration (B).
- 20 countries use certificates issued by doctors (C).
- 6 countries use none of these methods.

Below is a partially completed Venn diagram representing the above information.



- 10.1.1 Use the given information and the Venn diagram to determine the values of d , e , f and g . (4)
- 10.1.2 For a randomly selected country, calculate:
- $P(A \text{ and } B \text{ and } C)$ (1)
 - $P(A \text{ or } B \text{ or } C)$ (1)
 - $P(\text{only } C)$ (1)
 - $P(\text{that a country uses exactly two methods})$ (1)

10.2 Nametso may choose DVDs from three categories as listed in the table below:

Drama	Romance	Comedy
<ul style="list-style-type: none"> • <i>Last Hero</i> • <i>Midnight</i> • <i>Stranger Calls</i> • <i>Missing in Action</i> • <i>Only 40 Seconds Left</i> 	<ul style="list-style-type: none"> • <i>One Heart</i> • <i>You and Me</i> • <i>Love Song</i> • <i>Bird's First Nest</i> 	<ul style="list-style-type: none"> • <i>Laughing Dragon</i> • <i>Falling Down</i> • <i>Sitting on the Stairs</i>

10.2.1 Nametso must choose ONE DVD from the Drama category. What is the probability that she will choose *Midnight*? (2)

10.2.2 How many different selections are possible if her selection must include ONE drama, ONE romance and ONE comedy? (2)

10.2.3 Calculate the probability that she will have *Last Hero* and *Laughing Dragon* as part of her selection in QUESTION 10.2.2. (2)
[14]

DBE/MARCH 2015 (AFRIKAANS)

VRAAG 10

10.1 Navorsing is gedoen oor bestuur onder die invloed van alkohol. Inligting wat by die verkeersowerhede van 54 lande gekry is oor die metodes wat gebruik word om die alkoholvlak van 'n persoon te toets, is hieronder opgesom:

- 4 lande gebruik al drie metodes (A, B en C).
- 12 lande gebruik die alkoholinhoud van asem (A) en bloed-alkohol-konsentrasie (B).
- 9 lande gebruik bloed-alkoholkonsentrasie (B) en sertifikate uitgereik deur dokters (C).
- 8 lande gebruik die alkoholinhoud van asem (A) en sertifikate uitgereik deur dokters (C).
- 21 lande gebruik die alkoholinhoud van asem (A).
- 32 lande gebruik bloed-alkoholkonsentrasie (B).
- 20 lande gebruik sertifikate uitgereik deur dokters (C).
- 6 lande gebruik geeneen van hierdie metodes nie.

Hieronder is 'n gedeeltelik voltooide Venn-diagram wat die inligting hierbo voorstel.

Refer to English version above:

10.2

Drama	Liefdesverhaal	Komedie
<ul style="list-style-type: none"> • <i>Last Hero</i> • <i>Midnight</i> • <i>Stranger Calls</i> • <i>Missing in Action</i> • <i>Only 40 Seconds Left</i> 	<ul style="list-style-type: none"> • <i>One Heart</i> • <i>You and Me</i> • <i>Love Song</i> • <i>Bird's First Nest</i> 	<ul style="list-style-type: none"> • <i>Laughing Dragon</i> • <i>Falling Down</i> • <i>Sitting on the Stairs</i>

- 10.2.1 Nametso moet EEN DVD uit die Drama-kategorie kies. Wat is die waarskynlikheid dat sy *Midnight* sal kies? (2)
- 10.2.2 Hoeveel verskillende keuses is moontlik indien haar keuse EEN drama, EEN liefdesverhaal en EEN komedie moet insluit? (2)
- 10.2.3 Bereken die waarskynlikheid dat *Last Hero* en *Laughing Dragon* deel van haar keuse in VRAAG 10.2.2 sal wees. (2)

[14]

JUNE 2015 (AMENDED CAPS:SCE)

QUESTION 11

- 11.1 Zebra High School offers only two sporting activities, namely rugby and hockey.
- The following information is given:
- There are 600 learners in the school.
 - 372 learners play hockey.
 - 288 learners play rugby.
 - 56 of the learners play NO sport.
 - The number of learners that play both hockey and rugby is x .
- 11.1.1 Represent the given information in a Venn diagram, in terms of x . (3)
- 11.1.2 Calculate the value of x . (2)
- 11.1.3 Are the events playing rugby and playing hockey mutually exclusive? Justify your answer. (2)
- 11.2 Another school, Tulani High, has a sports awards ceremony. Tulani High has a basketball team consisting of 5 players and a volleyball team consisting of 6 players.
- 11.2.1 All the basketball players sit in a single row at the ceremony. There are no restrictions on who sits in which position. In how many different ways can they be seated? (1)
- 11.2.2 The decision is taken that the captain must sit in the first seat of the row. The two vice-captains have to be seated next to each other in any of the remaining seats. In how many different ways can the basketball players be seated now? (3)

JUNE 2015 (AMENDED CAPS:SCE) – (AFRIKAANS)

VRAAG 11

11.1 Zebra Hoërskool bied slegs twee sportaktiwiteite, naamlik rugby en hokkie, aan.

Die volgende inligting word gegee:

- Daar is 600 leerders in die skool.
- 372 leerders speel hokkie.
- 288 leerders speel rugby.
- 56 van die leerders speel GEEN sport nie.
- Die getal leerders wat beide hokkie en rugby speel, is x .

11.1.1 Stel die gegewe inligting in 'n Venn-diagram, in terme van x , voor. (3)

11.1.2 Bereken die waarde van x . (2)

11.1.3 Is die gebeurtenisse, speel rugby en speel hokkie, onderling uitsluitend? Motiveer jou antwoord. (2)

11.2 'n Ander skool, Tulani Hoër, het 'n prysuitdelingsereemonie vir sport. Tulani Hoër het 'n basketbalspan wat uit 5 spelers bestaan en 'n vlugbalspan wat uit 6 spelers bestaan.

11.2.1 Al die basketbalspelers sit in een ry by die sereemonie. Daar is geen beperking op wie in watter posisie sit nie. Op hoeveel verskillende maniere kan hulle sit? (1)

11.2.2 Daar is besluit dat die kaptein op die eerste sitplek van die ry moet sit. Die twee onderkapteine moet langs mekaar op enige van die oorblywende sitplekke sit. Op hoeveel verskillende maniere kan die basketbalspelers nou sit? (3)

11.2.3 Die basketbalspan en die vlugbalspan sit na die pouse by die sereemonie in dieselfde ry. Bereken die waarskynlikheid dat die basketbalspelers sal saamsit en dat die vlugbalspelers sal saamsit. Neem aan dat sitplekposisies willekeurig toegeken word. Gee jou antwoord in vereenvoudigde breukvorm. (3)

[14]

DBE/NOVEMBER 2015

QUESTION 11

11.1 For two events, A and B, it is given that:

$$\begin{aligned} P(A) &= 0,2 \\ P(B) &= 0,63 \\ P(A \text{ and } B) &= 0,126 \end{aligned}$$

Are the events, A and B, independent? Justify your answer with appropriate calculations. (3)

11.2 The letters of the word DECIMAL are randomly arranged into a new 'word', also consisting of seven letters. How many different arrangements are possible if:

11.2.1 Letters may be repeated (2)

11.2.2 Letters may not be repeated (2)

11.2.3 The arrangements must start with a vowel and end in a consonant and no repetition of letters is allowed (4)

11.3 There are t orange balls and 2 yellow balls in a bag. Craig randomly selects one ball from the bag, records his choice and returns the ball to the bag. He then randomly selects a second ball from the bag, records his choice and returns it to bag. It is known that the probability that Craig will select two balls of the same colour from the bag is 52%.

Calculate how many orange balls are in the bag. (6)
[17]

DBE/NOVEMBER 2015(AFRIKAANS)

VRAAG 11

11.1 Vir twee gebeurtenisse, A en B, word gegee dat:

$$\begin{aligned} P(A) &= 0,2 \\ P(B) &= 0,63 \\ P(A \text{ en } B) &= 0,126 \end{aligned}$$

Is die gebeurtenisse, A en B, onafhanklik? Motiveer jou antwoord met toepaslike berekeninge. (3)

11.2 Die letters van die woord DECIMAL word willekeurig gerangskik om 'n nuwe 'woord', wat ook uit sewe letters bestaan, te vorm. Hoeveel verskillende rangskikkings is moontlik as:

11.2.1 Letters herhaal mag word (2)

11.2.2 Letters nie herhaal mag word nie (2)

11.2.3 Die rangskikkings met 'n klinker moet begin en met 'n konsonant moet eindig en geen herhaling van letters toegelaat word nie (4)

11.3 Daar is t oranje balle en 2 geel balle in 'n sak. Craig kies een bal willekeurig uit die sak, teken sy keuse aan en plaas die bal terug in die sak. Hy kies daarna 'n tweede bal willekeurig uit die sak, teken sy keuse aan en plaas die bal terug in die sak. Dit is bekend dat die waarskynlikheid dat Craig twee balle van dieselfde kleur uit die sak sal kies, 52% is.

Bereken hoeveel oranje balle daar in die sak is. (6)
[17]

DBE/MARCH 2016**QUESTION 10**

- 10.1 Each passenger on a certain Banana Airways flight chose exactly one beverage from tea, coffee or fruit juice. The results are shown in the table below.

	MALE	FEMALE	TOTAL
Tea	20	40	60
Coffee	<i>b</i>	<i>c</i>	80
Fruit juice	<i>d</i>	<i>e</i>	20
TOTAL	60	100	<i>a</i>

- 10.1.1 Write down the value of *a*. (1)
- 10.1.2 What is the probability that a randomly selected passenger is male? (2)
- 10.1.3 Given that the event of a passenger choosing coffee is independent of being a male, calculate the value of *b*. (4)
- 10.2 A Banana Airways aeroplane has 6 seats in each row.
- 10.2.1 How many possible arrangements are there for 6 people to sit in a row of 6 seats? (2)
- 10.2.2 Xoliswa, Anees and 4 other passengers sit in a certain row on a Banana Airways flight. In how many different ways can these 6 passengers be seated if Xoliswa and Anees must sit next to each other? (2)
- 10.2.3 Mary and 5 other passengers are to be seated in a certain row. If seats are allocated at random, what is the probability that Mary will sit at the end of the row? (4)
- [15]

DBE/MARCH 2016 (AFRIKAANS)**VRAAG 10**

- 10.1 Elke passasier op 'n sekere Banana Lugdiens-vlug kies presies een drankie uit tee, koffie of vrugtesap. Die resultate word in die tabel hieronder aangedui.

	MANLIK	VROULIK	TOTAAL
Tee	20	40	60
Koffie	<i>b</i>	<i>c</i>	80
Vrugtesap	<i>d</i>	<i>e</i>	20
TOTAAL	60	100	<i>a</i>

- 10.1.1 Skryf die waarde van *a* neer. (1)
- 10.1.2 Wat is die waarskynlikheid dat 'n passasier wat willekeurig gekies word, manlik is? (2)
- 10.1.3 Gegee dat die geval van 'n passasier wat koffie kies, onafhanklik is van die feit dat hy manlik is, bereken die waarde van *b*. (4)
- 10.2 'n Vliegtuig van Banana Lugdiens het 6 sitplekke in elke ry.
- 10.2.1 Hoeveel moontlike rangskikkings is daar vir 6 mense om in 'n ry van 6 sitplekke te sit? (2)
- 10.2.2 Xoliswa, Anees en 4 ander passasiers sit in 'n sekere ry op 'n Banana Lugdiens-vlug. Op hoeveel verskillende maniere kan hierdie 6 passasiers geplaas word as Xoliswa en Anees langs mekaar moet sit? (2)
- 10.2.3 Mary en 5 ander passasiers moet in 'n sekere ry sit. As sitplekke willekeurig toegeken word, wat is die waarskynlikheid dat Mary op die punt van die ry sal sit? (4)
- [15]

DBE/JUNE 2016(AMENDED CAPS:SCE)**QUESTION 10**

- 10.1 A tournament organiser conducted a survey among 150 members at a local sports club to find out whether they play tennis or not. The results are shown in the table below.

	PLAYING TENNIS	NOT PLAYING TENNIS
Male	50	30
Female	20	50

- 10.1.1 What is the probability that a member selected at random is:
- (a) Female (2)
- (b) Female and plays tennis (1)
- 10.1.2 Is playing tennis independent of gender? Motivate your answer with the necessary calculations. (3)

- 10.2 The probability of events A and B occurring are denoted by $P(A)$ and $P(B)$ respectively.

For any two events A and B it is given that:

- $P(B') = 0,28$
- $P(B) = 3P(A)$
- $P(A \text{ or } B) = 0,96$

Are events A and B mutually exclusive? Justify your answer.

(4)
[10]

QUESTION 11

Five boys and four girls go to the movies. They are all seated next to each other in the same row.

- 11.1 One boy and girl are a couple and want to sit next to each other at any end of the row of friends. In how many different ways can the entire group be seated? (3)
- 11.2 If all the friends are seated randomly, calculate the probability that all the girls are seated next to each other. (3)

[6]

DBE/JUNE 2016(AMENDED CAPS:SCE) – (AFRIKAANS)

VRAAG 10

- 10.1 'n Toernooi-organiseerder het 'n opname onder 150 lede by 'n plaaslike sportklub gedoen om uit te vind of hulle tennis speel of nie. Die resultate word in die tabel hieronder getoon.

	SPEEL TENNIS	SPEEL NIE TENNIS NIE
Manlik	50	30
Vroulik	20	50

- 10.1.1 Wat is die waarskynlikheid dat 'n lid wat willekeurig gekies word:

- (a) Vroulik is (2)
- (b) Vroulik is en tennis speel (1)

- 10.1.2 Is tennisspeel onafhanklik van geslag? Motiveer jou antwoord met die nodige berekeninge. (3)

- 10.2 Die waarskynlikheid dat gebeurtenis A en B sal plaasvind, word deur $P(A)$ en $P(B)$ onderskeidelik aangetoon.

Vir enige twee gebeurtenisse A en B, word gegee dat:

- $P(B') = 0,28$
- $P(B) = 3P(A)$
- $P(A \text{ of } B) = 0,96$

- Is gebeurtenis A en B onderling uitsluitend? Motiveer jou antwoord. (4)
[10]

VRAAG 11

Vyf seuns en vier meisies gaan flied. Hulle sit almal langs mekaar in dieselfde ry.

- 11.1 Een seun en meisie is 'n paartjie en wil langs mekaar op enige punt van die ry vriende sit. Op hoeveel verskillende maniere kan die hele groep sit? (3)

- 11.2 Indien al die vriende willekeurig langs mekaar sit, bereken die waarskynlikheid dat al die meisies langs mekaar sal sit. (3)
[6]

DBE/NOVEMBER 2016

QUESTION 11

A survey was conducted among 100 boys and 60 girls to determine how many of them watched TV in the period during which examinations were written. Their responses are shown in the partially completed table below.

	WATCHED TV DURING EXAMINATIONS	DID NOT WATCH TV DURING EXAMINATIONS	TOTALS
Male	80	<i>a</i>	
Female	48	12	
Totals	<i>b</i>	32	160

- 11.1 Calculate the values of *a* and *b*. (2)
- 11.2 Are the events 'being a male' and 'did not watch TV during examinations' mutually exclusive? Give a reason for your answer. (2)
- 11.3 If a learner who participated in this survey is chosen at random, what is the probability that the learner:
- 11.3.1 Watched TV in the period during which the examinations were written? (2)
- 11.3.2 Is not a male and did not watch TV in the period during which examinations were written? (2)
- [8]**

QUESTION 12

The digits 1 to 7 are used to create a four-digit code to enter a locked room. How many different codes are possible if the digits may not be repeated and the code must be an even number bigger than 5 000?

[5]

DBE/NOVEMBER 2016(AFRIKAANS)

VRAAG 11

'n Opname is onder 100 seuns en 60 meisies gedoen om te bepaal hoeveel van hulle TV gekyk het in die tydperk waarin eksamen geskryf is. Hulle terugvoering word in die gedeeltelik voltooide tabel hieronder getoon.

	HET IN EKSAMENTYD TV GEKYK	HET NIE IN EKSAMENTYD TV GEKYK NIE	TOTALE
Manlik	80	<i>a</i>	
Vroulik	48	12	
Totale	<i>b</i>	32	160

- 11.1 Bereken die waardes van *a* en *b*. (2)
- 11.2 Is die gebeurtenisse 'om manlik te wees' en 'het nie in eksamentyd TV gekyk nie', onderling uitsluitend? Gee 'n rede vir jou antwoord. (2)
- 11.3 As 'n leerder wat aan die opname deelgeneem het, ewekansig gekies word, wat is die waarskynlikheid dat die leerder:
- 11.3.1 TV gekyk het in die tydperk waarin die eksamen geskryf is? (2)
- 11.3.2 Nie manlik is nie en nie TV gekyk het in die tydperk waarin eksamen geskryf is nie? (2)
- [8]

VRAAG 12

Die syfers 1 tot 7 word gebruik om 'n viersyferkode te skep om toegang tot 'n geslote vertrek te verkry. Hoeveel verskillende kodes is moontlik as die syfers nie herhaal mag word nie en die kode 'n ewe getal groter as 5 000 moet wees?

[5]

DBE/MARCH 2017**QUESTION 10**

10.1 The events S and T are independent.

- $P(S \text{ and } T) = \frac{1}{6}$
- $P(S) = \frac{1}{4}$

10.1.1 Calculate $P(T)$. (2)

10.1.2 Hence, calculate $P(S \text{ or } T)$. (2)

10.2 A FIVE-digit code is created from the digits 2 ; 3 ; 5 ; 7 ; 9.

How many different codes can be created if:

10.2.1 Repetition of digits is NOT allowed in the code (2)

10.2.2 Repetition of digits IS allowed in the code (1)

10.3 A group of 3 South Africans, 2 Australians and 2 Englishmen are staying at the same hotel while on holiday. Each person has his/her own room and the rooms are next to each other in a straight corridor.

If the rooms are allocated at random, determine the probability that the 2 Australians will have adjacent rooms and the 2 Englishmen will also have adjacent rooms. (4)
[11]

QUESTION 11

The success rate of the Fana soccer team depends on a number of factors. The fitness of the players is one of the factors that influence the outcome of a match.

- The probability that all the players are fit for the next match is 70%
- If all the players are fit to play the next match, the probability of winning the next match is 85%
- If there are players that are not fit to play the next match, the probability of winning the match is 55%

Based on fitness alone, calculate the probability that the Fana soccer team will win the next match. [5]

DBE/MARCH 2017(AFRIKAANS)

VRAAG 10

10.1 Die gebeurtenisse S en T is onafhanklik.

- $P(S \text{ en } T) = \frac{1}{6}$
- $P(S) = \frac{1}{4}$

10.1.1 Calculate P(T) (2)

10.1.2 Hence, calculate P(S or T) (2)

10.2 'n VYF-syfer kode word van die syfers 2 ; 3 ; 5 ; 7 ; 9 geskep.

Hoeveel verskillende kodes kan geskep word as:

10.2.1 herhaling van syfers in die kode NIE toegelaat word NIE. (2)

10.2.2 herhaling van syfers WORD in die kode toegelaat. (1)

10.3 'n Groep van 3 Suid-Afrikaners, 2 Australiërs en 2 Engelse bly in dieselfde hotel terwyl hulle op vakansie is. Elke persoon het hom/haar eie kamer en die kamers is langsaan mekaar in 'n reguit gang. As die kamers blindelings toegeken word, bepaal die waarskynlikheid dat die 2 Australiërs se kamers langs mekaar sal wees en dat die 2 Engelse se kamers ook langs mekaar sal wees. (4)

[11]

VRAAG 11

Die mate van sukses van die Fana sokkerspan is van 'n aantal faktore afhanklik.

Die fiksheid van die spelers is een van die faktore wat die uitslag van die wedstryd beïnvloed.

- Die waarskynlikheid dat al die spelers fiks sal wees vir die volgende wedstryd is 70%
- As al die spelers fiks is om die volgende wedstryd te speel, is die waarskynlikheid om die volgende wedstryd te wen 85%
- As daar spelers is wat nie fiks is om die volgende wedstryd te speel nie, is die waarskynlikheid om die wedstryd te wen 55%

Slegs op fiksheid gebaseer, bereken die waarskynlikheid dat die Fana sokkerspan die volgende wedstryd sal wen.

[5]

C. TRIGONOMETRY

TRIGONOMETRY: RATIOS, DEFINITIONS, REDUCTION, IDENTITIES, ETC.

NOVEMBER 2010

QUESTION 9

Given: $\tan \alpha = \frac{3}{4}$; where $\alpha \in [0^\circ ; 90^\circ]$

With the use of a sketch and without the use of a calculator, calculate:

9.1 $\sin \alpha$ (3)

9.2 $\cos^2(90^\circ - \alpha) - 1$ (2)

9.3 $1 - \sin 2\alpha$ (3)
[8]

QUESTION 10

(You may NOT use a calculator to answer this question.)

10.1 Simplify completely:

$$\frac{\sin(90^\circ + \theta) + \cos(180^\circ + \theta)\sin(-\theta)}{\sin 180^\circ - \tan 135^\circ} \quad (5)$$

10.2 Prove that for any angle A:

$$\frac{4 \sin A \cos A \cos 2A \sin 15^\circ}{\sin 2A (\tan 225^\circ - 2 \sin^2 A)} = \frac{\sqrt{6} - \sqrt{2}}{2} \quad (6)$$

10.3 Determine the general solution of:

$$6 \cos x - 5 = \frac{4}{\cos x} \quad ; \quad \cos x \neq 0 \quad (6)$$

[17]

FEB/MARCH 2011

QUESTION 10

10.1 If $\sin 28^\circ = a$ and $\cos 32^\circ = b$, determine the following in terms of a and/or b :

10.1.1 $\cos 28^\circ$ (2)

10.1.2 $\cos 64^\circ$ (3)

10.1.3 $\sin 4^\circ$ (4)

10.2 Prove without the use of a calculator, that if $\sin 28^\circ = a$ and $\cos 32^\circ = b$, then

$$b\sqrt{1-a^2} - a\sqrt{1-b^2} = \frac{1}{2} \quad (4)$$

10.3 Evaluate each of the following without using a calculator. Show ALL working.

10.3.1 $\frac{\sin 130^\circ \cdot \tan 60^\circ}{\cos 540^\circ \cdot \tan 230^\circ \cdot \sin 400^\circ}$ (7)

10.3.2 $(1 - \sqrt{2} \sin 75^\circ)(\sqrt{2} \sin 75^\circ + 1)$ (4)

10.4 Determine the general solution of: $\sin^2 x + \cos 2x - \cos x = 0$ (7)

10.5 Consider: $\frac{\cos 2x \tan x}{\sin^2 x}$

10.5.1 For which values of x , $x \in [0^\circ ; 180^\circ]$, will this expression be undefined? (3)

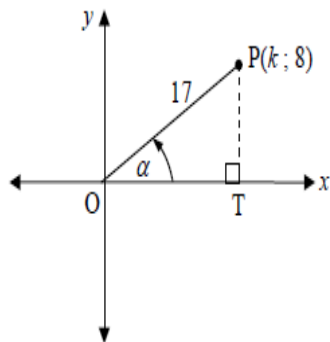
10.5.2 Prove that $\frac{\cos 2x \tan x}{\sin^2 x} = \frac{\cos x}{\sin x} - \tan x$ for all other values of x . (5)

[39]

QUESTION 8

Answer this question WITHOUT using a calculator.

- 8.1 The point $P(k; 8)$ lies in the first quadrant such that $OP = 17$ units and $\hat{T}OP = \alpha$ as shown in the diagram alongside.



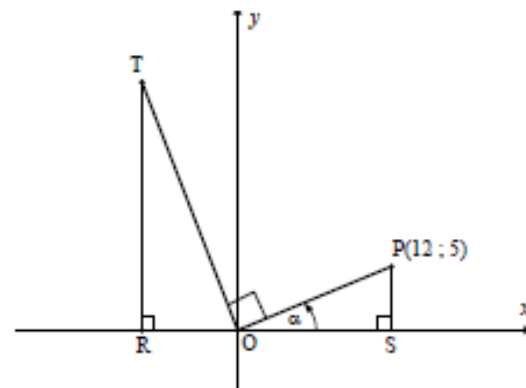
- 8.1.1 Determine the value of k . (2)
- 8.1.2 Write down the value of $\cos \alpha$. (1)
- 8.1.3 If it is further given that $\alpha + \beta = 180^\circ$, determine $\cos \beta$. (2)
- 8.1.4 Hence, determine the value of $\sin(\beta - \alpha)$. (4)
- 8.2 Consider the expression: $\frac{1 - \cos 2x - \sin x}{\sin 2x - \cos x}$
- 8.2.1 Prove that: $\frac{1 - \cos 2x - \sin x}{\sin 2x - \cos x} = \tan x$ (4)
- 8.2.2 The above expression is undefined if $\sin 2x - \cos x = 0$. Solve this equation in the interval $0^\circ \leq x \leq 360^\circ$. (4)

[17]

QUESTION 9

- 9.1 Answer this question without using a calculator.

In the diagram, P is the point $(12; 5)$. $OT \perp OP$. PS and TR are perpendicular to the x -axis. $\hat{P}OS = \alpha$ and $OR = 7,5$ units.



Determine:

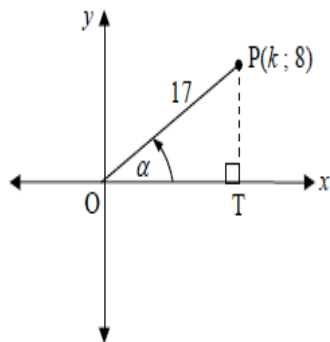
- 9.1.1 $\cos \alpha$ (2)
- 9.1.2 $\hat{T}OR$, in terms of α (2)
- 9.1.3 The length of OT (4)
- 9.2 Show that $\frac{\sin(90^\circ + x) \cdot \cos x \cdot \tan(-x)}{\cos(180^\circ + x)} = \sin x$. (4)

[12]

QUESTION 8

Answer this question WITHOUT using a calculator.

- 8.1 The point $P(k; 8)$ lies in the first quadrant such that $OP = 17$ units and $\hat{T}OP = \alpha$ as shown in the diagram alongside.



- 8.1.1 Determine the value of k . (2)
- 8.1.2 Write down the value of $\cos \alpha$. (1)
- 8.1.3 If it is further given that $\alpha + \beta = 180^\circ$, determine $\cos \beta$. (2)
- 8.1.4 Hence, determine the value of $\sin(\beta - \alpha)$. (4)
- 8.2 Consider the expression: $\frac{1 - \cos 2x - \sin x}{\sin 2x - \cos x}$
- 8.2.1 Prove that: $\frac{1 - \cos 2x - \sin x}{\sin 2x - \cos x} = \tan x$ (4)
- 8.2.2 The above expression is undefined if $\sin 2x - \cos x = 0$. Solve this equation in the interval $0^\circ \leq x \leq 360^\circ$. (4)

[17]

QUESTION 9

- 9.1 Simplify as far as possible: $\frac{\sin^2 \theta}{\sin(180^\circ - \theta) \cdot \cos(90^\circ + \theta) + \tan 45^\circ}$ (5)
- 9.2 Simplify without the use of a calculator: $\frac{\sin 104^\circ (2 \cos^2 15^\circ - 1)}{\tan 38^\circ \cdot \sin^2 412^\circ}$ (8)

[13]

QUESTION 6

- 6.1 If $\sin 23^\circ = p$, write down the following in terms of p . Do NOT use a calculator.
- 6.1.1 $\cos 113^\circ$ (2)
- 6.1.2 $\cos 23^\circ$ (2)
- 6.1.3 $\sin 46^\circ$ (2)
- 6.2 It is known that $13 \sin \alpha - 5 = 0$ and $\tan \beta = -\frac{3}{4}$ where $\alpha \in [90^\circ; 270^\circ]$ and $\beta \in [90^\circ; 270^\circ]$. Determine, without using a calculator, the values of the following:
- 6.2.1 $\cos \alpha$ (3)
- 6.2.2 $\cos(\alpha + \beta)$ (5)
- 6.3 Solve for $x \in [0^\circ; 360^\circ]$ if $\frac{1}{2} \cos x = 0,435$. (3)

[17]

TRIGONOMETRY: FUNCTIONS

QUESTION 8

Consider the functions $f(x) = \cos 3x$ and $g(x) = \sin x$ for $x \in [-90^\circ; 180^\circ]$.

QUESTION 8

Consider the functions defined by $f(x) = \sin 2x$ and $g(x) = \frac{1}{2} \tan x$ for $x \in [-90^\circ; 180^\circ]$.

QUESTION 12

Given: $g(x) = 2\cos(x - 30^\circ)$

- 12.1 Sketch the graph of g for $x \in [-90^\circ; 270^\circ]$ on DIAGRAM SHEET 4. (2)
- 12.2 Use the symbols A and B to plot the two points on the graph of g for which $\cos(x - 30^\circ) = 0,5$ (2)
- 12.3 Calculate the x -coordinates of the points A and B. (3)

QUESTION 12

Given: $f(x) = 2\cos x$ and $g(x) = \tan 2x$

- 12.1 Sketch the graphs of f and g on the same system of axes provided on DIAGRAM SHEET 4, for $x \in [-90^\circ; 90^\circ]$ (6)
- 12.2 Solve for x if $2\cos x = \tan 2x$ and $x \in [-90^\circ; 90^\circ]$. Show ALL working details. (8)

QUESTION 11

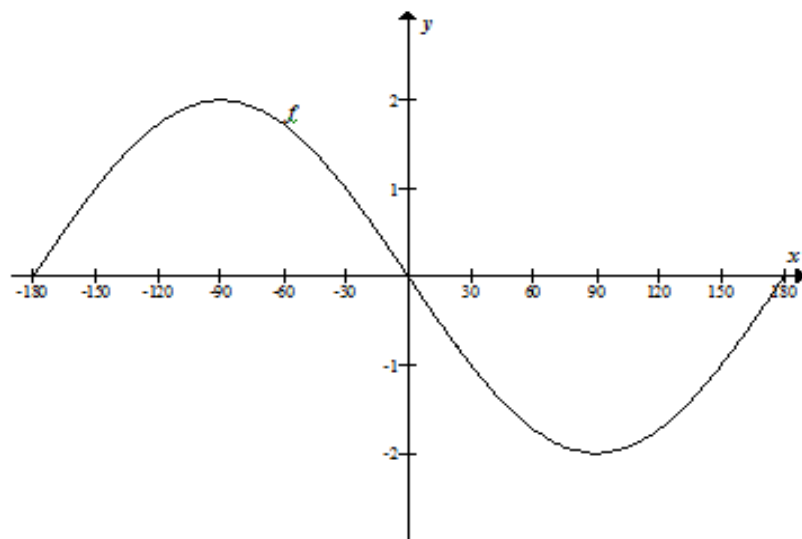
Given: $f(x) = 1 + \sin x$ and $g(x) = \cos 2x$

- 11.1 Calculate the points of intersection of the graphs f and g for $x \in [180^\circ; 360^\circ]$. (7)
- 11.2 Draw sketch graphs of f and g for $x \in [180^\circ; 360^\circ]$ on the same system of axes provided on DIAGRAM SHEET 3. (4)

NOTES

QUESTION 12

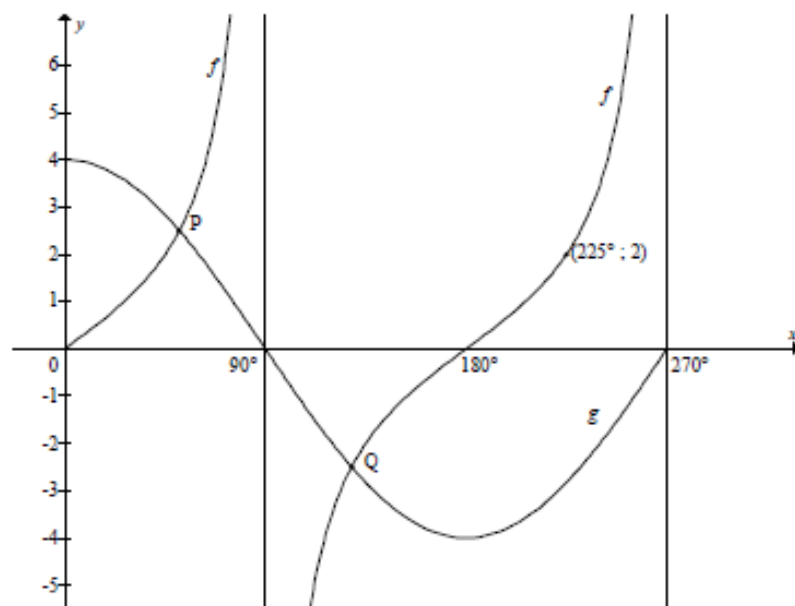
The graph of $f(x) = -2 \sin x$ is drawn below.



- 12.1 Write down the period of f . (1)
- 12.2 Write down the amplitude of h if $h(x) = \frac{f(x)}{4}$. (2)
- 12.3 Draw the graph of $g(x) = \cos(x - 30^\circ)$ for $x \in [-180^\circ; 180^\circ]$ on the grid provided on DIAGRAM SHEET 5. (3)
- 12.4 Use the graph to determine the number of solutions for $-2 \sin x = \cos(x - 30^\circ)$, $x \in [-180^\circ; 180^\circ]$. (1)

QUESTION 10

The graphs of the functions $f(x) = a \tan x$ and $g(x) = b \cos x$ for $0^\circ \leq x \leq 270^\circ$ are shown in the diagram below. The point $(225^\circ; 2)$ lies on f . The graphs intersect at points P and Q.

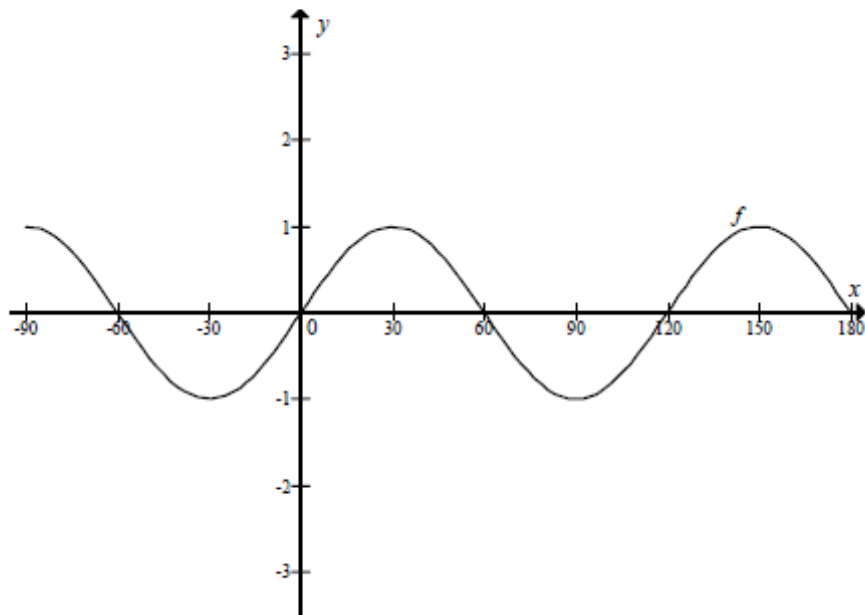


- 10.1 Determine the numerical values of a and b . (4)
- 10.2 Determine the minimum value of $g(x) + 2$. (2)
- 10.3 Determine the period of $f\left(\frac{1}{2}x\right)$. (2)
- 10.4 Show that, if the x -coordinate of P is θ , then the x -coordinate of Q is $(180^\circ - \theta)$. (4)

[12]

QUESTION 10

The graph of $f(x) = \sin 3x$ is drawn below for $x \in [-90^\circ; 180^\circ]$.

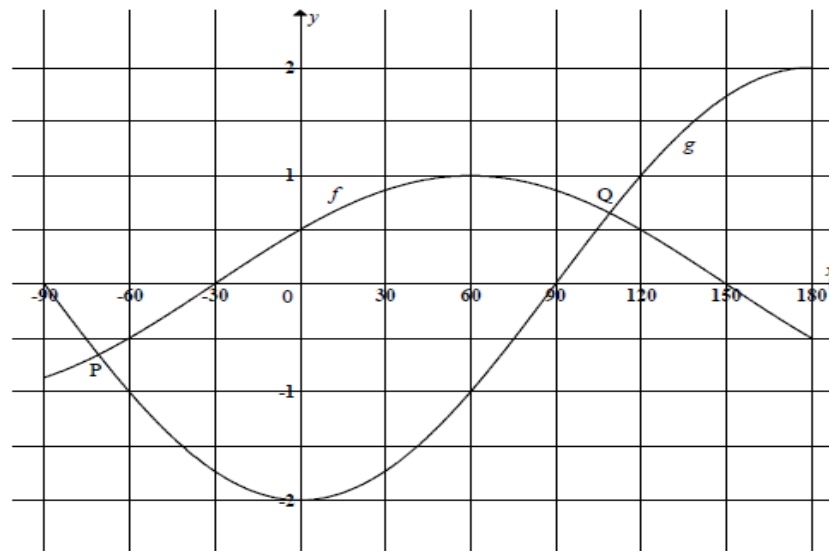


- 10.1 Write down the period of f . (1)
- 10.2 Write down the solutions for $\sin 3x = -1$ on the interval $[-90^\circ; 180^\circ]$. (2)
- 10.3 Give the maximum value of h if $h(x) = f(x) - 1$. (2)
- 10.4 Draw the graph of $g(x) = 3 \cos x$ for $x \in [-90^\circ; 180^\circ]$ on the grid on DIAGRAM SHEET 4. (3)

NOTES

QUESTION 10

The graphs of $f(x) = \sin(x + 30^\circ)$ and $g(x) = -2 \cos x$ for $-90^\circ \leq x \leq 180^\circ$ are given below. The graphs intersect at point P and point Q.

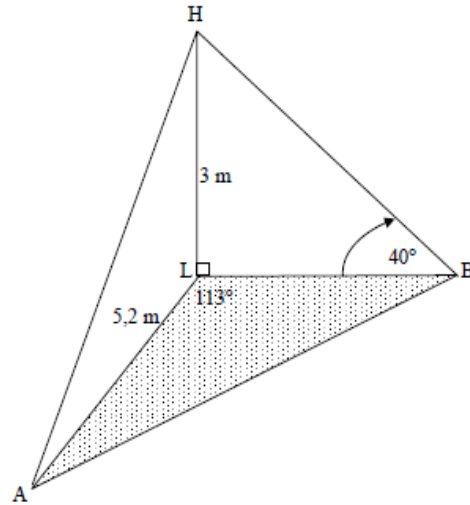


- 10.1 Calculate $f(0) - g(0)$. (1)
- 10.2 Calculate the x -coordinates of point P and point Q. (7)

NOTES

QUESTION 7

A, B and L are points in the same horizontal plane, HL is a vertical pole of length 3 metres, $AL = 5,2$ m, the angle $\hat{A}LB = 113^\circ$ and the angle of elevation of H from B is 40° .

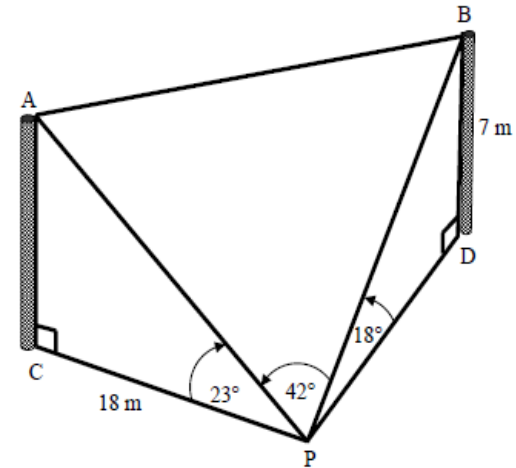


- 7.1 Calculate the length of LB. (2)
- 7.2 Hence, or otherwise, calculate the length of AB. (4)
- 7.3 Determine the area of $\triangle ABL$. (4)
- [10]

NOTES

QUESTION 7

Thandi is standing at point P on the horizontal ground and observes two poles, AC and BD, of different heights. P, C and D are in the same horizontal plane. From P the angles of inclination to the top of the poles A and B are 23° and 18° respectively. Thandi is 18 m from the base of pole AC. The height of pole BD is 7 m.



Calculate, correct to TWO decimal places:

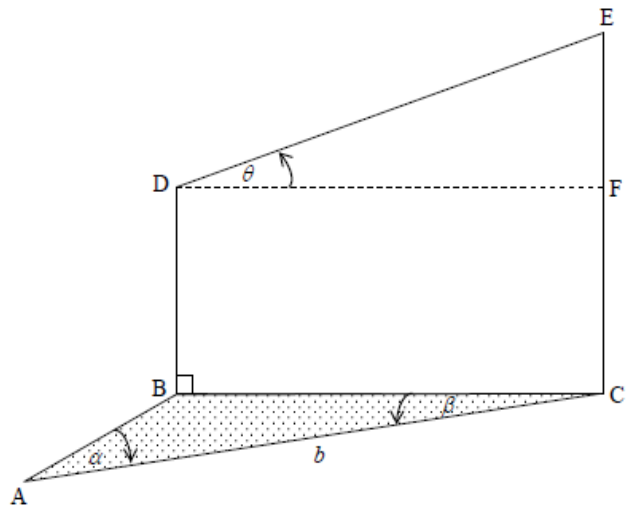
- 7.1 The distance from Thandi to the top of pole BD (2)
- 7.2 The distance from Thandi to the top of pole AC (2)
- 7.3 The distance between the tops of the poles, that is the length of AB, if $\hat{A}PB = 42^\circ$ (4)
- [8]

NOTES

QUESTION 12

In the diagram below A, B and C are three points in the same horizontal plane. D is vertically above B and E is vertically above C. The angle of elevation of E from D is θ° . F is a point on EC such that $DF \parallel BC$.

$\hat{BAC} = \alpha$, $\hat{ACB} = \beta$ and $AC = b$ metres.

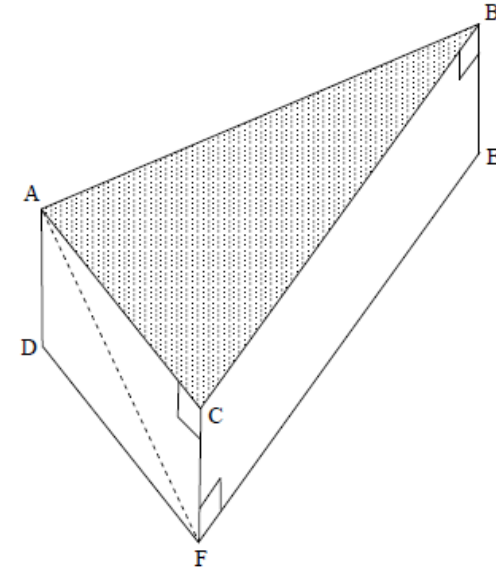


12.1 Prove that $DE = \frac{b \sin \alpha}{\sin(\alpha + \beta) \cos \theta}$ (6)

12.2 Calculate DE if $b = 2\,000$ metres, $\alpha = 43^\circ$, $\beta = 36^\circ$ and $\theta = 27^\circ$. (3)
[9]

QUESTION 11

The figure below represents a triangular right prism with $BA = BC = 5$ units, $\hat{ABC} = 50^\circ$ and $\hat{FAC} = 25^\circ$.



11.1 Determine the area of $\triangle ABC$. (2)

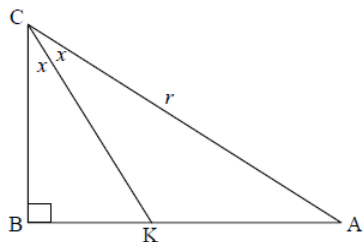
11.2 Calculate the length of AC. (3)

11.3 Hence, determine the height FC of the prism. (3)

[8]

QUESTION 11

In the diagram below, ABC is a right-angled triangle. KC is the bisector of \hat{ACB} .
 $AC = r$ units and $\hat{BCK} = x$.

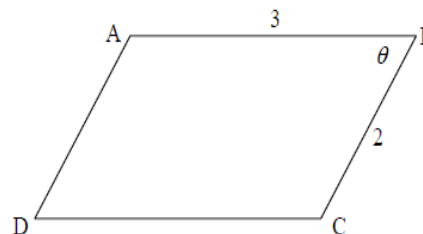


- 11.1 Write down AB in terms of x and r . (2)
- 11.2 Give the size of \hat{AKC} in terms of x . (1)
- 11.3 If it is given that $\frac{AK}{AB} = \frac{2}{3}$, calculate the value of x . (8)
- [11]

NOTES

QUESTION 11

$ABCD$ is a parallelogram with $AB = 3$ units, $BC = 2$ units and $\hat{ABC} = \theta$ for $0^\circ < \theta \leq 90^\circ$.



- 11.1 Prove that the area of parallelogram $ABCD$ is $6 \sin \theta$. (3)
- 11.2 Calculate the value of θ for which the area of the parallelogram is $3\sqrt{3}$ square units. (3)

NOTES

D. EUCLIDEAN GEOMETRY

NOVEMBER 2008

QUESTION 7

7.1 Complete the statements below by filling in the missing word(s) so that the statements are CORRECT:

7.1.1 The angle subtended by a chord at the centre of a circle is (1)

7.1.2 The angle between the tangent and a chord is (1)

7.1.3 The opposite angles of a cyclic quadrilateral are (1)

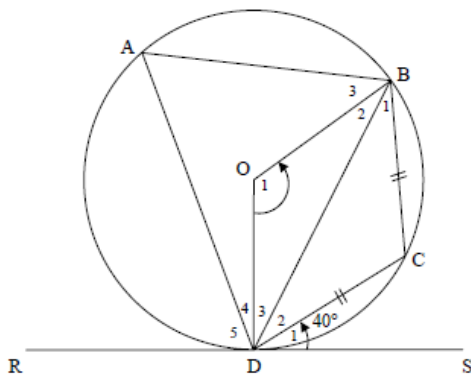
7.2 In the figure below, RDS is a tangent to circle O at D. If $BC = DC$ and $\hat{CDS} = 40^\circ$, calculate, with reasons, the measures of:

7.2.1 \hat{BDC} (2)

7.2.2 \hat{C} (1)

7.2.3 \hat{A} (1)

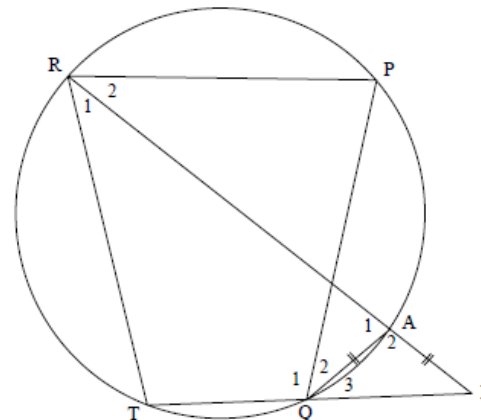
7.2.4 \hat{O}_1 (2)



[9]

QUESTION 8

In the diagram below, points R, P, A, Q and T lie on a circle. RA bisects \hat{R} and $AB = AQ$. RA and TQ produced meet at B.



Prove that:

8.1 AQ bisects \hat{PQB} (3)

8.2 $TR = TB$ (2)

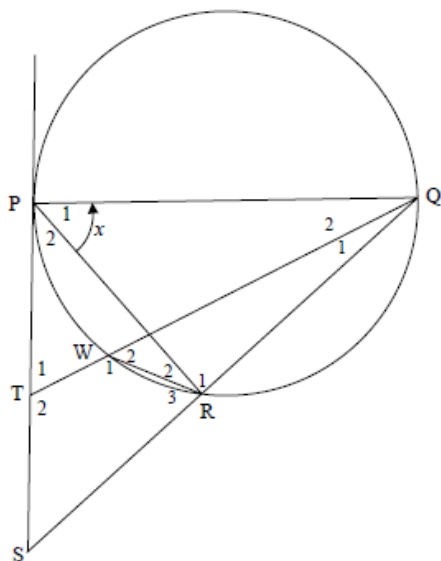
8.3 $\hat{P} = \hat{TRP}$ (3)

[8]

NOTES

QUESTION 9

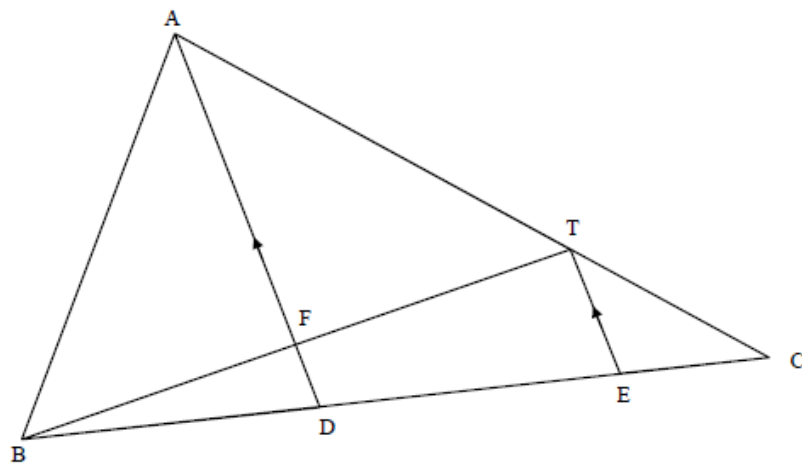
In the figure below, PQ is a diameter to circle PWRQ. SP is a tangent to the circle at P.
Let $\hat{P}_1 = x$



- 9.1 Why is $\hat{PRQ} = 90^\circ$? (1)
 - 9.2 Prove that $\hat{P}_1 = \hat{S}$. (3)
 - 9.3 Prove that SRWT is a cyclic quadrilateral. (3)
 - 9.4 Prove that $\triangle QWR \parallel \triangle QST$. (3)
 - 9.5 If $QW = 5$ cm, $TW = 3$ cm, $QR = 4$ cm and $WR = 2$ cm, calculate the length of:
 - 9.5.1 TS (3)
 - 9.5.2 SR (3)
- [16]

QUESTION 10

In the figure below, $\triangle ABC$ has D and E on BC. $BD = 6$ cm and $DC = 9$ cm.
 $AT : TC = 2 : 1$ and $AD \parallel TE$.



- 10.1 Write down the numerical value of $\frac{CE}{ED}$
- 10.2 Show that D is the midpoint of BE.
- 10.3 If $FD = 2$ cm, calculate the length of TE.
- 10.4 Calculate the numerical value of:
 - 10.4.1 $\frac{\text{Area of } \triangle ADC}{\text{Area of } \triangle ABD}$
 - 10.4.2 $\frac{\text{Area of } \triangle TEC}{\text{Area of } \triangle ABC}$

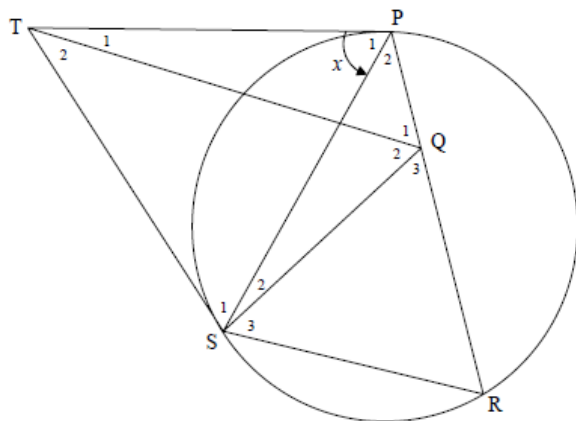
QUESTION 7

In the figure TP and TS are tangents to the given circle. R is a point on the circumference.

Q is a point on PR such that $\hat{Q}_1 = \hat{P}_1$.

SQ is drawn.

Let $\hat{P}_1 = x$.

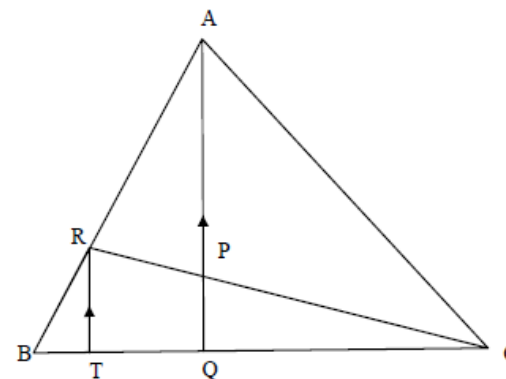


Prove that:

- 7.1 TQ \parallel SR (4)
 - 7.2 QPTS is a cyclic quadrilateral (4)
 - 7.3 TQ bisects \hat{SQP} (3)
- [11]

QUESTION 8

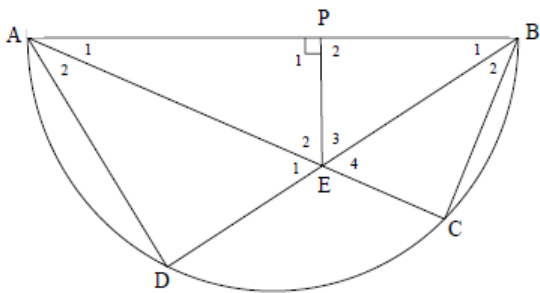
In the figure $AQ \parallel RT$, $\frac{BQ}{QC} = \frac{3}{5}$ and $\frac{BR}{RA} = \frac{1}{2}$.



- 8.1 If $BT = k$, calculate TQ in terms of k . (3)
 - 8.2 Hence, or otherwise, calculate the numerical value of:
 - 8.2.1 $\frac{CP}{PR}$ (3)
 - 8.2.2 $\frac{\text{Area } \Delta RCT}{\text{Area } \Delta ABC}$ (4)
- [10]

QUESTION 9

In the accompanying figure, AB is the diameter of circle ADCB. Chords AC and BD intersect at E. EP is perpendicular to AB.



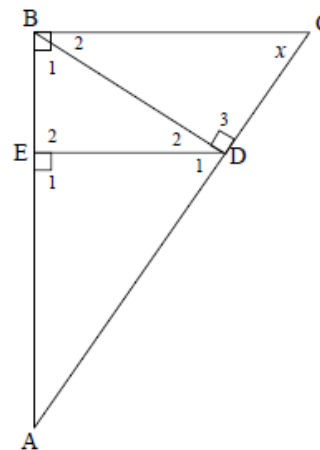
- 9.1 Prove that $\triangle BPE \parallel \triangle BDA$.
- 9.2 Hence show that $\frac{BP}{BD} = \frac{PE}{AD}$.
- 9.3 Prove that $AB^2 = BD^2 + \frac{BD^2 \cdot PE^2}{BP^2}$.

(3)
(2)
(5)
[10]

QUESTION 11

$\triangle ABC$ is a right-angled triangle with $\hat{B} = 90^\circ$. D is a point on AC such that $BD \perp AC$ and E is a point on AB such that $DE \perp AB$. E and D are joined.

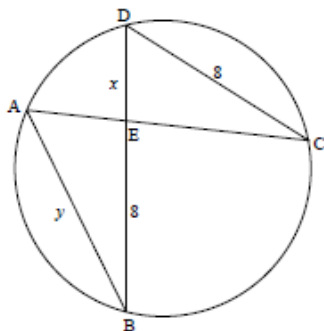
AD : DC = 3 : 2.
AD = 15 cm.



- 11.1 Prove that $\triangle BDA \parallel \triangle CDB$.
- 11.2 Calculate BD (Leave your answer in surd form).
- 11.3 Calculate AE (Leave your answer in surd form).

QUESTION 9

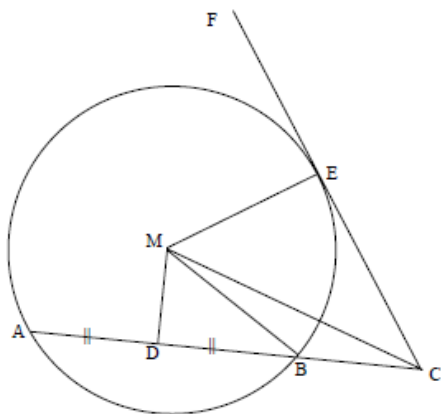
In the diagram below A, B, C and D are points on the circumference of the circle. BD and AC intersect at E. Also, EB = 8 cm, DC = 8 cm and AE : EC = 4 : 7.



If DE = x units and AB = y units, calculate x and y .

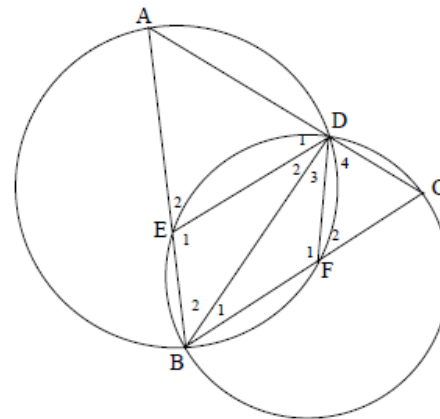
QUESTION 10

In the diagram below M is the centre of the circle. FEC is a tangent to the circle at E. D is the midpoint of AB.



- 10.1 Prove MDCE is a cyclic quadrilateral. (3)
 - 10.2 Prove that $MC^2 = MB^2 + DC^2 - DB^2$. (3)
 - 10.3 Calculate CE if AB = 60 mm, ME = 40 mm and BC = 20 mm. (4)
- [10]

- 9.2 In the diagram below two circles intersect one another at D and B. AB is a straight line such that it intersects the circle BCD at point E. BC is a straight line such that it intersects the circle ABD at F. DE, DB and DF are joined.
- $\hat{F}_2 = 180^\circ - 2x$
 $FC = FD$

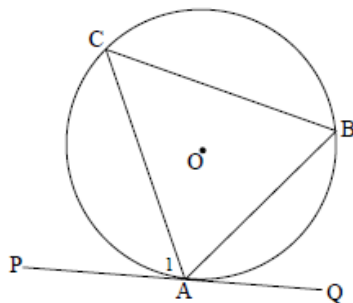


- 9.2.1 Calculate, with reasons, in terms of x :
 - (a) $\hat{D}EB$ (3)
 - (b) \hat{A} (2)
 - 9.2.2 Hence, or otherwise, prove $ED \parallel BC$. (3)
- [13]

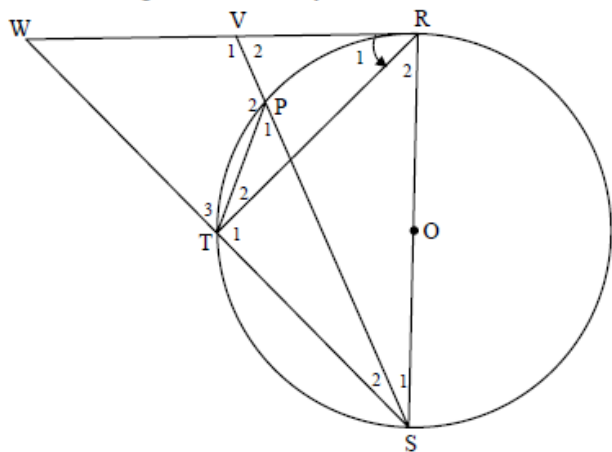
QUESTION 8

8.1 In the diagram below, O is the centre of the circle. PQ is a tangent to the circle at A. B and C are points on the circumference of the circle. AB, AC and BC are joined.

Prove the theorem that states $\hat{C}AP = \hat{A}BC$.



8.2 RS is a diameter of the circle with centre O. Chord ST is produced to W. Chord SP produced meets the tangent RW at V. $\hat{R}_1 = 50^\circ$.



Calculate the size of:

8.2.1 \hat{WRS}

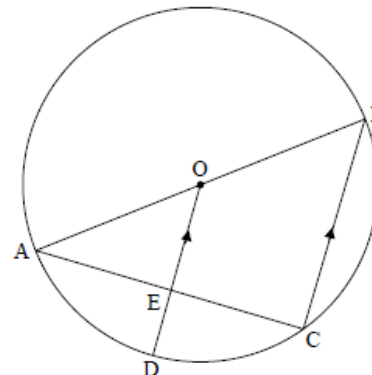
8.2.2 \hat{W}

8.2.3 \hat{P}_1

8.2.4 Prove that $\hat{V}_1 = \hat{PTS}$.

QUESTION 9

AB is a diameter of the circle ABCD. OD is drawn parallel to BC and meets AC in E.



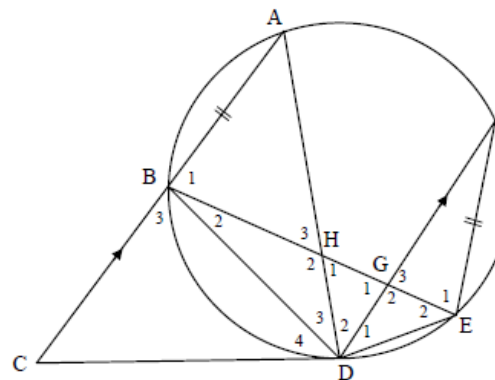
(5)

If the radius is 10 cm and $AC = 16$ cm, calculate the length of ED.

[5]

QUESTION 10

CD is a tangent to circle ABDEF at D. Chord AB is produced to C. Chord BE cuts chord AD in H and chord FD in G. $AC \parallel FD$ and $FE = AB$. Let $\hat{D}_4 = x$ and $\hat{D}_1 = y$.



10.1 Determine THREE other angles that are each equal to x .

(6)

(1) 10.2 Prove that $\triangle BHD \parallel \triangle FED$.

(5)

(2) 10.3 Hence, or otherwise, prove that $AB \cdot BD = FD \cdot BH$.

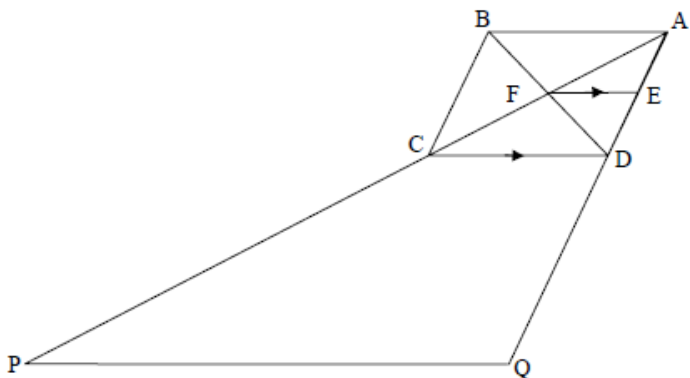
(2)

[13]

(4)
[15]

QUESTION 11

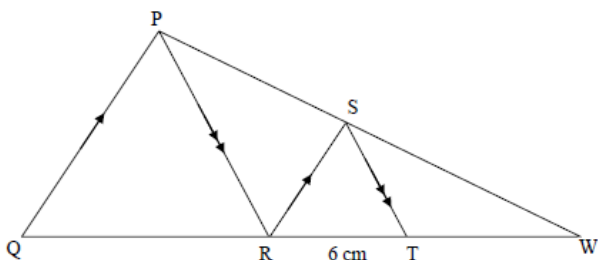
ABCD is a parallelogram with diagonals intersecting at F. FE is drawn parallel to CD. AC is produced to P such that $PC = 2AC$ and AD is produced to Q such that $DQ = 2AD$.



- 11.1 Show that E is the midpoint of AD.
- 11.2 Prove $PQ \parallel FE$.
- 11.3 If PQ is 60 cm, calculate the length of FE.

QUESTION 10

In ΔPQW , S is a point on PW and R is a point on QW such that $SR \parallel PQ$. T is a point on QW such that $ST \parallel PR$. $RT = 6$ cm. $WS : SP = 3 : 2$

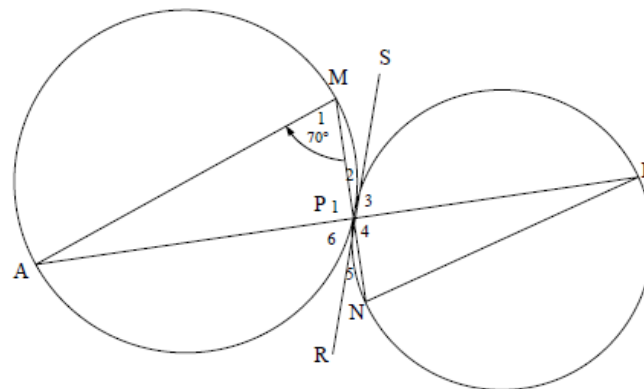


Calculate:

- 10.1 WT (3)
 - 10.2 WQ (4)
- [7]

QUESTION 8

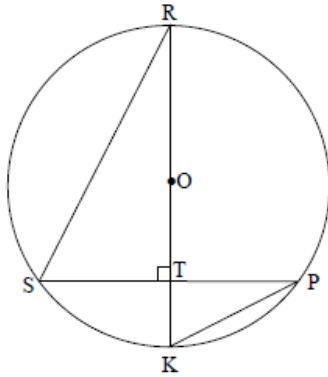
In the diagram below, AM is the diameter of the bigger circle AMP. RPS is a common tangent to both circles at P. APB and MPN are straight lines.



- 8.1 State the size of \hat{P}_1 . (1)
 - (2)
 - 8.2 Hence, show that BN is the diameter of the smaller circle. (2)
 - (3)
 - 8.3 If $\hat{M}_1 = 70^\circ$, calculate the size of each of the following angles: (5)
 - 8.3.1 \hat{A} (1)
 - 8.3.2 \hat{P}_6 (1)
 - 8.3.3 \hat{B} (2)
- [7]

QUESTION 9

In the diagram below, O is the centre of the circle with diameter RK .
 $PS \perp RK$.
 RK intersects PS at T .

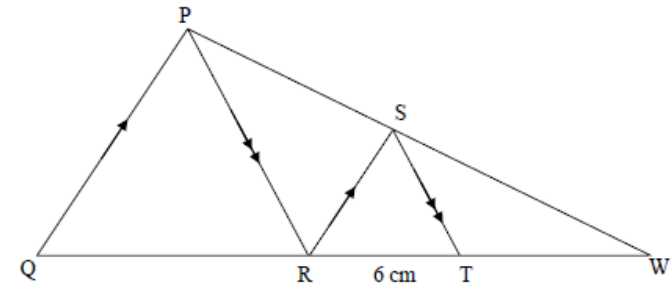


- 9.1 If $PS = 4x$, write down the length of ST in terms of x .
- 9.2 Prove that $\triangle RST \parallel \triangle PKT$.
- 9.3 If it is further given that $TK = x$ and $RT = 320$ mm, calculate the value of x .

(1)
 (3)
 (3)
 [7]

QUESTION 10

In $\triangle PQW$, S is a point on PW and R is a point on QW such that $SR \parallel PQ$.
 T is a point on QW such that $ST \parallel PR$.
 $RT = 6$ cm
 $WS : SP = 3 : 2$



Calculate:

- 10.1 WT
- 10.2 WQ

(3)
 (4)
 [7]

QUESTION 8

8.1 Complete the following statement:

The angle between the tangent and the chord is equal ...

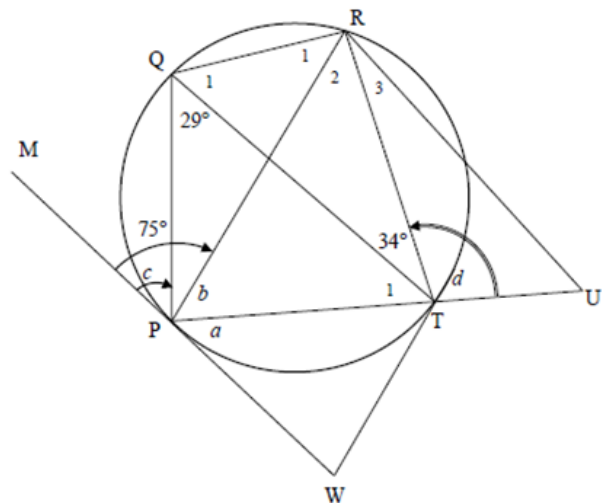
8.2 In the diagram points P, Q, R and T lie on the circumference of a circle. MW and TW are tangents to the circle at P and T respectively. PT is produced to meet RU at U.

$$\hat{MPR} = 75^\circ$$

$$\hat{PQT} = 29^\circ$$

$$\hat{QTR} = 34^\circ$$

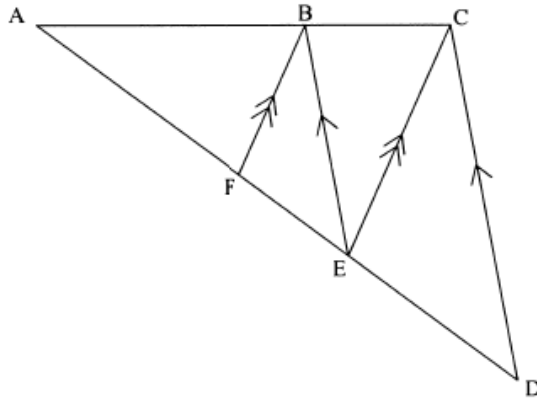
Let $\hat{TPW} = a$, $\hat{RPT} = b$, $\hat{MPQ} = c$ and $\hat{RTU} = d$, calculate the values of a , b , c and d .



(9)
[10]

QUESTION 9

In $\triangle ADC$, E is a point on AD and B is a point on AC such that $EB \parallel DC$. F is a point on AD such that $FB \parallel EC$. It is also given that $AB = 2BC$.



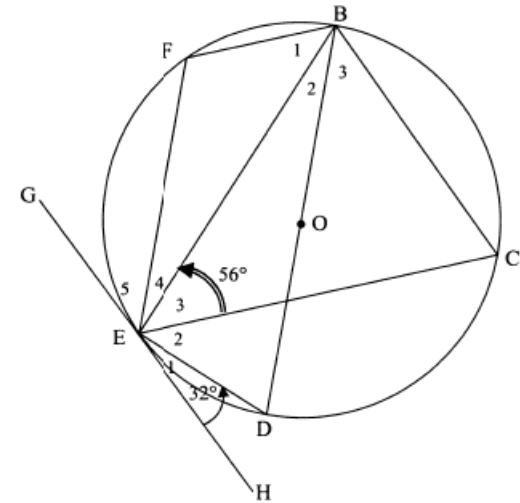
- 9.1 Determine the value of $AF : FE$
- 9.2 Calculate the length of ED if $AF = 8$ cm.

(2)
(4)
[6]

QUESTION 10

In the diagram below, O is the centre of the circle. BD is a diameter of the circle. GEH is a tangent to the circle at E. F and C are two points on the circle and FB, FE, BC, CE and BE are drawn.

$\hat{E}_1 = 32^\circ$ and $\hat{E}_3 = 56^\circ$.



Calculate, with reasons, the values of:

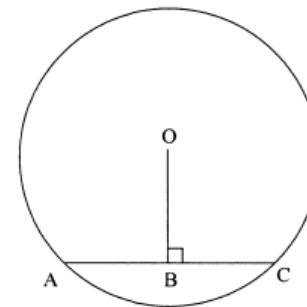
- 10.1 \hat{E}_2
- 10.2 $\hat{E}BC$
- 10.3 \hat{F}

(4)
[6]

QUESTION 11

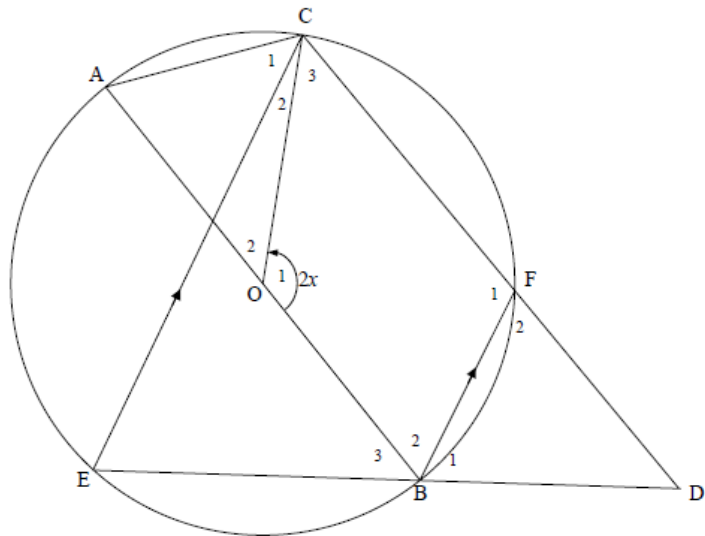
In the diagram below, O is the centre of the circle and OB is perpendicular to the chord AC.

Prove, using Euclidean geometry methods, the theorem that states $AB = BC$.



[5]

- 11.2 In the diagram below, O is the centre of the circle. AB is a diameter of the circle. Chord CF produced meets chord EB produced at D . Chord EC is parallel to chord BF . CO and AC are joined. Let $\hat{O}_1 = 2x$



- 11.2.1 Determine, in terms of x , the size of \hat{F}_1 . (4)
- 11.2.2 Prove that $DF = BD$. (4)
- 11.2.3 Show that $\hat{C}_1 = \hat{C}_3$. (4)
- 11.2.4 If $DF = 5$ cm and $OA = 6$ cm, calculate area ΔBFD : area ΔAOC . (4)
- [22]

QUESTION 12

In the diagram below, two circles intersect at K and Y . The larger circle passes through O , the centre of the smaller circle. T is a point on the smaller circle such that KT is a tangent to the larger circle. TY produced meets the larger circle at W . WO produced meets KT at E .

Let $\hat{W}_1 = x$

