Province of the
EASTERN CAPE
EDUCATION

CURRICULUM FET – MATHEMATICS

MATHEMATICS

REVISION MATERIAL – FROM 2019 ONWARDS

A COMPILATION OF 2019 TRIAL EXAMINATIONS FROM PROVINCES GROUPED ACCORDING TO TOPICS!

It does not matter what percentage you got in the past, you can always improve if you practice!

Practice makes perfect! Collect the marks!

YES YOU CAN!

NOTE: PLEASE USE THIS BOOKLET TOGETHER WITH OTHER REVISION MATERIAL LIKE BOOKLET 1 OF 2017, BOOKLET 2 OF 2018, PREVIOUS YEARS QUESTION PAPERS' REVISION MATERIAL (WITH QUESTION PAPERS FROM 2015 – 2019 JUNE)

TO BE USED FROM OCTOBER 2019 ONWARDS!

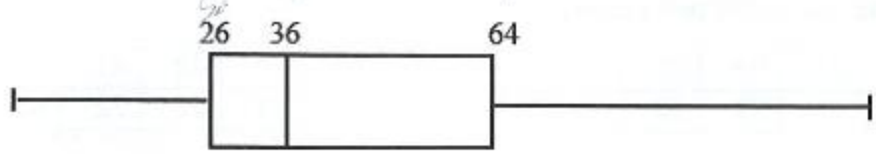
BOOKLET 2 OF 2019

STATISTICS

PREP 2019_LIMPOPO

QUESTION 1

Some of the test results of 21 learners are given below. There was only one result of 26 marks and only one result of 64 marks.



1.1 What information is omitted on the diagram above? (1)

1.2 The results were read to the learners in ascending order. If the 5th learner's result was 26, which learner obtained a result of 64? (2)

1.3 One of the learners was arguing that the distribution of the data wasn't symmetrical. Is the learner correct? Give a reason for the learner's remark. (3)

1.4 The class calculated the following, using the test results:

$\bar{x} = 45,5$ and $\delta\bar{x} = 19,2$.

Use the above information and determine the number of learners whose results fall outside ONE standard deviation of the mean. (3)

1.5 If the marks of each learner would increase by 5 marks, what effect would it have on the mean and standard deviation? (2)

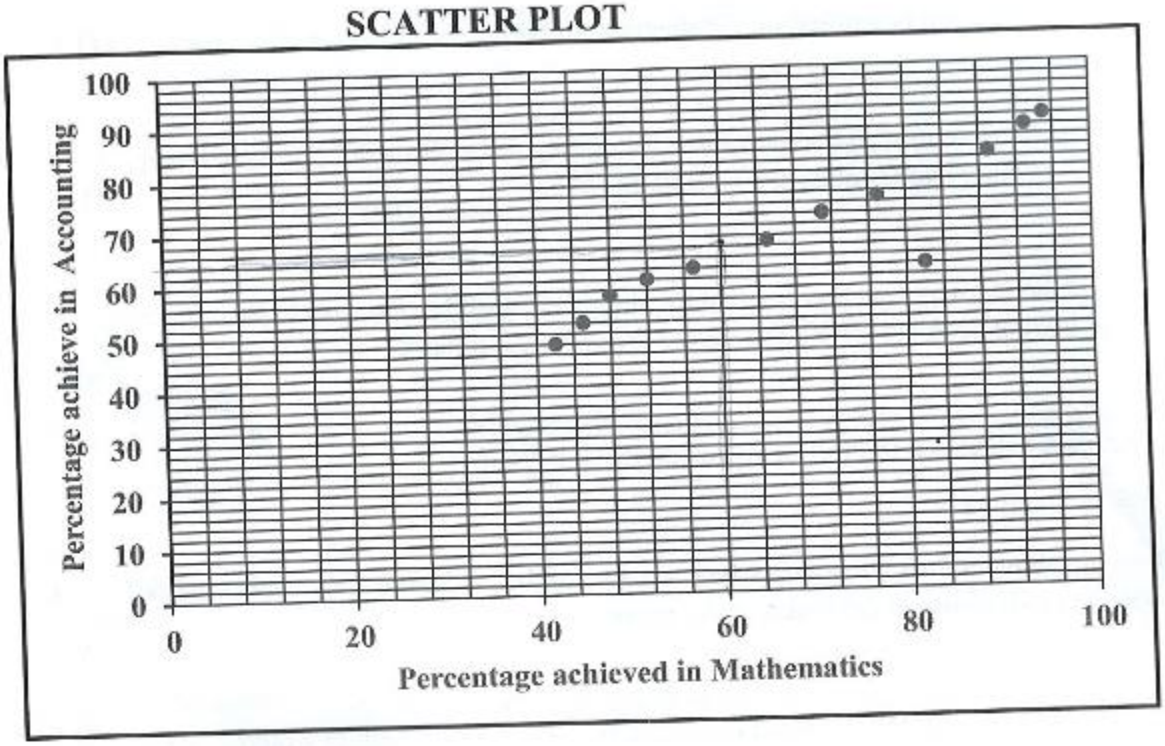
[11]

QUESTION 2

At a certain High School only 12 candidates take Mathematics and Accounting. The marks as a percentage scored by these candidates in the Trial Examinations for these two subjects, are shown in the table and scatter plot below.

Mathematics	52	82	93	95	71	65	77	42	89	48	45	57
Accounting	60	62	88	90	72	67	75	48	83	57	52	62

SCATTER PLOT



2.1 Use the scatter plot to comment on the relationship between the learner's performance in Mathematics and Accounting. Provide a reason for your answer. (2)

2.2 Calculate the value of the correlation coefficient. (1)

2.3 Calculate the equation for the least squares regression line for the data. (3)

2.4 If a candidate from this group scored 60% in the Mathematics examination, but was absent for the Accounting examination, predict the percentage he would have scored in the Accounting examination. (2)

2.5 Use the scatter plot and identify any outlier(s) in the data. (1)

[9]

PREP 2019_EASTERN CAPE

QUESTION 1

The table below shows the height, in metres, of 80 giraffes.

Height in m	Frequency
$4,6 \leq h < 4,8$	4
$4,8 \leq h < 5,0$	7
$5,0 \leq h < 5,2$	15
$5,2 \leq h < 5,4$	33
$5,4 \leq h < 5,6$	17
$5,6 \leq h < 5,8$	4

[Image from *Stuff You Should Know*]

1.1 Calculate the estimated mean height of the giraffes. (2)

1.2 The height range of $4,8 \leq h < 5,0$ m, consisted entirely of 7 young male giraffes. The mean of the heights of the 7 young males was calculated to be 4,86 m. However, it was noticed that the height of one of the young males was incorrectly recorded as 4,98 m but should have been 4,89 m. Calculate the new mean of the 7 young males. (3)

1.3 Would you expect the standard deviation of the individual heights of the 80 giraffes to be more or less than the standard deviation of the individual heights of the 7 young male giraffes? Explain your answer. (2)

[7]

QUESTION 2

Cocoa solids are used to make chocolate.
A student is investigating the relationship between the percentage of cocoa solids in a 100 g slab of chocolate and the price of the slab (in rands). The data obtained is shown in the table below.

Chocolate brand	A	B	C	D	E	F	G	H
x (% of cocoa)	10	20	30	35	40	50	60	70
y (price in rand)	6,50	10,20	9,40	24,00	11,20	16,80	20,50	24,20

2.1 Use the grid in the ANSWER BOOK to represent this data in a scatter plot. (2)

2.2 Determine the equation of the least squares regression line for this data. (3)

2.3 Draw the least squares regression line in the ANSWER BOOK on the same grid used in QUESTION 2.1. (2)

2.4 Calculate the correlation coefficient for the percentage of cocoa solids and the price of the chocolate slab. (1)

2.5 Comment on the relationship between the percentage of cocoa solids and the price of the chocolate slab. (1)

2.6 The student believes one brand of chocolate is overpriced.

2.6.1 Identify the chocolate brand which is over-priced. (1)

2.6.2 Estimate by how much more this slab of chocolate is overpriced. (3)

[13]

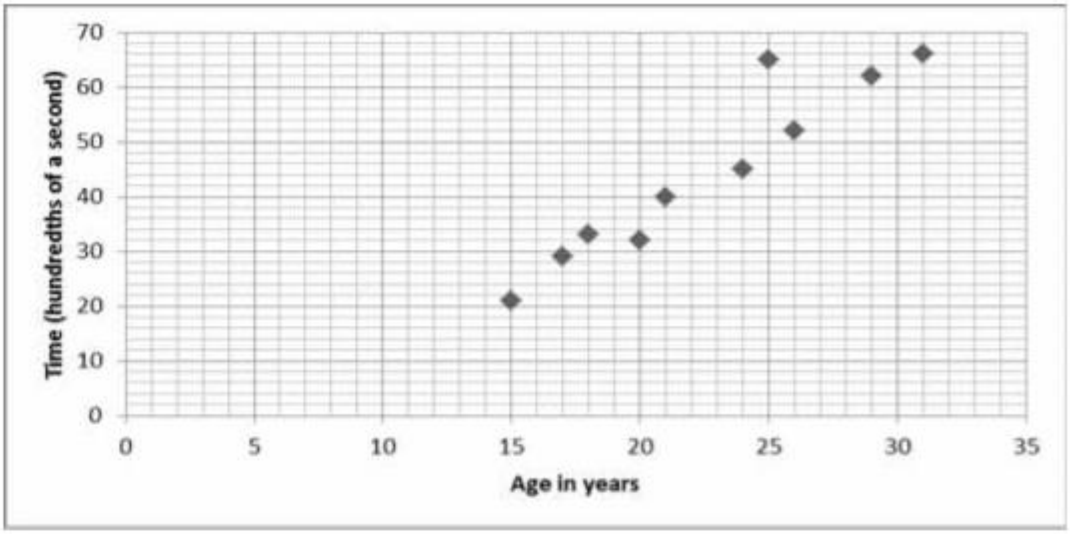
PREP 2019_GAUTENG

QUESTION 1

A hospital carries out a survey to compare the reaction time of patients of different ages, to a specific medication taken. The results are shown in the table and scatter plot below.

Age in years	15	17	18	20	21	24	25	26	29	31
Time (hundredths of a second)	21	29	33	32	40	45	65	52	62	66

Age in years	15	17	18	20	21	24	25	26	29	31
Time (hundredths of a second)	21	29	33	32	40	45	65	52	62	66



1.1 Draw the line of best fit on the scatter plot on DIAGRAM SHEET 1. (2)

1.2 One of these patients reaction time is an outlier.

1.2.1 How old is this patient? (1)

1.2.2 Explain why this patient is an outlier. (1)

1.3 Calculate the equation of the least squares regression line (line of best fit) for the data. (3)

1.4 Calculate the correlation coefficient of the data. Comment on the strength of the relationship between the variables. (2)

1.5 Hospital records for this reaction time test give the following information.

	15 year olds Time (hundredths of a second)	30 year olds Time (hundredths of a second)
Lower quartile	20	61
Median	22	65
Upper quartile	25	76

Comment on the reaction time of the different age groups on this test. Motivate your answer by referring to the values given. (2)

[11]

QUESTION 2

The ages of 500 people attending a concert are given in the table below.

Age in years	Number of people	Cumulative frequency
$0 \leq A < 10$	20	
$10 \leq A < 20$	130	
$20 \leq A < 30$	152	
$30 \leq A < 40$	92	
$40 \leq A < 60$	86	
$60 \leq A < 80$	18	
$80 \leq A < 100$	2	

2.1 Complete the cumulative frequency column on DIAGRAM SHEET 2. (1)

2.2 Draw an ogive (cumulative frequency graph) of the above data on DIAGRAM SHEET 2. (3)

2.3 Use your cumulative frequency graph to estimate:

2.3.1 the median age (1)

2.3.2 the percentage of people at the concert who are 16 years and older. (3)
[8]

PREP 2019_NORTH WEST

QUESTION 1

The time (in seconds)between the consecutive landings of aeroplanes at an airporton day 1 was recorded. The data is given in the Cumulative Frequency table below.

Time in seconds	**Number of aeroplanes (Frequency)**	**Cumulative Frequency**
$60 < t \leq 90$	2	2
$90 < t \leq 120$	16	18
$120 < t \leq 150$	28	46
$150 < t \leq 180$	17	63
$180 < t \leq 210$	k	p
$210 < t \leq 240$	7	80

1.1 Show that $k = 10$. (1)

1.2 Write down the value of p. (1)

1.3 Calculate the estimated mean time between the landings of two consecutive aeroplanes. (3)

1.4 It is given that (q ; 186,89) is the interval of the landing time between aeroplanes within ONE standard deviation from the estimated mean.

1.4.1 Write down the estimated standard deviation of the time between the consecutive landings of the aeroplanes. (2)

1.4.2 Calculate the value of q. (1)

1.5 On day 2, the same number of aeroplanes that landed on day 1, land at the airport.The elapsed time between all the consecutive landings of allthe aeroplanes is m seconds shorter than the time that is given in the table above.

If an ogive is to be drawn of the data of day 2, the following will be true:

- The ogive will be grounded at (57 ; 0)
- The maximum value of the ogive will be at (237 ; 80)

Determine the average time between the landing of two aeroplanes on DAY 2, if it is given that the frequency distribution of the two days are the same. (2)

[10]

QUESTION 2

The marks, in percentage, obtained in an Accounting and Mathematics test by a group of ten Grade 12 learners is shown in the table below.

Accounting Test	76	65	88	68	70	79	51	66	59	74
Mathematics Test	80	69	93	19	76	85	57	79	62	78

2.1 Identify an outlier of the above data. (1)

2.2 Determine the equation for the least squares regression line after ignoring the outlier in the above data. (3)

2.3 Another learner in the same class obtained 83% in the Accounting test, but due to illness could not write the Mathematics test. Use the equation established in 2.2 to predict the learner's mark for the Mathematics test. (2)

2.4 The teacher decided to award the learner who was absent the predicted mark obtained in 2.3 for the Mathematics test. Other learners in the class felt that it was unfair.

Motivate to these learners why the predicted mark is a good indication of what the learner may have scored in the Mathematics test. (2)

2.5 After the Mathematics subject advisor has moderated the answer books of the Mathematics tests, she decides to lower every test mark by p%. Explain, **without any calculations,** what influence the lowering in the marks of the Mathematics test has on the slope of the least squares regression line of the above data when the outlier is ignored. (2)

[10]

PREP 2019_FREE STATE

QUESTION 1

The following table shows the test marks (in %) of Grade 11 learners in Frances Baard High School.

INTERVAL OF TEST MARKS	NUMBER OF LEARNERS
$0 \le x < 20$	4
$20 \le x < 40$	5
$40 \le x < 60$	9
$60 \le x < 80$	13
$80 \le x < 100$	10
Totals	**41**

1.1 Write down the modal class. (1)

1.2 Calculate the estimated mean. (3)

1.3 Complete the cumulative frequency table provided in the ANSWER BOOK. (2)

1.4 Draw a cumulative frequency curve (ogive) to represent the data on the grid provided in the ANSWER BOOK. (3)

1.5 Use the cumulative frequency curve (ogive) to determine the interquartile range for the data. (3)

[12]

QUESTION 2

Research is done to determine if the number of hours reading over a certain period of time has an effect on the results of a candidate's mark in a general knowledge test (out of 120).

Number of hours	15	20	22	25	32	40	44	50	55	58
Mark out of 120	40	30	55	80	70	75	100	105	98	79

2.1 Determine the equation of the least squares regression line. (3)

2.2 Estimate the mark that a person that reads 36 hours in that same period will obtain in the test. (2)

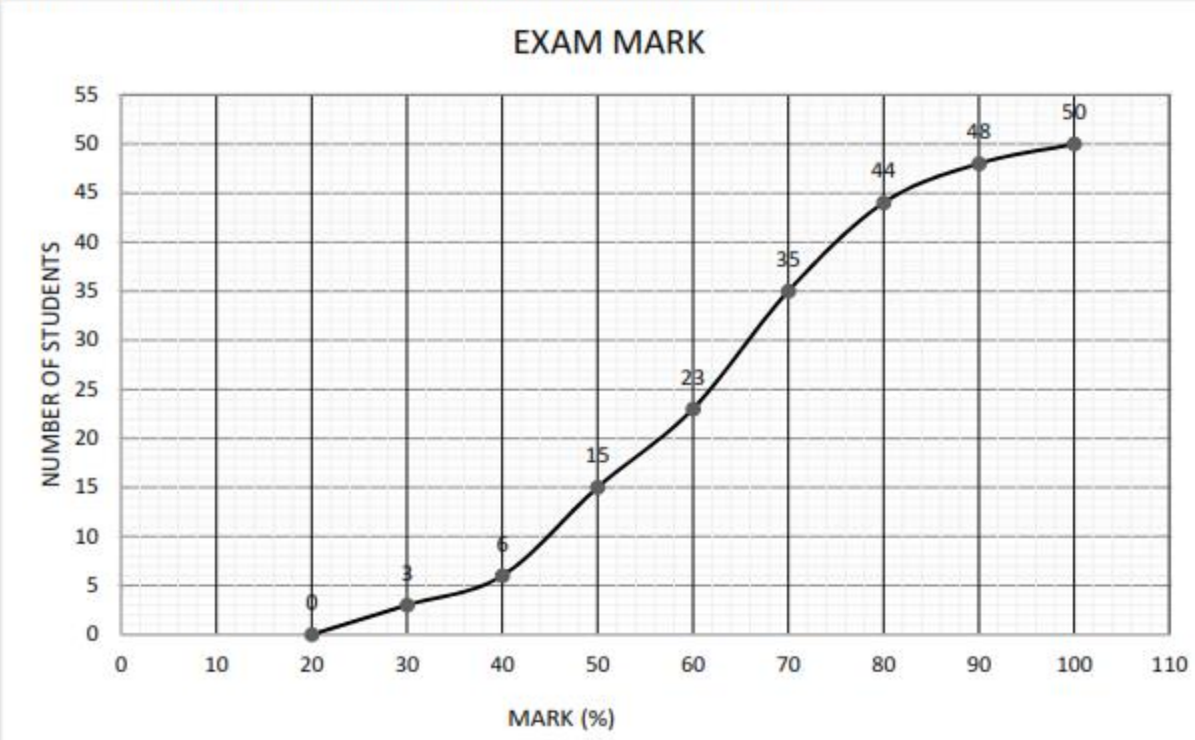
2.3 What is the correlation between the number of hours reading and the mark a person scores for the test? Motivate your answer. (3)

[8]

PREP 2019_WESTERN CAPE

QUESTION 1

1.1 Several learners' marks for a certain exam were requested in a survey. Marks from the exam are represented in the cumulative frequency curve (ogive) below:



1.1.1 How many learners participated in this survey? (1)

1.1.2 Complete the frequency column in the table given in the ANSWER BOOK.

Interval	Frequency	Cumulative frequency
$20 \leq x < 30$		3
$30 \leq x < 40$		6
$40 \leq x < 50$		15
$50 \leq x < 60$		23
$60 \leq x < 70$		35
$70 \leq x < 80$		44
$80 \leq x < 90$		48
$90 \leq x < 100$		50

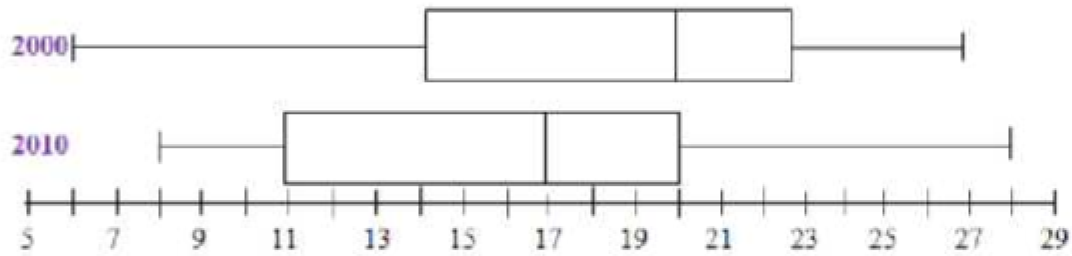
(2)

1.1.3 Use the ogive to determine the median mark. (2)

1.1.4 Write down the modal class. (1)

1.1.5 If a learner needs 70% to qualify for a reward, how many learners qualified? (2)

1.2 The results from a test for students for the year 2000 and the year 2010 are illustrated in the box and whisker plot. The total mark for the test was 30.



1.2.1 Determine the interquartile range for 2010. (2)

1.2.2 In which year did students perform better in the test? Motivate your answer. (2)

[12]

QUESTION 2

A group of 12 learners have been asked to measure their resting heart rate (beats per minute) and the time (in minutes) that they exercise in a week. The data below was gathered.

Minutes of exercise per week	Resting heart rate (BPM)
30	82
40	77
60	75
90	70
140	68
180	67
270	60
350	58
360	52
420	50
440	48
500	45

2.1 Represent the data as a scatter plot on the grid provided in the ANSWER BOOK. (3)

2.2 Calculate the correlation coefficient for the given data and describe the relationship between minutes of exercise per week and resting heart rate. (2)

2.3 Use the data to calculate the equation of the least squares regression line. (3)

2.4 If a learner has a resting heart rate of 65 beats per minute, how many hours would you expect him to exercise per week? (3)

[11]

PREP 2019_KZN

QUESTION 1

1. The following information represents the amount of maize exported to other countries over 11 years in 1000 tons.

39	42	48	54	62	68	78	78	82	91	93

1.1 Calculate the mean amount of maize exported over the 11 years. (2)

1.2 Calculate the standard deviation of the data. (2)

1.3 Calculate the number of years that are within one standard deviation of the mean. (2)

1.4 Draw a box and whisker diagram to represent the data. (4)

1.5 Comment on the skewness of the data. (1)

1.6 There was an error in the data. The mean amount of maize exported over the 11 years should increase by 1,25 thousands of tons. What impact will this error have on the:

1.6.1 yearly data provided in the above table? (1)

1.6.2 on the interquartile range of the given data above? (1)

[13]

QUESTION 2

The following information (in %) represent contributions made by the Agricultural and Mining industries in order to evaluate the GDP(GROSS DOMESTIC PRODUCT) of a certain country.

YEAR	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Agriculture (x)	4.2	3,4	3,1	2,7	2,9	3,0	2,9	3,0	2,6	2,5	2,6
Mining (y)	19,2	19,4	19,2	18,5	17,5	17,0	16,8	15,2	14,2	12,8	12,4

2.1 Determine the equation of the least squares regression line. (3)

2.2 Estimate the percentage that the mining industry will contribute if the agriculture industry dropped to 1,2 %. (2)

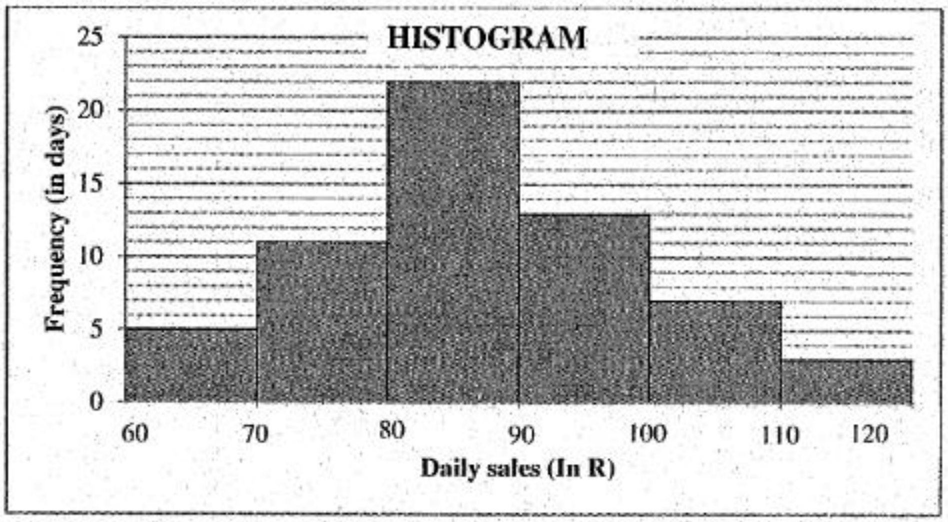
2.3 Comment on the strength of the correlation between the contributions made by these two industries. Motivate your answer. (2)

[7]

PREP 2019_MPUMALANGA

QUESTION 1

A street vendor has kept record of his daily sales for November and December 2017. The daily sales(in Rand) is shown in the histogram below.



1.1 A table summarising the daily sales for November and December 2017 is drawn in the answer book. Complete the cumulative frequency column in the table. (3)

1.2 Draw an ogive (cumulative frequency graph) of the daily sales for November and December 2017 on the grid provided IN THE ANSWER BOOK. (4)

1.3 On how many days did the daily sales exceed 100? (2)

1.4 Use the ogive IN THE ANSWER BOOK to estimate the median value of the daily sales. (2)

[11]

QUESTION 2

2.1 A learner conducted an experiment to investigate the relationship between the age and resting heart rate (in beats per minute). He sought the assistance of the local clinic. The information for 12 people is shown in the table below:

Age	59	32	42	50	22	39	21	20	27	40	29	47
Resting heart rate (beats per minute)	88	74	74	93	85	71	78	82	70	75	95	75

2.1.1 Determine the equation of the least squares regression line. (3)

2.1.2 Predict the resting heart rate of a person aged 57 years. (2)

2.1.3 Write down the correlation coefficient for the data. (1)

2.1.4 Use the correlation coefficient to comment on the prediction done in Question 2.1.2. (1)

2.2 A group of students wrote a statistics test. The following information was obtained from the marks obtained in this test:

$$\sum_{n=1}^{22} x_n = 1320$$

2.2.1 How many students wrote the test? (1)

2.2.2 Calculate the mean mark for the test. (2)

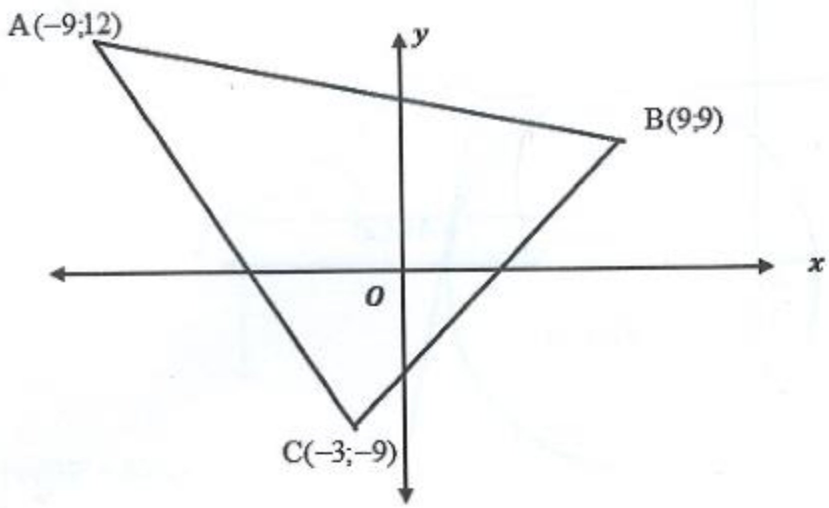
[10]

ANALYTICAL GEOMETRY

PREP 2019_LIMPOPO

QUESTION 3

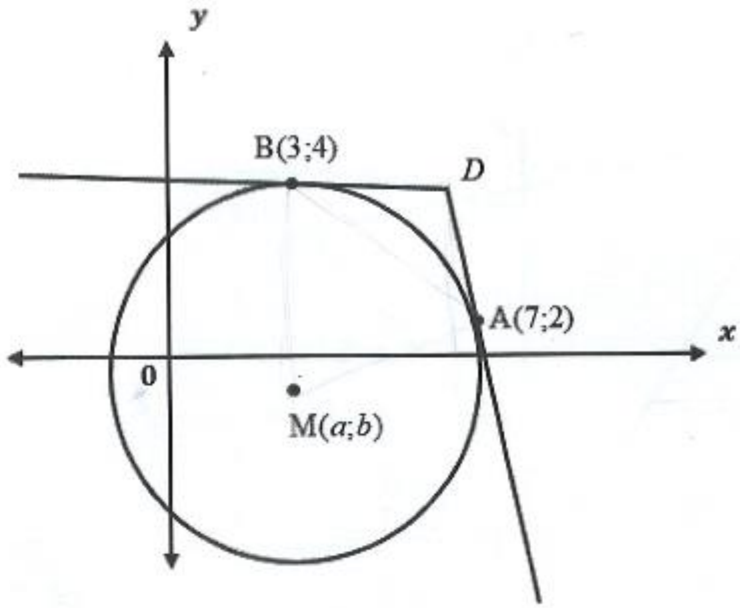
In the diagram $A(-9;12)$, $B(9;9)$ and $C(-3;-9)$ are the vertices of Δ ABC. K $(m;n)$ is a point in the second quadrant.



3.1	Calculate the gradient of AB.	(2)
3.2	Calculate the size of $\hat{B}$, rounded off to two decimal digits.	(5)
3.3	Determine the coordinates of M, the midpoint of BC.	(2)
3.4	Determine the equation of the median AM.	(3)
3.5	Determine the coordinates of K, if A, K and M are collinear and BK = $5\sqrt{5}$ units.	(8)
		[20]

QUESTION 4

In the diagram a circle $x^2 - 6x + y^2 + 2y = 15$ passes through the points A(7;2) and B(3;4).
AD and BD are tangents to the circle, with M the centre of the circle



4.1 Determine the:

4.1.1 Coordinates of the centre M, and the radius of the circle. (5)

4.1.2 Equation of tangent AD. (4)

4.1.3 Coordinates of D, if BD and AD meet at D. (4)

4.2 Identify the type of quadrilateral ADBM is, and provide a valid reason. (2)

4.3 Determine the area of ADBM. (4)

[19]

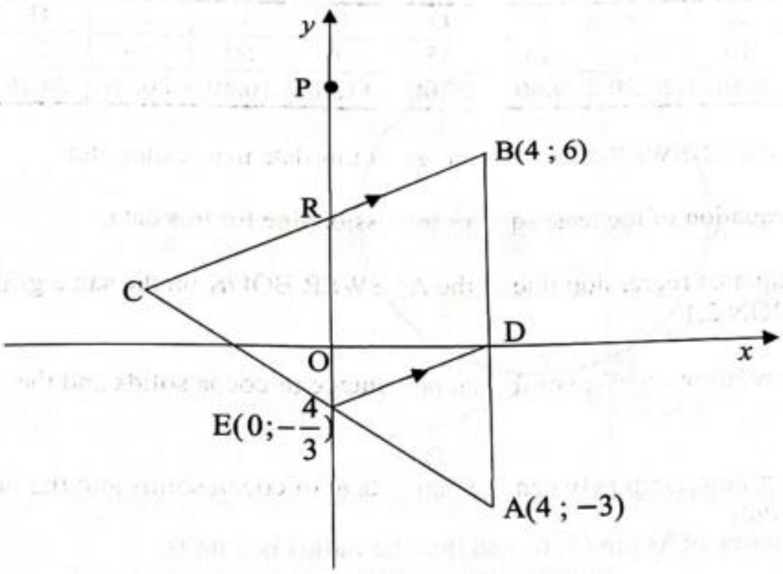
PREP 2019_EASTERN CAPE

QUESTION 3

In the diagram below, ΔABC is shown with coordinates of A $(4; -3)$ and B $(4 ; 6)$ given. AB intersects the x-axis at D and AC intersects the y-axis at E.

The coordinates of E are $(0;-\frac{4}{3})$. BC intersects the y-axis at R. P is a point on the y-axis.

DE ∥ BC.



3.1 Write down the coordinates of D. (1)

3.2 Determine the gradient of DE. (2)

3.3 Calculate $P\hat{R}B$. (4)

3.4 Calculate the length of DE in simplest surd form. (2)

3.5 Determine the ratio AD: AB in its simplest form. (2)

3.6 Hence, or otherwise, calculate the length of BC. (3)

3.7 Determine:

3.7.1 The midpoint of DE (2)

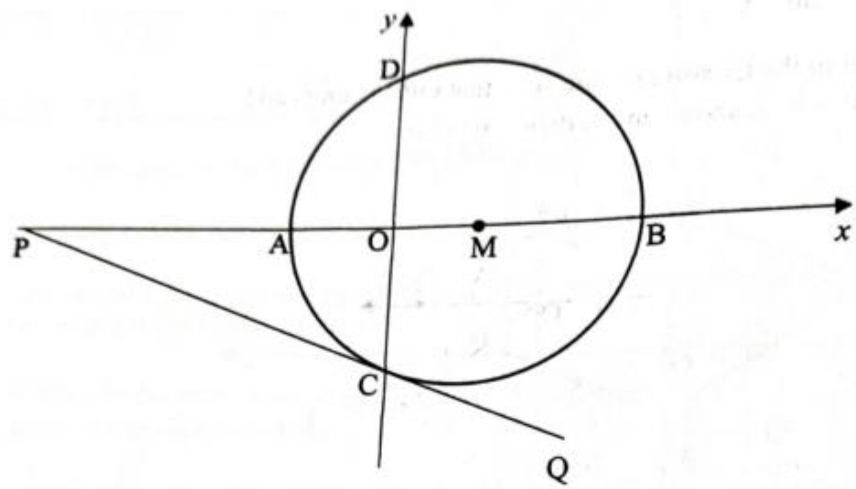
3.7.2 The equation of the perpendicular bisector of DE in the form $y = mx + c$ (3)

3.8 Does this perpendicular bisector pass through A? Justify your answer. (2)

[21]

QUESTION 4

The diagram below shows a circle having centre M which intersects the x-axis at A and B and the y-axis at D and C. PCQ is a tangent to the circle at C, the point of contact on the y-axis. P lies on the x-axis.
The equation of the circle is: $x^2 + y^2 - 6x - 16 = 0$



4.1 Show that the coordinates of M are (3; 0) and that the radius is 5 units. (3)

4.2 Determine:

4.2.1 The coordinates of B (3)

4.2.2 The coordinates of C (2)

4.3 If the length of PM is $8\frac{1}{3}$ units, calculate the length of PC. (3)

4.4 Calculate the angle subtended by the chord DC at B, i.e. find $D\hat{B}C$. (4)

4.5 If the given circle is moved 2 units right and 1 unit up, determine the equation of the tangent to the circle in its new position passing through point C'. (4)

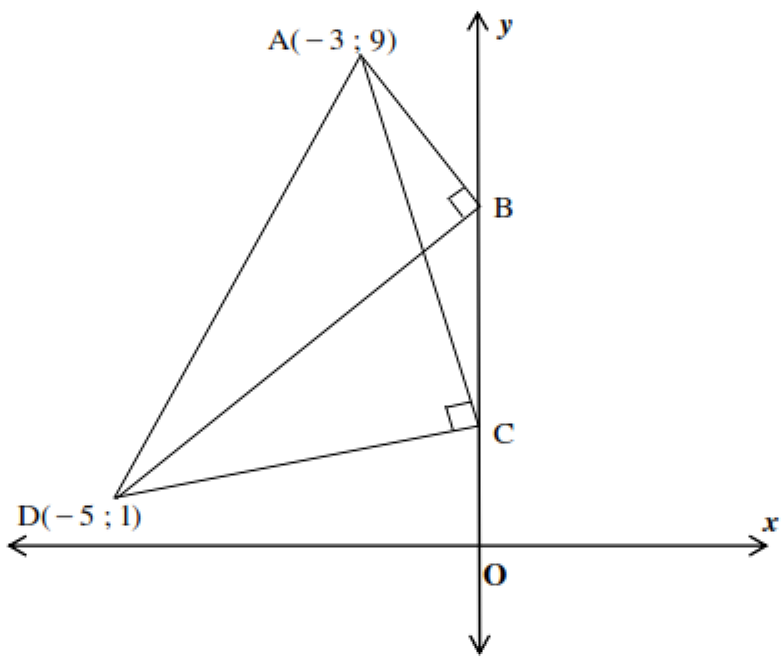
[19]

PREP 2019_GAUTENG

QUESTION 3

In the diagram sketched below, $A(-3\ ;\ 9)$ and $D(-5\ ;\ 1)$ are points on ΔABD and ΔACD.

B and C are points on the y-axis such that $A\hat{B}D = A\hat{C}D = 90^{\circ}$.



3.1	Calculate the coordinates of M, the midpoint of AD.	(2)
3.2	Calculate the length of the radius of the circle passing through A, B and D.	(2)
3.3	Will point C lie on circle ABD? Give a reason for your answer.	(2)
3.4	Calculate the coordinates of B.	(5)
3.5	Determine the equation of the straight line passing through D and which is parallel to AB.	(3)
3.6	Calculate the size of $B\hat{D}A$. Round off the answer to the nearest degree.	(6)
		[20]

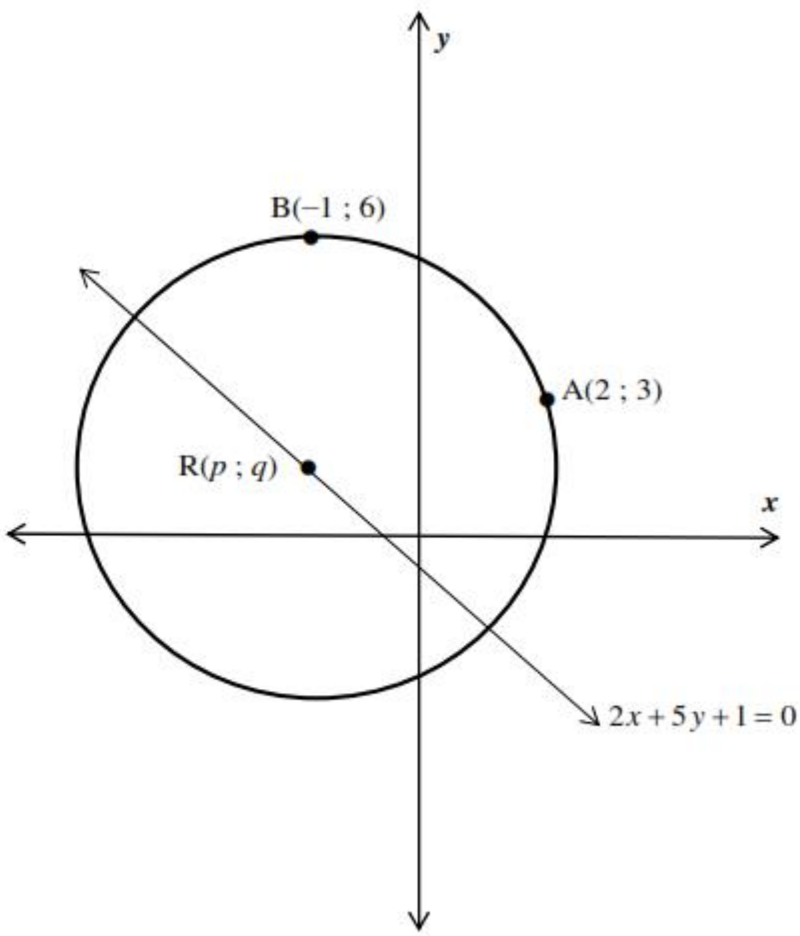
QUESTION 4

4.1 Given: $x^2 + y^2 - 2x + 6y = 0$.

4.1.1 Determine the coordinates of the centre of the circle and the length of the radius of the circle. (4)

4.1.2 Determine the equation of the tangent to the circle at $(-2 ; -4)$. (4)

4.2 Points A(2 ; 3) and B(−1 ; 6) lie on the circumference of the given circle. R(p ; q) is the centre of the circle and lies on the line $2x + 5y + 1 = 0$.



4.2.1 Show that $p - q = -4$. (4)

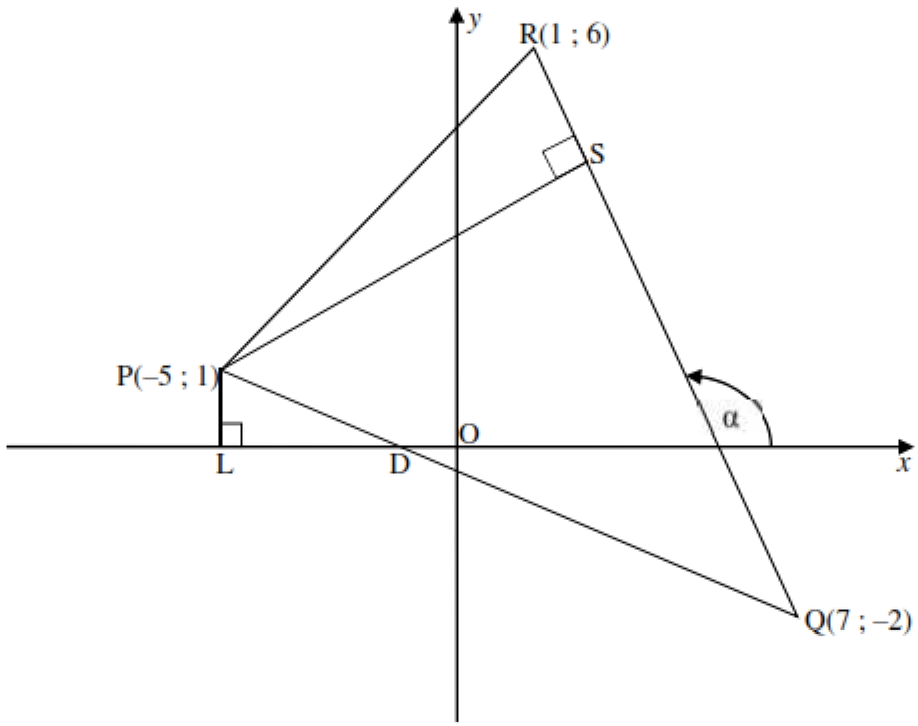
4.2.2 Hence, determine the equation of the circle. (7)

[19]

PREP 2019_NORTH WEST

QUESTION 3

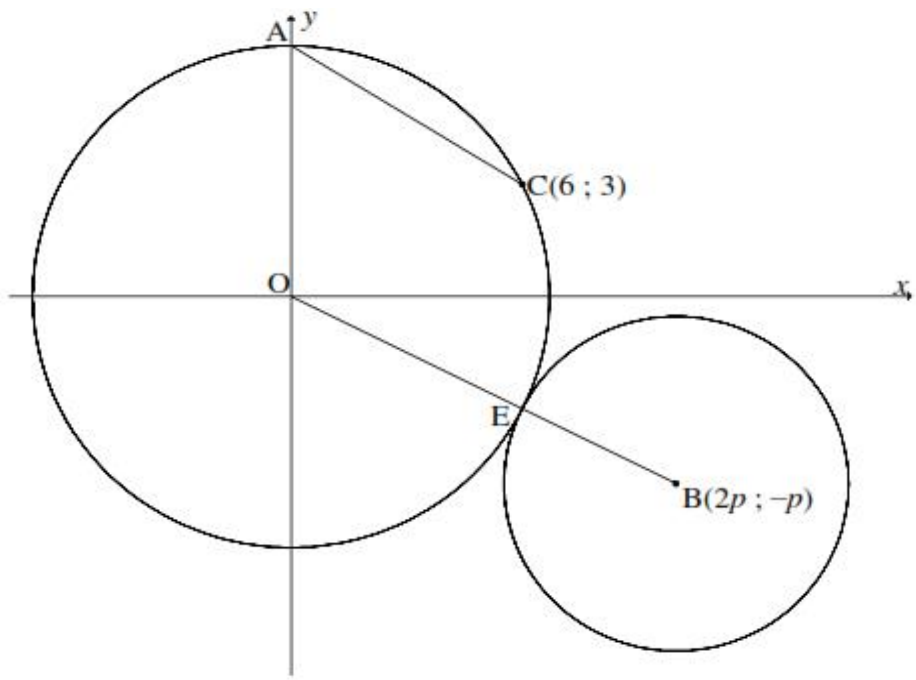
In the diagram below,P $(-5;\ 1)$, Q$(7;\ -2)$ and R$(1;\ 6)$ are the vertices of Δ PQR. PQ intersects the x-axis at D. The angle of elevation of QR is $\boldsymbol{a}$.
PS $\perp$ RQ and L lies on the x-axis such that PL $\perp$ $x-$axis.



3.1	Write down the equation of the line PL.	(1)
3.2	Calculate the gradient of QR.	(2)
3.3	Determine the equation of the linePS.	(4)
3.4	Calculate the size of the angle of inclination of PQ.	(3)
3.5	Calculate the size of $P\hat{Q}S$.	(4)
3.6	It is given that the areas of Δ PRS $= 4x^2$ and Δ PQS $= 16x^2$. Calculate the length of SQ, WITHOUT calculating the coordinates of S.	(5) **[19]**

QUESTION 4

In the diagram below, two circles are given. Circle O, having the origin as centre, intersects the y-axis at A and passes through the point C(6 ; 3). The circle having centre B($2p$;$-p$) touches circle O externally in point E. The centres of the two circles are joined by the line OB.



4.1 Determine the equation of the circle having centre O. (2)

4.2 Determine the coordinates of A. (2)

4.3 Determine the equation of AC. (3)

4.4 Calculate the value(s) of k for which the line $y = \frac{1-\sqrt{5}}{2}x + k$ will intersect the circle having centre O at two points, one of which has a positive x-value and the other a negative x-value. (2)

4.5 It is given that the length of $\text{EB} = \sqrt{20}$.

4.5.1 Write down, in terms of p, the equation of circle B in the form $(x-a)^2 + (y-b)^2 = r^2$. (2)

4.5.2 Determine the value of p if $p > 0$. (5)

4.6 Suppose a third circle with the following equation is given:
$x^2 + y^2 + 4x\cos\theta + 8y\sin\theta + 3 = 0$

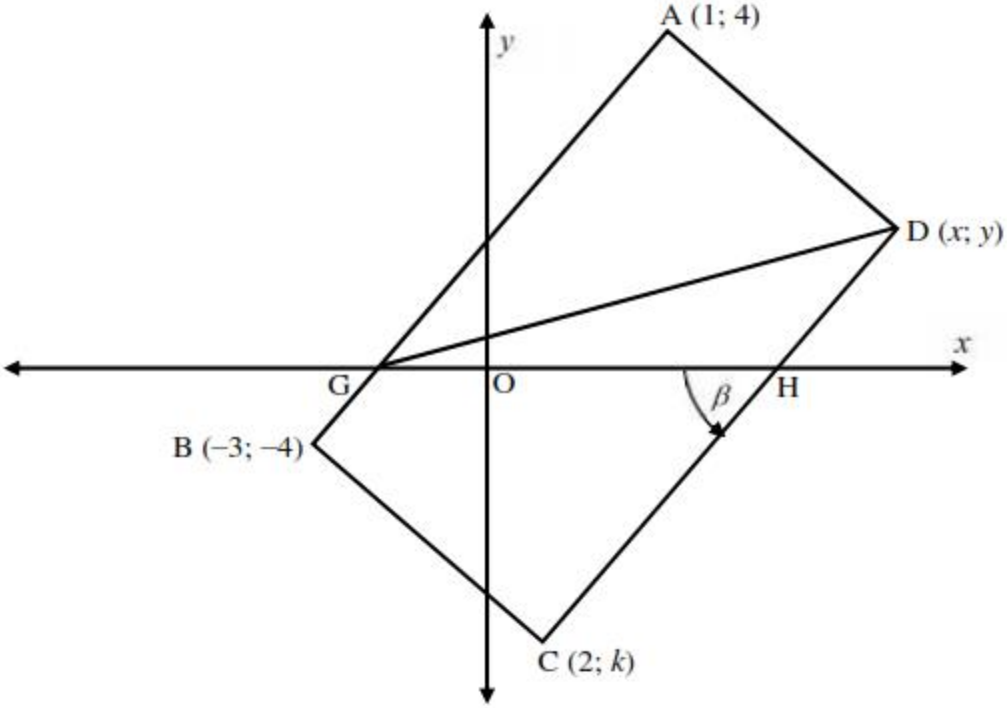
Determine the maximum length that the radius of this circle can be for any value of θ . (6)

[22]

PREP 2019_FREE STATE

QUESTION 3

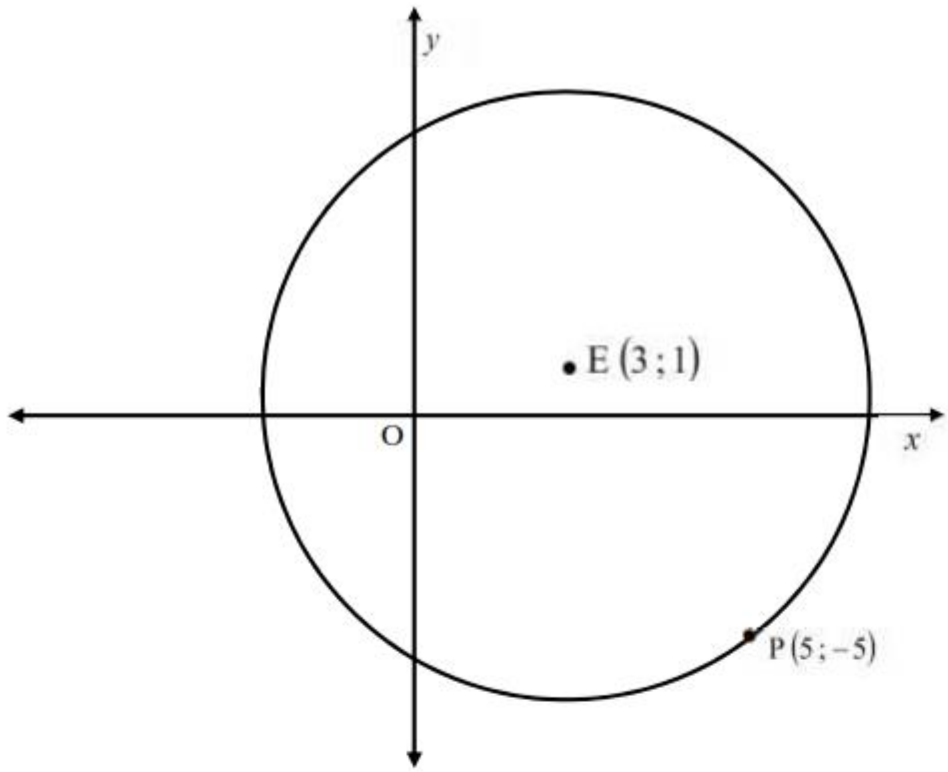
In the diagram below, A $(1; 4)$, B $(-3; -4)$, C $(2; k)$ and D $(x; y)$ are the vertices of a rectangle. AB and DC cuts the x-axis at G and H respectively. GD is drawn. $G\hat{H}C = \boldsymbol{b}$.



3.1	Calculate the gradient of BG.	(2)
3.2	Determine the equation of AB in the form $y = mx + c$.	(2)
3.3	Calculate the:	
3.3.1	Value of k (y-coordinate of C).	(4)
3.3.2	Coordinates of D.	(3)
3.3.3	Size of $\boldsymbol{b}$.	(3)
3.3.4	Area of Δ DHG.	(7)
		[21]

QUESTION 4

In the diagram below, the circle centred at $E(3;1)$ passes through point $P(5;-5)$.



4.1 Determine the equation of:

4.1.1 The circle in the form $x^2 + y^2 + Ax + By + C = 0$. (4)

4.1.2 The tangent to the circle at $P(5;-5)$ in the form $y = mx + c$. (5)

4.2 A smaller circle is drawn inside the circle. Line EP is a diameter of the small circle. Determine the:

4.2.1 Coordinates of the centre of the smaller circle. (3)

4.2.2 Length of the radius. (3)

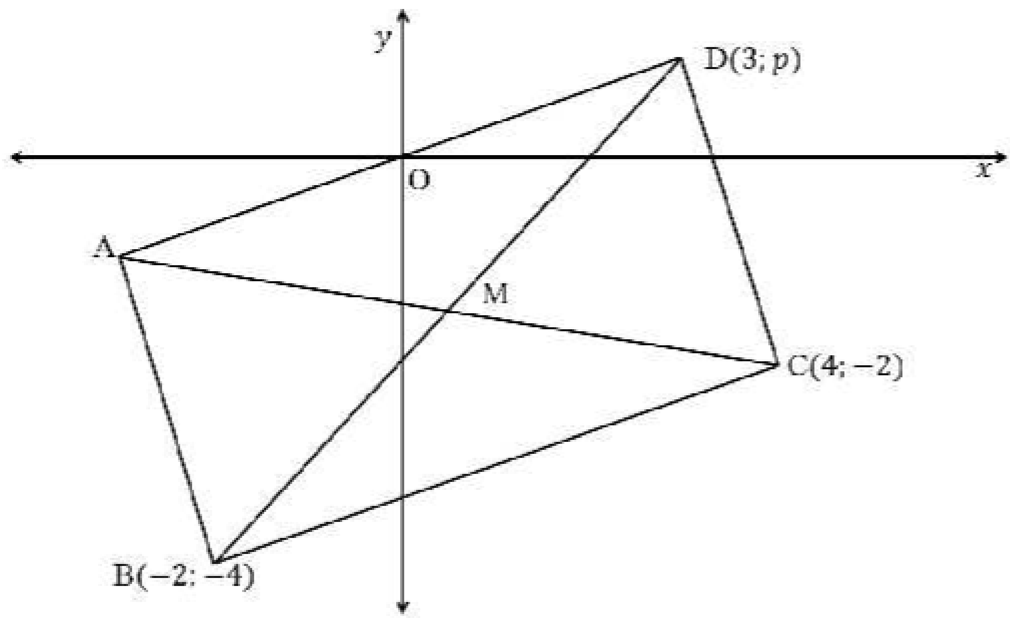
4.3 Hence, or otherwise, determine whether point $C(9;3)$ lies inside or outside the circle centre at E. (3)

[18]

PREP 2019_WESTERN CAPE

QUESTION 3

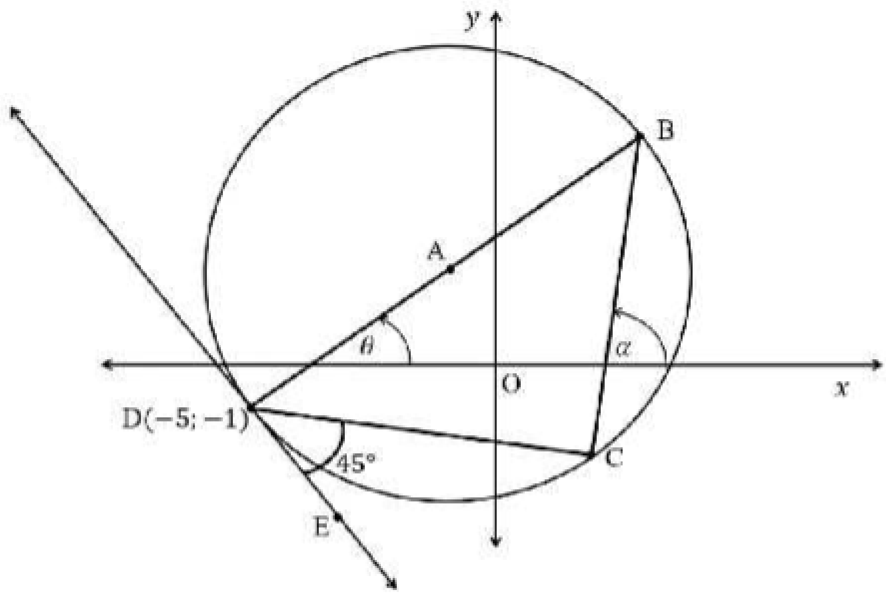
In the diagram A, B$(-2;-4)$, C$(4;-2)$ and D$(3;\ p)$ are the vertices of a rectangle. The diagonals AC and BD intersect at M.



3.1	If the length of AC is $\sqrt{50}$ units, show that $p = 1$.	(4)
3.2	Determine the coordinates of M.	(2)
3.3	Calculate the gradient of CD.	(2)
3.4	Determine the equation of line AB in the form $y = m \quad + c$.	(2)
3.5	Determine the coordinates of A	(2)
		[12]

QUESTION 4

In the diagram is the circle with equation $(x+1)^2+(y-2)^2=25$.
DB is a diameter and A the centre of the circle. DE is a tangent to the circle at D$(-5;-1)$. $E\hat{D}C$ = 45°. The inclination angles of AD and BC are θ and α respectively.
B and C are points on the circumference of the circle.



4.1 Determine:

4.1.1 The coordinates of A, the centre of the circle. (2)

4.1.2 The coordinates of B (3)

4.1.3 The value of θ, the inclination angle of AD. (4)

4.1.4 The equation of the tangent DE. (3)

4.2 Calculate the gradient of BC. (4)

4.3 Another circle with equation $x^2+y^2-6x+2y=8$ and centre M is given.

Show that:

4.3.1 The coordinates of M $(3;\ -1)$ and give the length of the radius. (4)

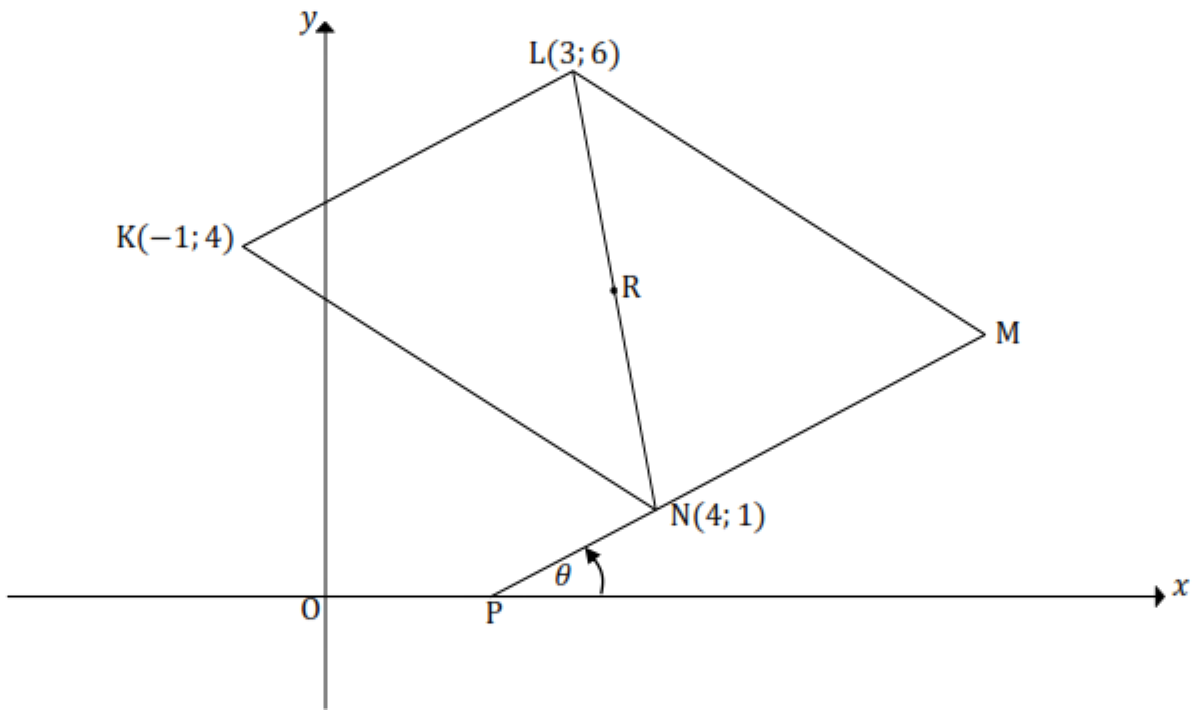
4.3.2 The two circles will intersect each other. Show all calculations. (4)

[24]

PREP 2019_KZN

QUESTION 3

In the diagram, K $(-1 ; 4)$; L$(3 ; 6)$; M and N$(4 ; 1)$ are vertices of a parallelogram. R is the midpoint of LN. P is the x-intercept of the line MN produced.



3.1 Calculate the:

3.1.1 gradient of KL. (2)

3.1.2 coordinates of R. (3)

3.1.3 coordinates of M. (4)

3.2 Determine the equation of NM in the form $y = mx + c$. (3)

3.3 Calculate the:

3.3.1 coordinates of P. (2)

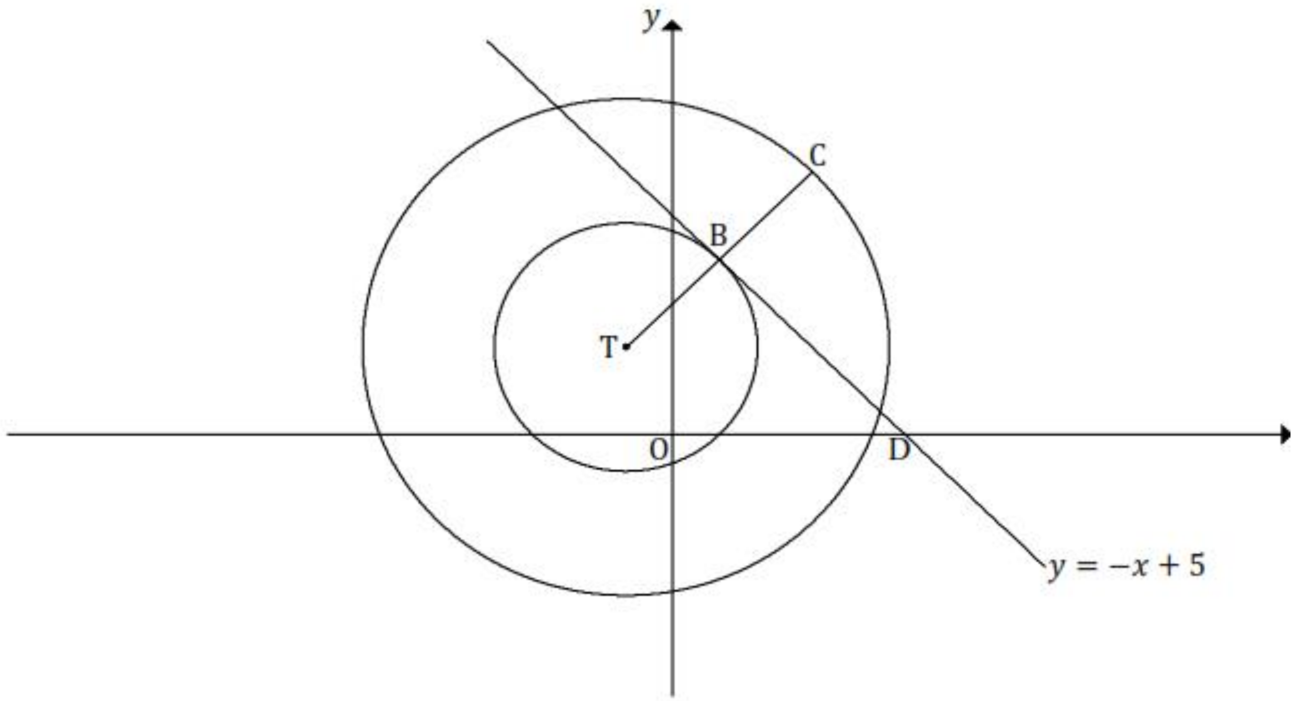
3.3.2 size of Θ, the inclination of PM. (2)

3.3.3 size of $K\hat{P}N$. (4)

[20]

QUESTION 4

In the diagram, T is the centre of two concentric circles. The larger circle has equation $x^2 + y^2 - 4y + 2x - 27 = 0$. The smaller circle touches the straight line $y = -x + 5$ at point B. BD is a tangent to smaller circle T. D is the x- intercept of the straight line. C is a point on the larger circle such that TBC is a straight line.

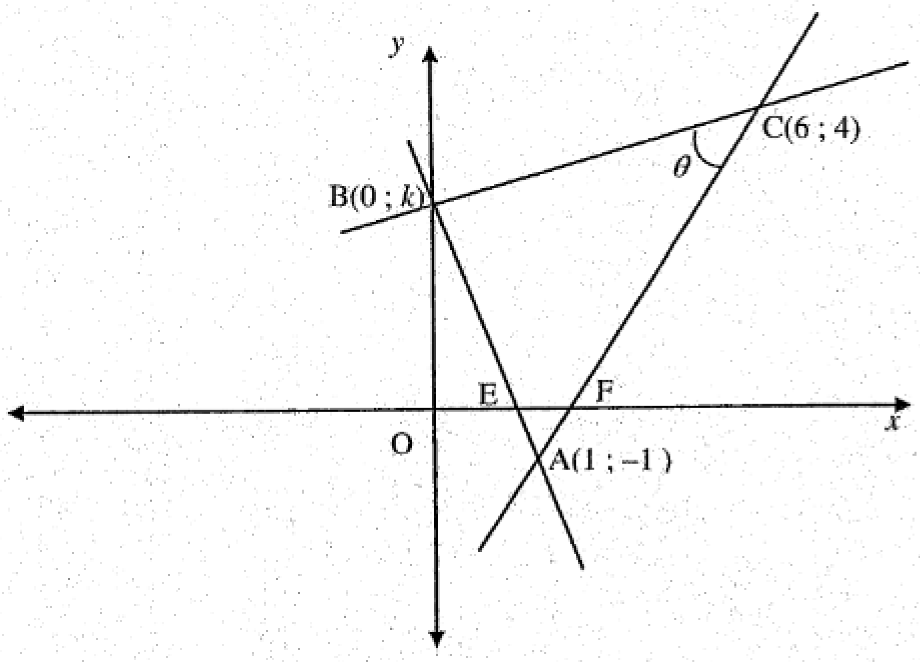


4.1	Calculate the coordinates of T.	(4)
4.2	Show that equation of TB is given by $y = x + 3$.	(3)
4.3	Calculate the coordinates of B	(3)
4.4	Determine the equation of the smaller circle in the form $(x-a)^2 + (y-b)^2 = r^2$.	(3)
4.5	Calculate the area of quadrilateral OTBD.	(7) **[20]**

PREP 2019_MPUMALANGA

QUESTION 3

In the diagram, A(1 ; −1), B(0 ; k) and C(6 ; 4) are the vertices of ΔABC. The equations of the sides AB and AC are $y + 3x - 2 = 0$ and $y = x - 2$ respectively. AB cuts the x-axis at E and AC cuts the x-axis at F.



3.1 Write down the value of k. (2)

3.2 Calculate the length of AC and leave your answer in simplest surd form. (2)

3.3 Prove that $A\hat{B}C = 90°$. (3)

3.4 Calculate the size of θ. (5)

3.5 Determine the equation of the circle passing through A, B and C in the form $(x-a)^2 + (y-b)^2 = r^2$. (4)

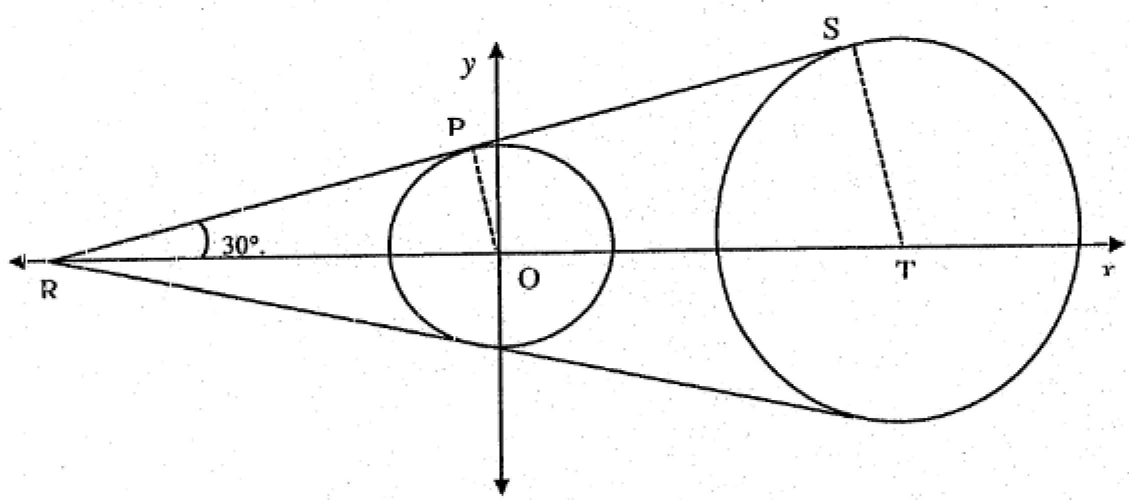
3.6 If D is a point in the first quadrant, calculate the coordinates of D such that ABCD in that order, forms a parallelogram. (4)

[20]

QUESTION 4

The diagram below shows a representation of the chain of a bicycle attached to two circular cogs as represented in the Cartesian plane. The equations of the circles are given by $x^2+y^2=1$ and $(x-6)^2+y^2=9$.

RPS is a common tangent to the smaller and larger circles at P and S respectively, with R a point on the negative x-axis. T is the centre of the larger circle. The angle of inclination between the tangent RPS and the horizontal axis is $30°$.



4.1	Write down the coordinates of T.	(1)
4.2	Write down the length of the radius of the larger circle.	(1)
4.3	If $R(-3\,;0)$, determine the equation of tangent PRS in the form $y=mx+c$. Leave answer in surd form.	(4)
4.4	Determine the equation of the radius ST in the form $y=mx+c$. Leave answer in surd form.	(3)
4.5	Determine the coordinates of S.	(5)
4.6	Determine the distance between the two circular cogs.	(4)

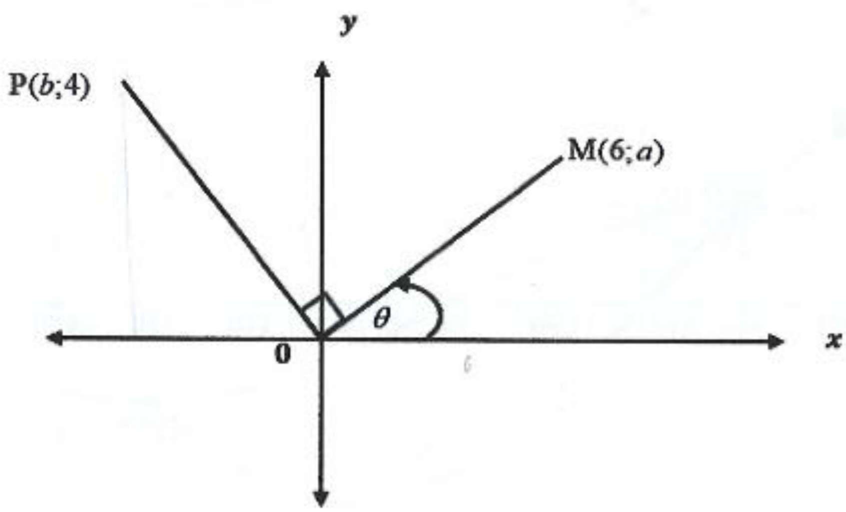
[18]

TRIGONOMETRY

PREP 2019_LIMPOPO

QUESTION 5

In the diagram alongside $P\hat{O}M = 90^\circ$, $M\hat{O}X = \theta$, $M(6;a)$, $P(b;4)$ and $\sqrt{5}\cos\theta - 2 = 0$.



Hint: $\tan(90^\circ + \theta) = -\dfrac{1}{\tan\theta}$.

5.1 Determine, without the use of a calculator, the values of:

5.1.1 a **(3)**

5.1.2 b **(4)**

5.2 Simplify the following expression to a single trigonometric ratio of θ:

5.2.1 $\dfrac{\sin 163^\circ}{\cos 73^\circ} - \dfrac{\sin(90^\circ + \theta).\tan^2(\theta - 360^\circ)}{\cos(\cancel{360^\circ - \theta})}$ $(180-\theta)$ $(180-\theta)$ **(7)**

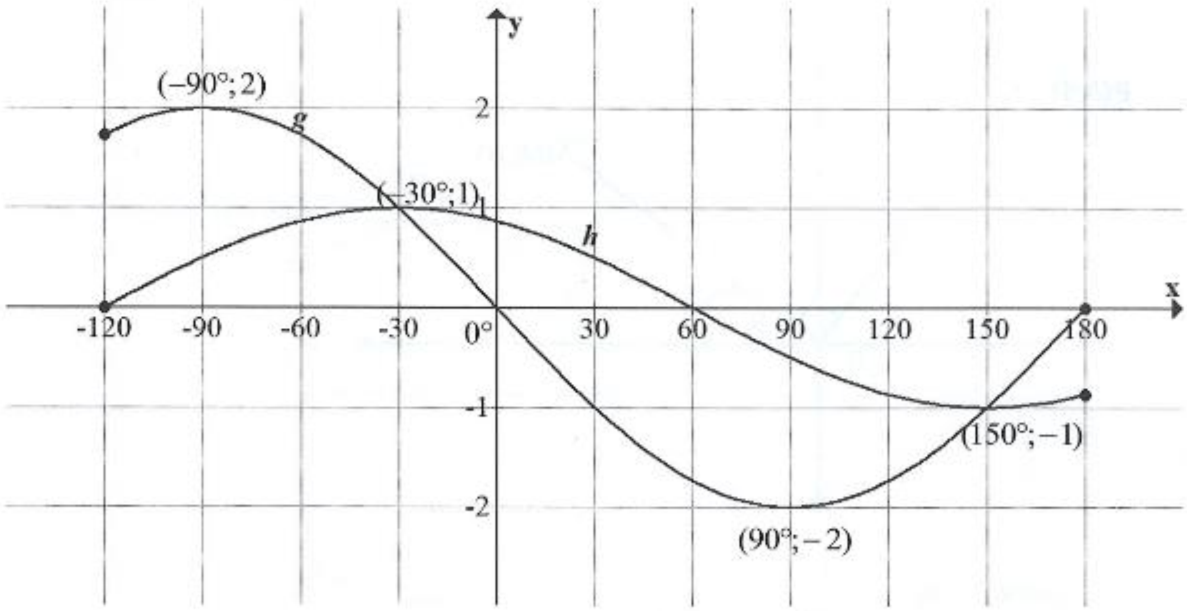
5.2.2 $\dfrac{\sin^2\theta\cos\theta}{\tan 2\theta(\sin^2\theta - \cos^2\theta)}$ **(5)**

5.3 Determine the general solution of $\cos(x - 60^\circ) = 3\cos x$. **(6)**

[25]

QUESTION 6

The graphs of $h(x) = \cos(x + 30°)$ and $g(x) = -2\sin x, x \in [-120°;180°]$ is drawn below.



Use the graphs above to determine for which values of x:

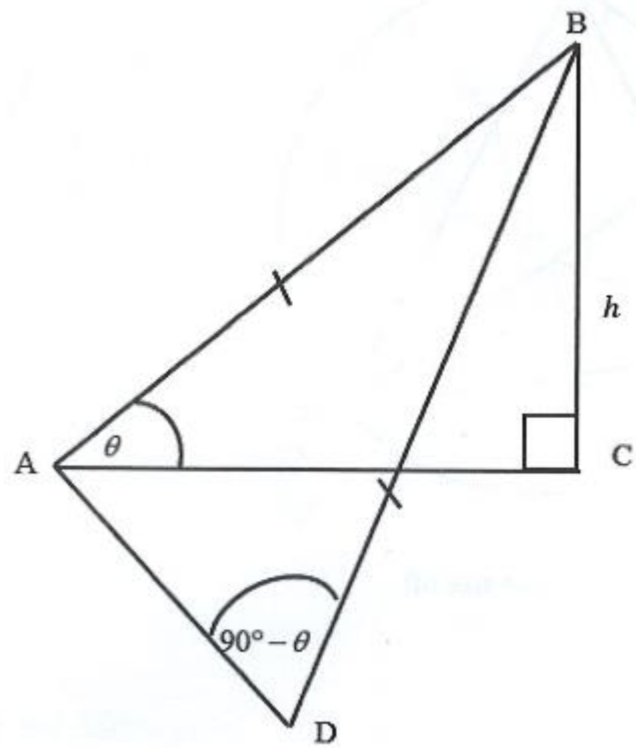
6.1 $2\sin x + \cos x \cos 30° \geq \sin x \sin 30°$, $x \in [-120°;180°]$ (4)

6.2 Will both $g(x)$ and $h(x)$ increase as x increases for $x \in [-120°;180°]$ (4)

[8]

QUESTION 7

In the diagram below, BC is a pole anchored by two cables at A and D. A, D and C are in the same horizontal plane. The height of the pole is h and the angle of elevation from A to the top of the pole B, is θ. BA = BD and $B\hat{D}A = 90° - \theta$.



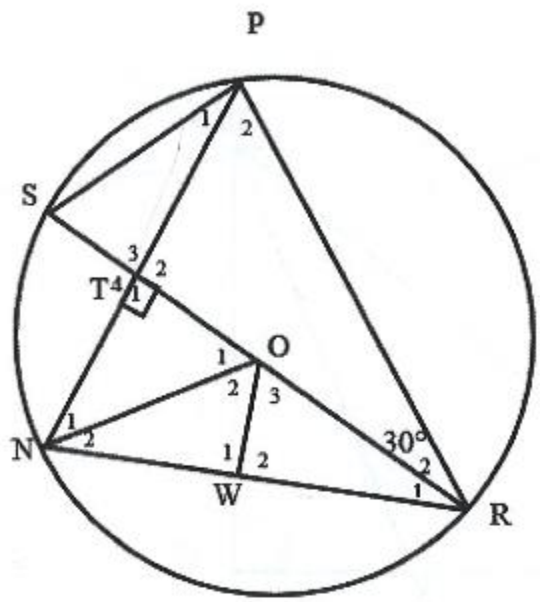
7.1 Express AB in terms of h and a trigonometric ratio of θ. (1)

7.2 Determine the magnitude of $A\hat{B}D$ in terms of θ. (3)

7.3 Determine the distance between the two anchors in terms of h. (3)
[7]

QUESTION 8

In the diagram the vertices of Δ PNR lie on the circle with centre O. Diameter SR and chord NP intersect at T. $OT \perp NP$ and $\hat{R}_2 = 30°$.



8.1 Determine, stating reasons, the size of:

8.1.1 $\hat{S}$ (3)

8.1.2 $\hat{R}_1$ (3)

8.1.3 $\hat{N}_1$ (2)

8.2 If it is further given that NW = WR, prove that TNWO is a cyclic quadrilateral. (4)

[12]

PREP 2019_EASTERN CAPE

QUESTION 5

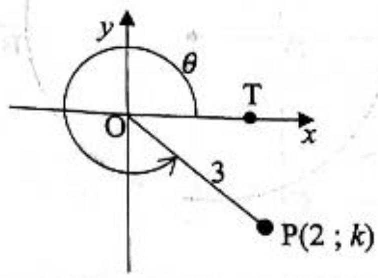
DO NOT USE A CALCULATOR FOR THIS QUESTION.

5.1 Complete the following identities:

5.1.1 $\cos^2 A + \sin^2 A = \dots$ (1)

5.1.2 $\cos^2 A - \sin^2 A = \dots$ (1)

5.2 P(2 ; k) is a point in the Cartesian plane such that OP = 3 units and reflex angle $T\hat{O}P = \theta$, as shown in the diagram below.



5.2.1 Calculate the value of k (leave your answer in surd form). (2)

5.2.2 Hence, determine the value of the following:

(a) $\tan(\theta - 180°)$ (2)

(b) $\dfrac{1-\sin^2 2\theta}{1-2\sin^2\theta}$ (4)

5.3 Determine, **without the use of a calculator**, the value of:

$\sin(-200°).\cos 310° + \tan(-135°).\cos 380° : \sin 230°$ (6)

5.4 Prove the following identity:

$\sin 2\theta + \cos(2\theta - 90°) = 4\sin\theta\cos\theta$ (3)

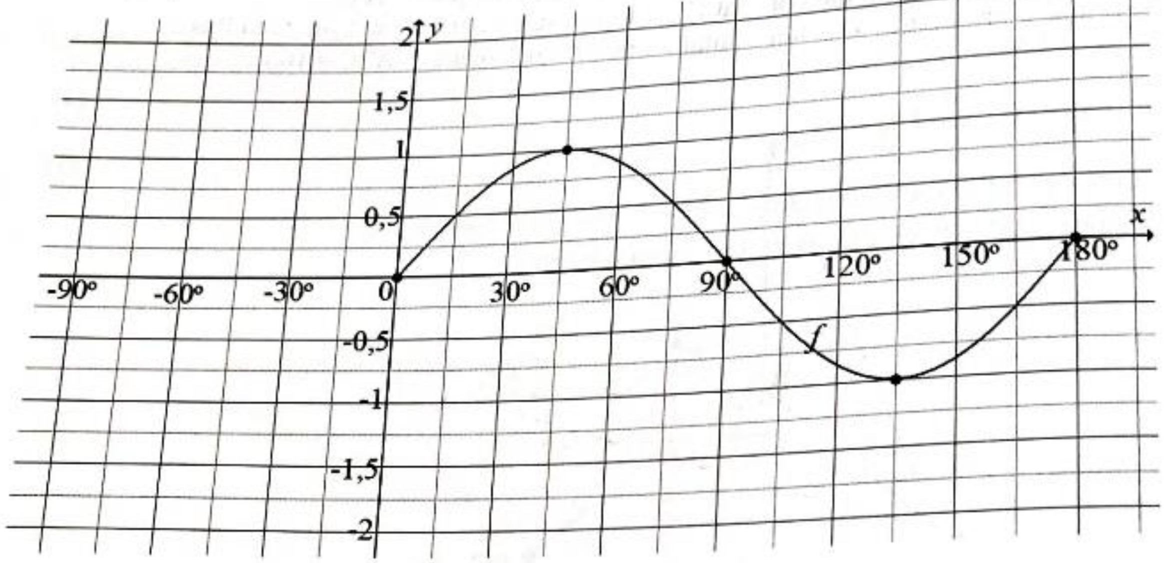
5.5 Solve for x if:

$10^{\sin x} + 10^{\sin x + 1} = 110 \qquad \text{for } -360° \le x \le 360°$ (5)

[24]

QUESTION 6

The graph below shows part of the function $f(x) = \sin 2x$ for $0° \leq x \leq 180°$.



6.1 On the grid provided in the ANSWER BOOK, complete the graph of f for the interval $-90° \leq x \leq 180°$. (1)

6.2 On the same grid, draw the graph of $g(x) = \cos(x - 30°)$ for the interval $-90° \leq x \leq 180°$. Clearly show the intercepts with the axes, the coordinates of the turning points and the end points of the graph. (4)

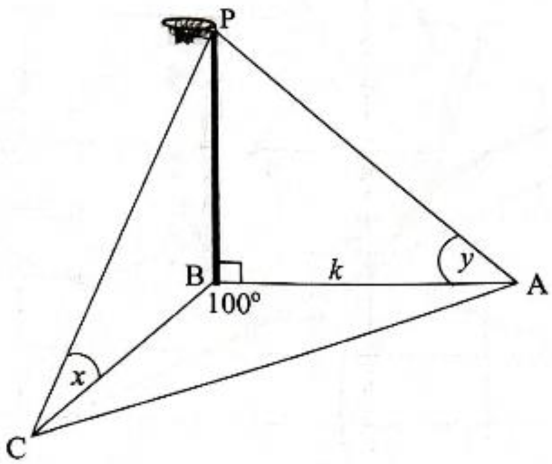
6.3 Calculate the solutions to the equation:

$$\sin 2x = \cos(x - 30°) \quad \text{for} \quad -90° \leq x \leq 90°$$ (6)

[11]

QUESTION 7

The diagram below shows a vertical netball pole PB. Player A is standing on the base line of the court and the angle of elevation from A to the top of the pole P is y°.
A second player is standing in the court at C, and the angle of elevation from C to P is x°.
Points A, B and C are in the same horizontal plane. BA is k metres; $A\hat{B}C = 100^{\circ}$.



7.1 Show that $BC = \dfrac{k.\tan y}{\tan x}$ (3)

7.2 Calculate the length of AC if $BC = 4{,}73$ m and $k = 3$ m. (3)

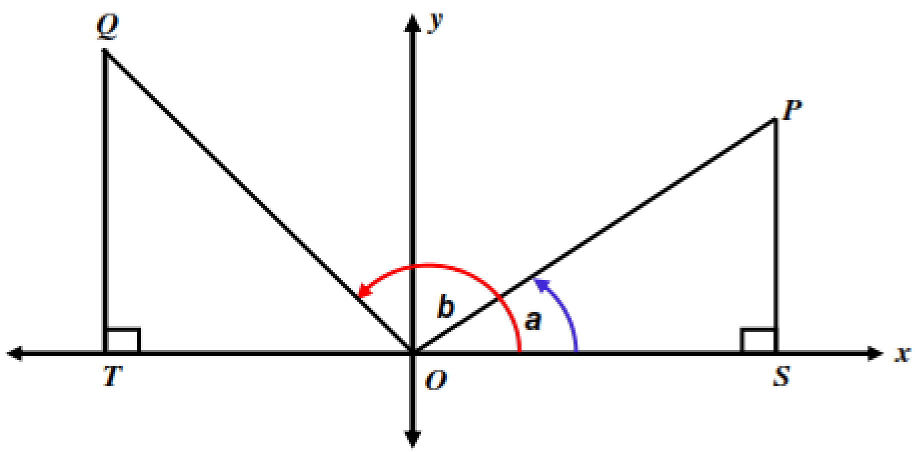
[6]

PREP 2019_GAUTENG

QUESTION 5

This question is to be done without the use of a calculator.

In the diagram below the equation of OP is given by $3y - 2x = 0$. S is a point on the x-axis such that $PS \perp x$-axis. $S\hat{O}P = \boldsymbol{a}$. The line segment OQ is drawn such that $S\hat{O}Q = \boldsymbol{b}$. T is a point on the x-axis such that $QT \perp x$-axis.



5.1 Show that $\tan \boldsymbol{a} = \frac{2}{3}$. (2)

5.2 Calculate the value of $\sin \boldsymbol{a}$. (2)

5.3 Write down $Q\hat{O}P$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$. (1)

5.4 If it is given that $\sin \boldsymbol{b} = \frac{3}{5}$, calculate the value of $\sin Q\hat{O}P$. (4)

[9]

QUESTION 6

6.1 Simplify fully WITHOUT the use of a calculator:

$$\frac{\cos(40^\circ - x)\cdot\cos x - \sin(40^\circ - x)\cdot\sin x}{\sin 205^\circ.\cos 25^\circ}$$ (5)

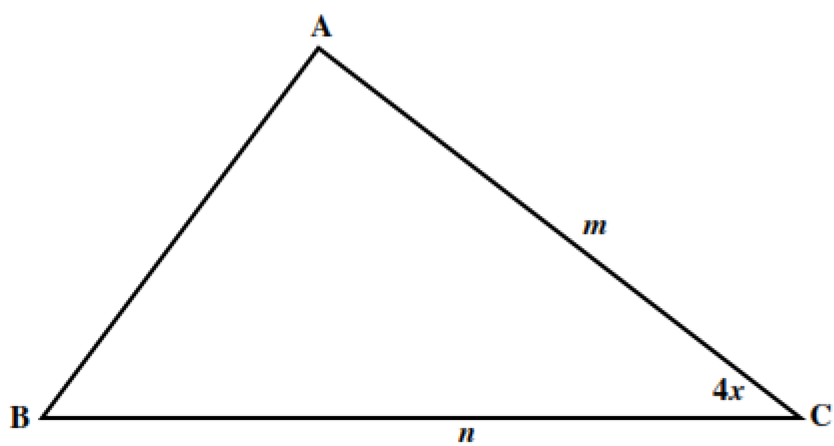
6.2 Given: $\dfrac{\cos 2x}{\cos x + \sin x}$

6.2.1 Show that $\dfrac{\cos 2x}{\cos x + \sin x} = \cos x - \sin x$. (2)

6.2.2 Show that $\cos x\left(\dfrac{\cos 2x}{\cos x + \sin x}\right) = \dfrac{1}{2}$ can be simplified to $\cos 2x = \sin 2x$. (4)

6.2.3 Hence, determine the general solution of $\cos x\left(\dfrac{\cos 2x}{\cos x + \sin x}\right) = \dfrac{1}{2}$. (3)

6.3 In $\Delta\,\mathrm{ABC}$, $\mathrm{AC} = m$, $\mathrm{BC} = n$ and $\hat{\mathrm{C}} = 4x$.



6.3.1 Write down an expression for the area of $\Delta\mathrm{ABC}$. (1)

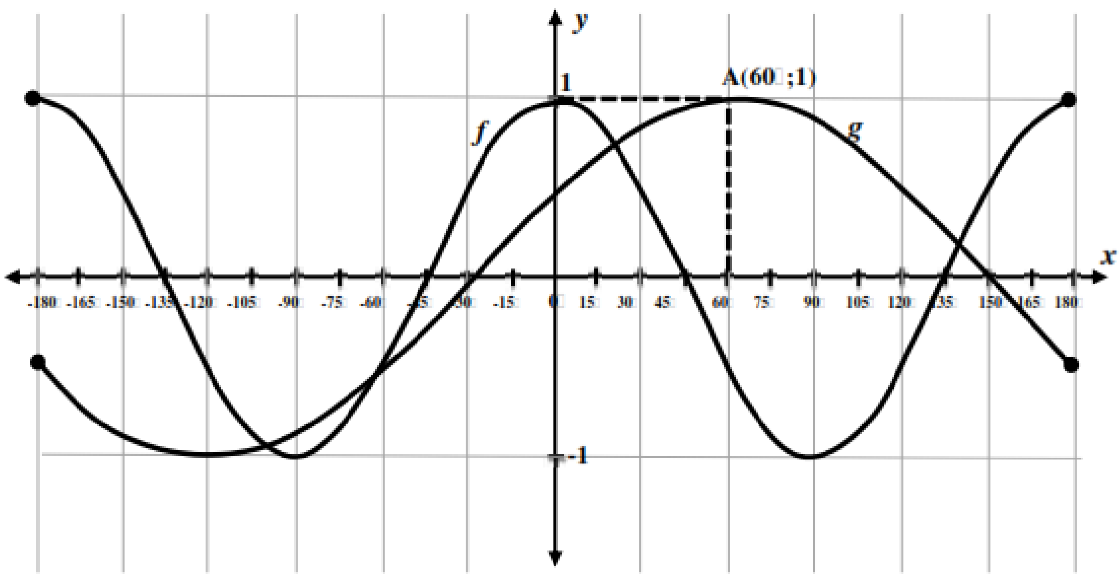
6.3.2 Calculate the value of x for which the area of the triangle will be a maximum. (2)

6.3.3 What conclusion can you make about the type of triangle formed when the area of the triangle is a maximum? (1)

[18]

QUESTION 7

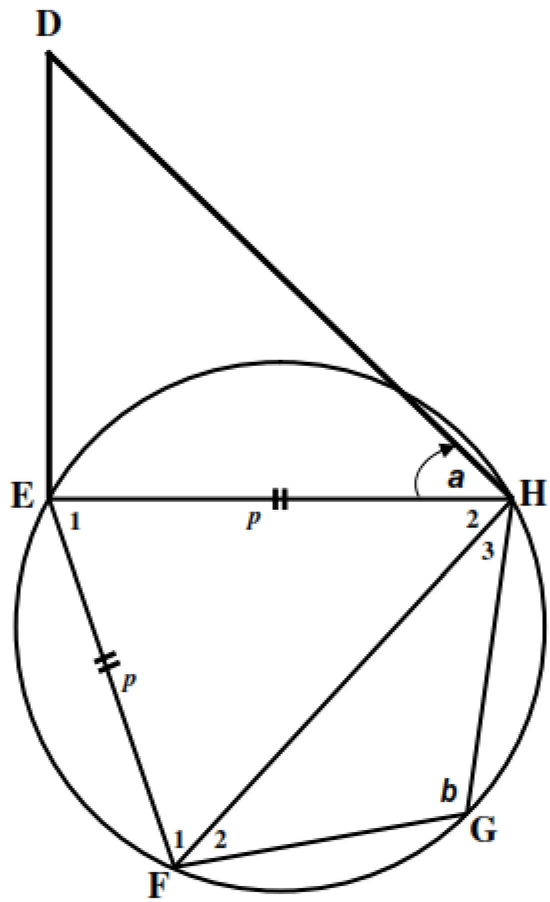
The sketch below shows the graphs of $f(x) = \cos 2x$ and $g(x) = \sin(x - \boldsymbol{q})$ for $x \in [-180° ; 180°]$. A $(60° ; 1)$ is a point on the graph of g. Use the graph to answer the questions that follow.



7.1	Write down the value of $\boldsymbol{q}$.	(1)
7.2	Determine the period of f.	(1)
7.3	If $h(x) = f(x) - 1$, write down the range of h.	(1)
7.4	Determine the values of x, where $x \in [0° ; 180°]$ for which	
7.4.1	$f(x).g(x) < 0$.	(3)
7.4.2	$f'(x).g(x) > 0$.	(2)
		[8]

QUESTION 8

In the diagram, DE represents a vertical cell phone tower positioned on one corner of a field. The field is shaped as a cyclic quadrilateral EFGH and E, F, G, and H are all on the same horizontal plane. From H, the angle of elevation to D, the top of the cell phone tower, is $\boldsymbol{a}$.
$EH = EF = p$ units. $\hat{G} = \beta$.



8.1 Write down DE in terms of $\boldsymbol{a}$. (1)

8.2 Show that:

8.2.1 $\hat{H}_2 = \frac{1}{2}\boldsymbol{b}$ (Give reasons for your answers) (3)

8.2.2 $p = \dfrac{FH}{2\cos\frac{1}{2}\boldsymbol{b}}$ (3)

8.2.3 $FH = p\sqrt{2(1+\cos\boldsymbol{b})}$ (3)

[10]

PREP 2019_NORTH WEST

QUESTION 5

5.1 Simplify each of the following **without the use of a calculator.** Show ALL calculations.

5.1.1 $\dfrac{\sin 110°.\tan 60°}{\cos 540°.\tan 250°.\sin 380°}$ (7)

5.1.2 $(1-\sqrt{2}\sin 22{,}5°)(\sqrt{2}\sin 22{,}5° + 1)$ (4)

5.2 Given the expression: $\dfrac{\cos 2x.\tan x}{\sin^2 x}$

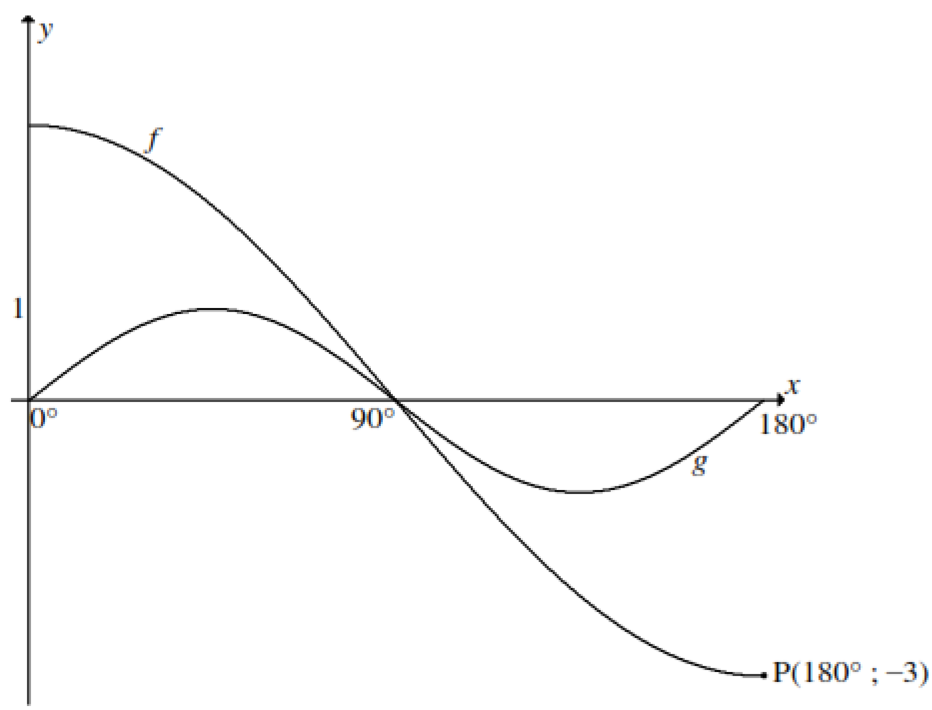
5.2.1 For which value(s) of x, in the interval $x \in [0° ; 180°]$, will this expression be undefined? (3)

5.2.2 Prove that $\dfrac{\cos 2x.\tan x}{\sin^2 x} = \dfrac{\cos x}{\sin x} - \tan x$ (5)

[19]

QUESTION 6

In the diagram below, the graphs of $f(x) = a\cos x$ and $g(x) = \sin bx$ are drawn for the interval $x \in [0° ; 180°]$. The point P(180° ; −3) is on the graph of f.



6.1 Write down the values of a and b. (2)

6.2 Write down the period of f. (1)

6.3 Write down the range of $g(x) + 3$ (2)

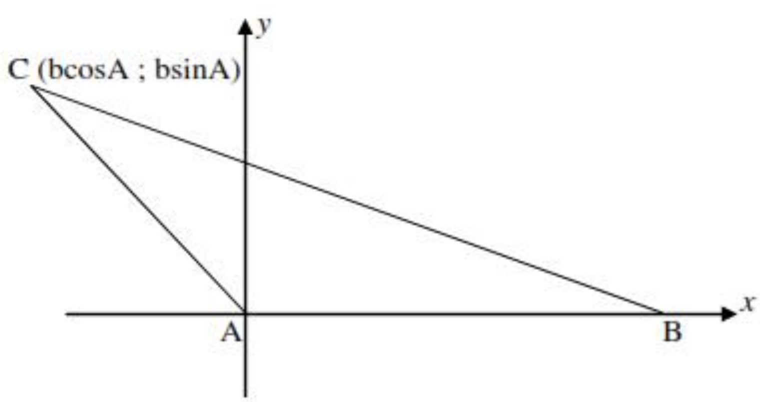
6.4 For which values of x, in the given interval, is $f(x).g'(x) > 0$ (3)

6.5 When the graph of g is shifted $q°$ to the left, it coincides with the function $y - \cos^2 x = -\sin^2 x$. Determine the value of q. (3)

[11]

QUESTION 7

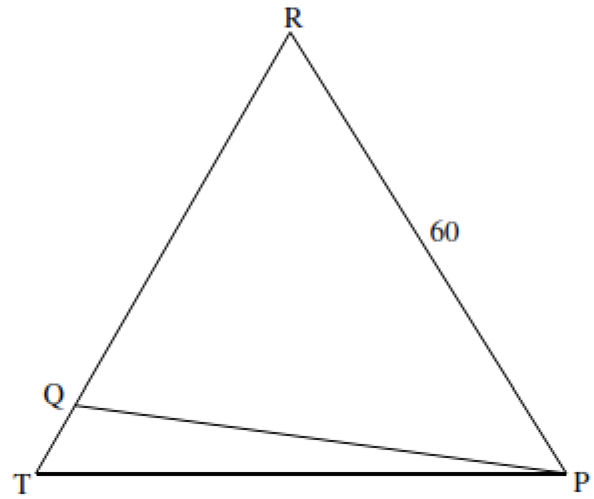
7.1 In the diagram below, Δ ABC is drawn having A at the origin, B on the x-axis and the vertex C has the coordinates(bcosA ; bsinA).



Use the above diagram to prove that $a^2 = b^2 + c^2 - 2bc\cos A$ (4)

7.2 In the diagram below, Δ TPR is equilateral with PR = 60 units. Q is a point on RT such that RQ:QT = 5:1.

7.2 In the diagram below, Δ TPR is equilateral with PR = 60 units. Q is a point on RT such that RQ:QT = 5:1.



7.2.1 Show, by calculations, that PQ = 55,68 units. (4)

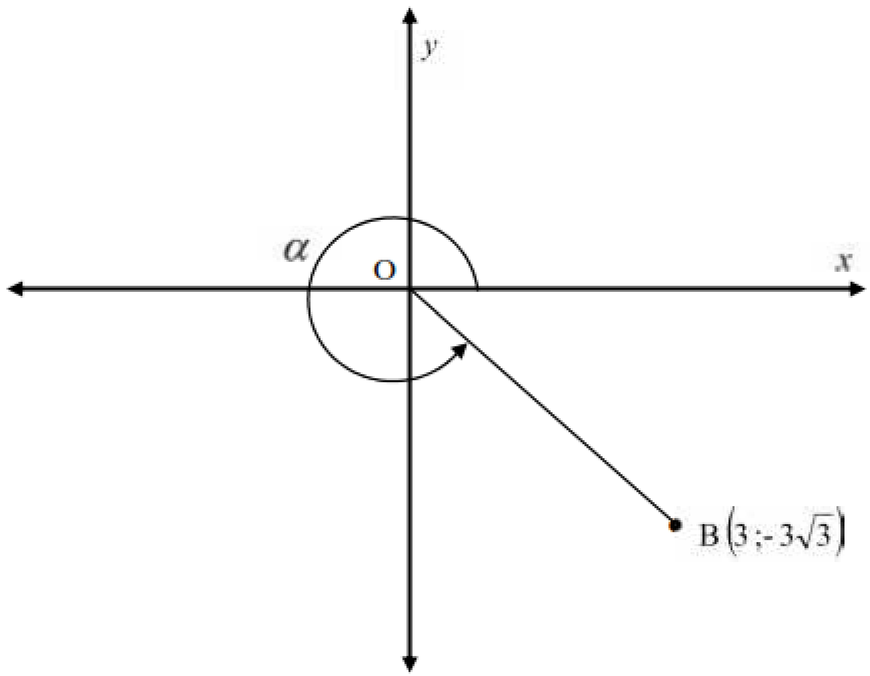
7.2.2 It is given that S is any point on the straight line PQ. Calculate the distance QS when S is the nearest to R. (4)

[12]

PREP 2019_FREE STATE

QUESTION 5

5.1 In the Cartesian plane below, the point $B\left(3\,;-3\sqrt{3}\right)$ and the reflex angle, α , are shown.



Determine **(without using a calculator)** the value of:

5.1.1 OB (2)

5.1.2 $\cos\left(\alpha + 30^\circ\right)$ (4)

5.2 Simplify:

$$\frac{\sin^2\left(90^\circ - x\right)\tan\left(360^\circ - x\right)}{\sin\left(-x\right)}$$ (4)

5.3 Prove that:

$$\cos\left(60^\circ + \theta\right) - \cos\left(60^\circ - \theta\right) = -\sqrt{3}\sin\theta$$ (3)

5.4 Consider the identity: $\dfrac{1 - \sin 2A}{\sin A - \cos A} = \sin A - \cos A$

5.4.1 Prove the identity. (4)

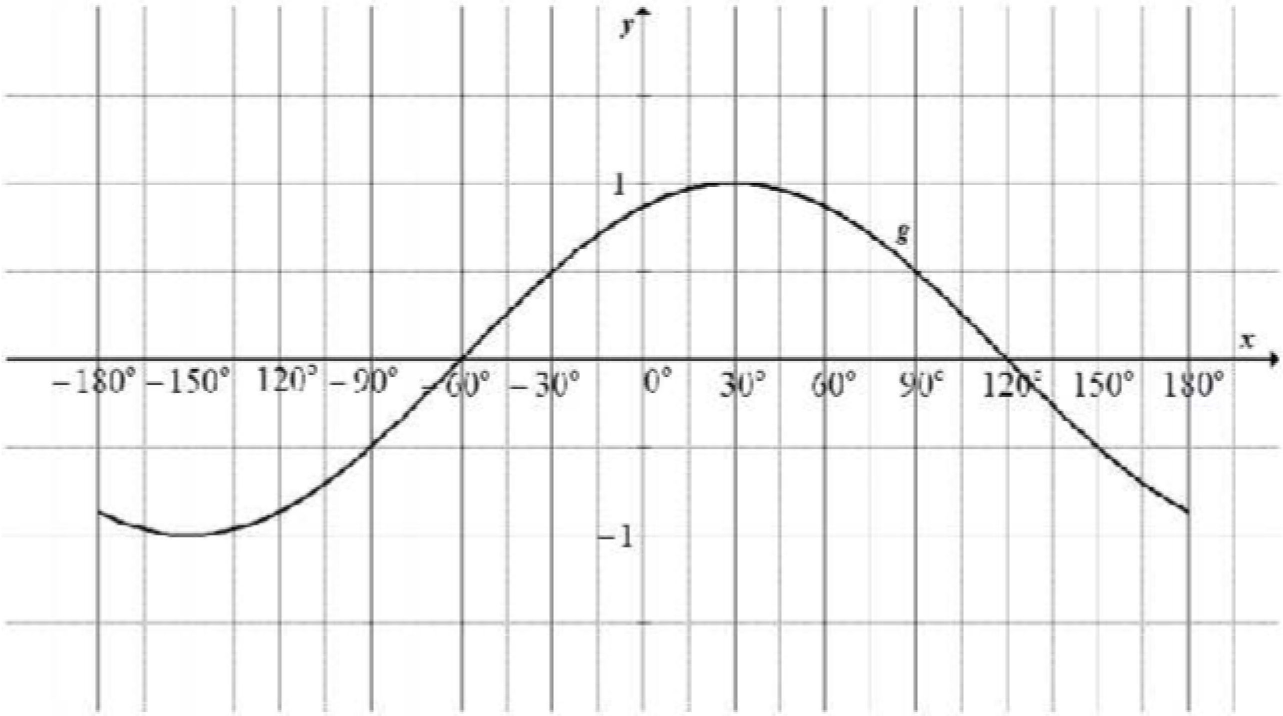
5.4.2 For which values of A in the interval $0^\circ < A < 180^\circ$ will the identity be undefined? (2)

[19]

QUESTION 6

6.1 Determine the general solution for $\sin 2x = \cos(x - 30^\circ)$. (5)

6.2 The diagram below shows the graph of $g(x) = \cos(x - 30^\circ)$ for the interval $x \in [-180^\circ ; 180^\circ]$.



6.2.1 Write down the period of g. (1)

6.2.2 Determine the values of x for which the graph of g increasing. (2)

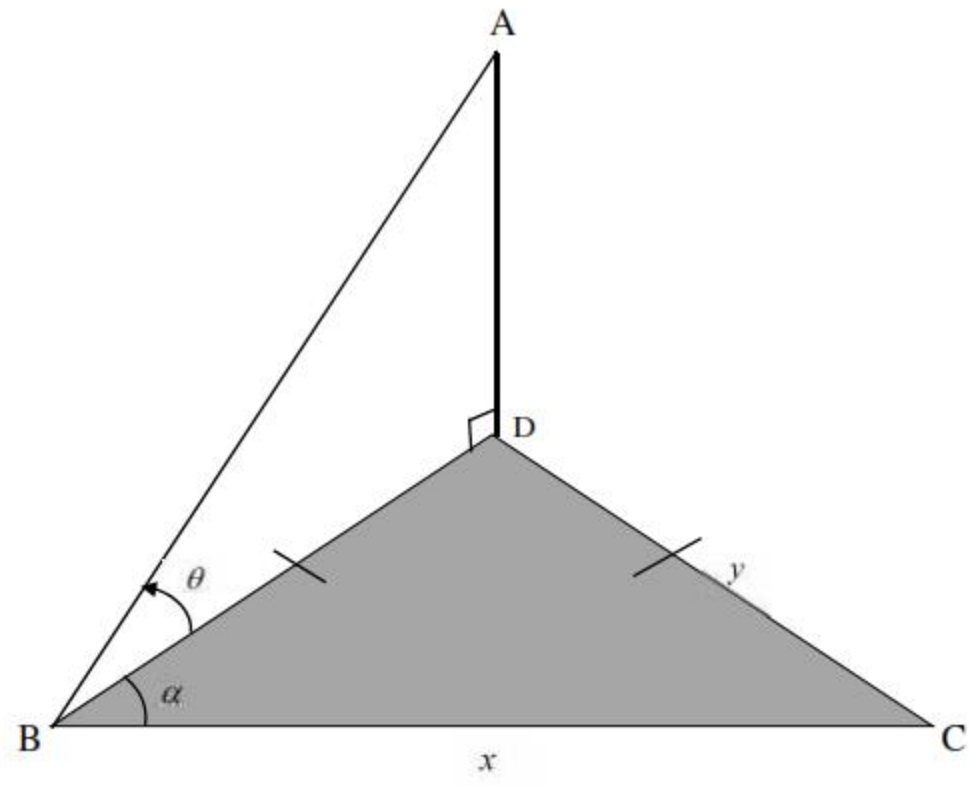
6.2.3 On the same system of axes draw the graph of $f(x) = \sin 2x$ for $x \in [-180^\circ ; 180^\circ]$ in your ANSWER BOOK. (3)

6.2.4 Hence or otherwise, determine the values of x in the interval $-180^\circ \leq x \leq 180^\circ$ for which $f(x).g(x) < 0$. (3)

[14]

QUESTION 7

In the diagram below, B, C and D are three points on the same horizontal plane such that BD = DC = y . $C\hat{B}D = \alpha$ and $A\hat{B}D = \theta$. Line BC = x .



Prove that $AB = \dfrac{x}{2\cos\alpha\ \cos\theta}$ **[7]**

PREP 2019_WESTERN CAPE

QUESTION 5

5.1 If $\sin 40°.\cos 22° + \cos 40°.\ \sin 22° = k$, determine without the use of a calculator, the value of the following in terms of k.

5.1.1 $\sin 62°$ (2)

5.1.2 $\tan 118°$ (3)

5.1.3 $\sin 14°.\ \cos 14°$ (3)

5.2 Prove the following identity:

$$\frac{1-\cos 2\theta}{\sin 2\theta \times \tan\theta} = 1$$ (4)

5.3 For which value(s) of A will the following expression be real?

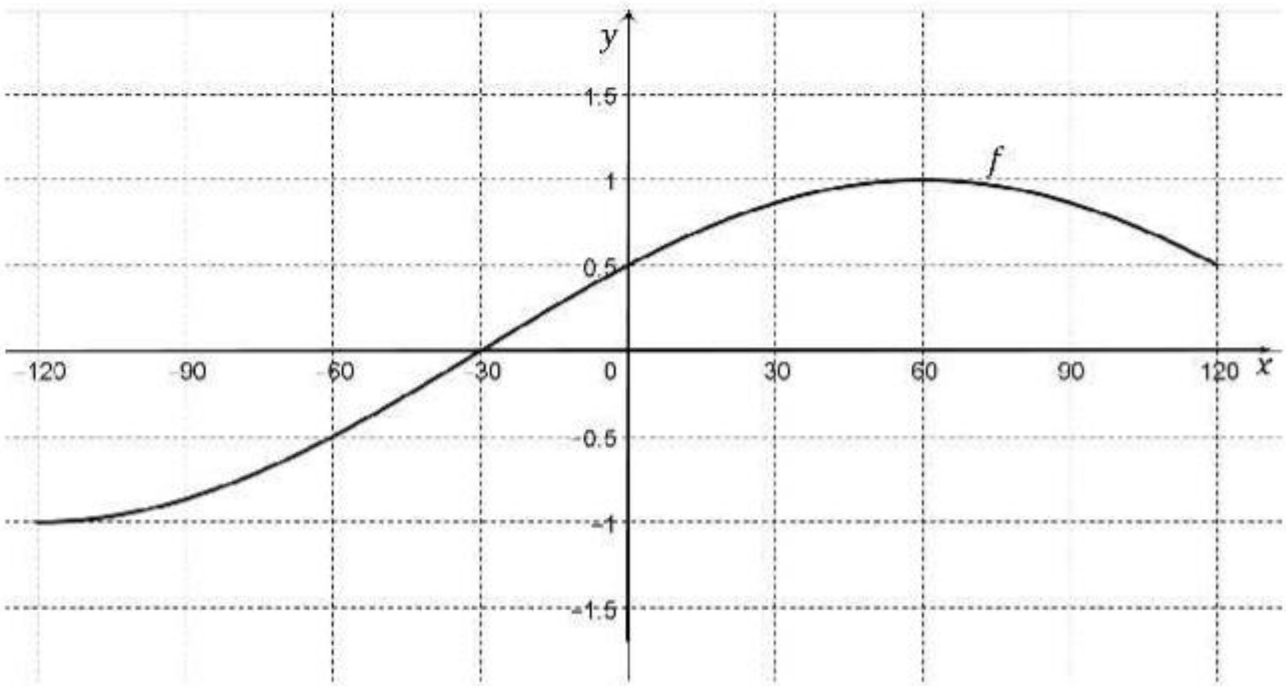
$$\sqrt{\sin(180° + \mathrm{A}).\cos(90° + \mathrm{A}) - \tan 45°}$$

(6)

[18]

QUESTION 6

In the diagram is the graph of $f(x) = \sin(x + a)$ for the interval [-120°;120°]



6.1 Determine the numerical value of a. (1)

6.2 On the grid provided in the ANSWER BOOK, draw the graph of $g(x) = \cos(3x)$ for the interval $x \in [-120°; 120°]$. Clearly show ALL intercepts with the axes, the turning point(s) and endpoint(s) of the graph. (4)

6.3 Determine the general solution for $f(x) = g(x)$ (5)

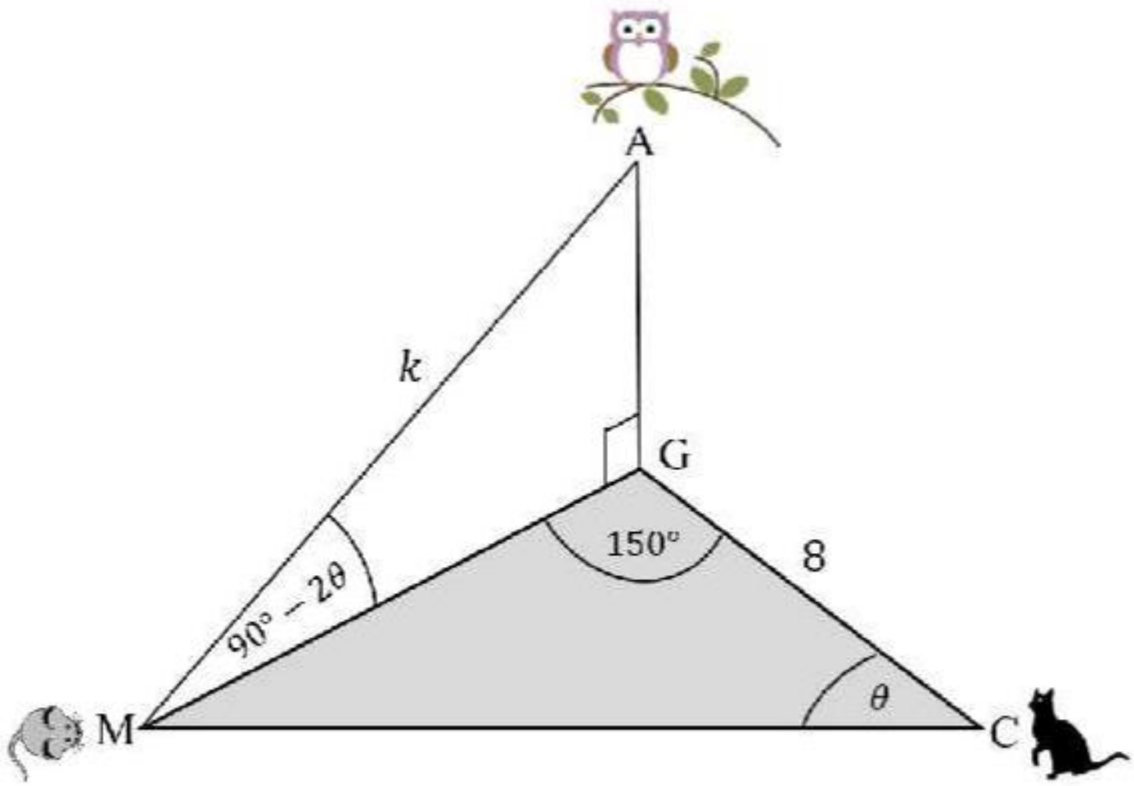
6.4 Determine the values of x in the interval $x \in [0°; 120°]$, for which $f(x) > g(x)$. (2)

6.5 Describe the transformation from graph g to the graph of $k(x) = \cos(60° - 3x)$. (2)

[14]

QUESTION 7

A mouse on the ground is looking up to an owl in a tree and a cat to his right on the ground. The angle of elevation from the mouse to the owl is $(90° - 2\theta)$.
$AM = k\ u$, $GC = 8\ u$, $M\hat{G}C = 150°$ and $M\hat{C}G = \theta$

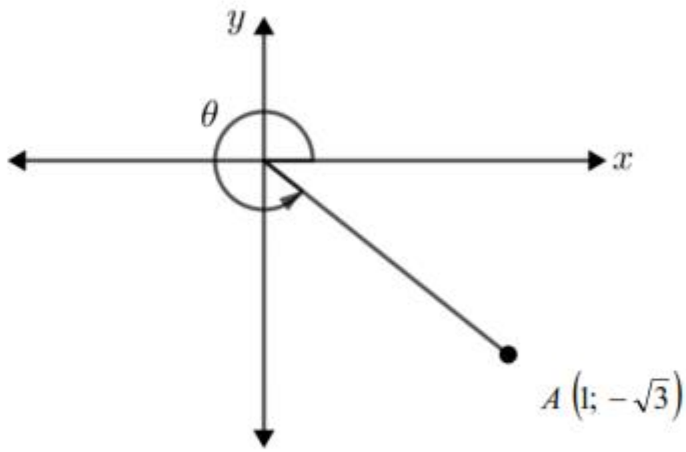


7.1	Give the size of $M\hat{A}G$ in terms of θ.	(1)
7.2	Show that $MG = k \sin 2\theta$	(2)
7.3	Show that $MC = k \cos \theta$	(4)
7.4	Show that the area of $\Delta MGC = 2k \sin 2\theta$	(2)
		[9]

PREP 2019_KZN

QUESTION 5

5.1 Use the diagram below to calculate, **without the use of a calculator,** the following



5.1.1 $\tan\theta$ (1)

5.1.2 $\sin(-\theta)$ (3)

5.1.3 $\sin(\theta - 60°)$ (4)

5.2 Determine the value of the following trigonometric expression:

$$\frac{\tan(180° - \theta)\sin(90° + \theta)}{\cos 300° \sin(\theta - 360°)}$$ (6)

5.3 Consider: $\dfrac{\cos 2x - 1}{\sin 2x} = -\tan x$

5.3.1 Prove the identity (3)

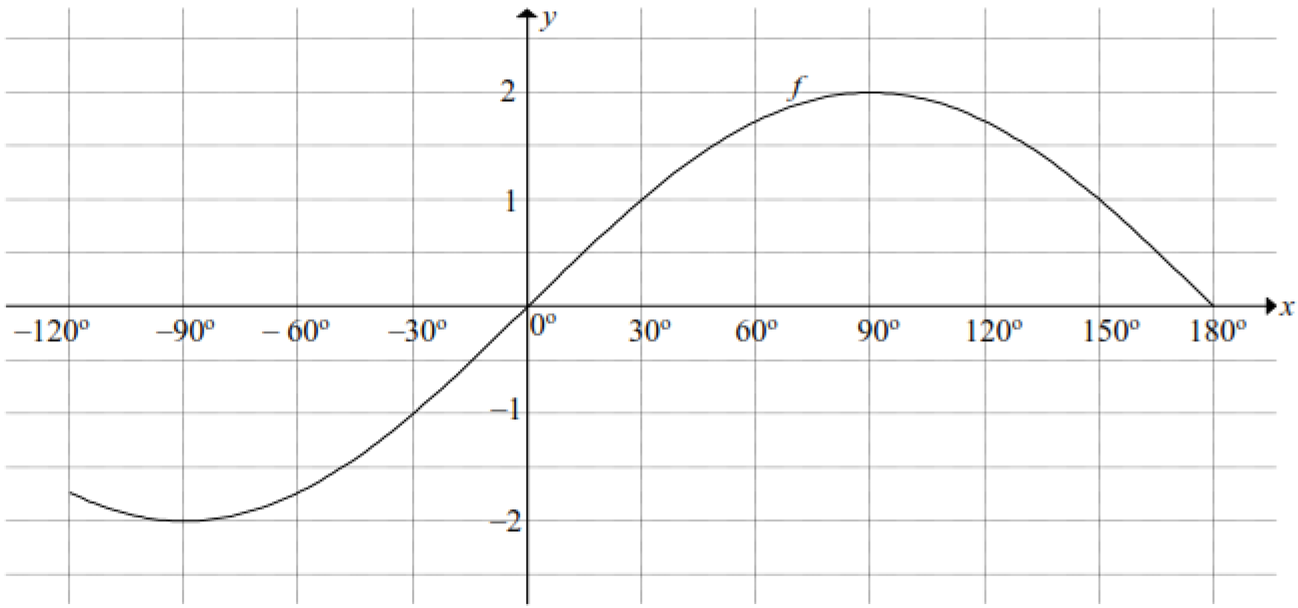
5.3.2 For which value(s) of x, $0° < x < 360°$, is this identity undefined? (3)

5.3.3 Hence or otherwise, find the general solution of $\dfrac{\sin 4x}{\cos 4x - 1} = 4$. (4)

[24]

QUESTION 6

In the diagram below, the graph of $f(x) = 2\sin x$ is drawn for the interval $x \in [-120° \,;\, 180°]$.



6.1 Draw on the same system of axes the graph of $g(x) = \cos(x + 30°)$, for the interval $x \in [-120° \,;\, 180°]$. Show all intercepts with the axes as well as the turning and end Points of the graph. (4)

6.2 Write down the period of f. (1)

6.3 For which values of x in the interval $x \in [-120° \,;\, 180°]$ is:

6.3.1 The graph of g decreasing? (2)

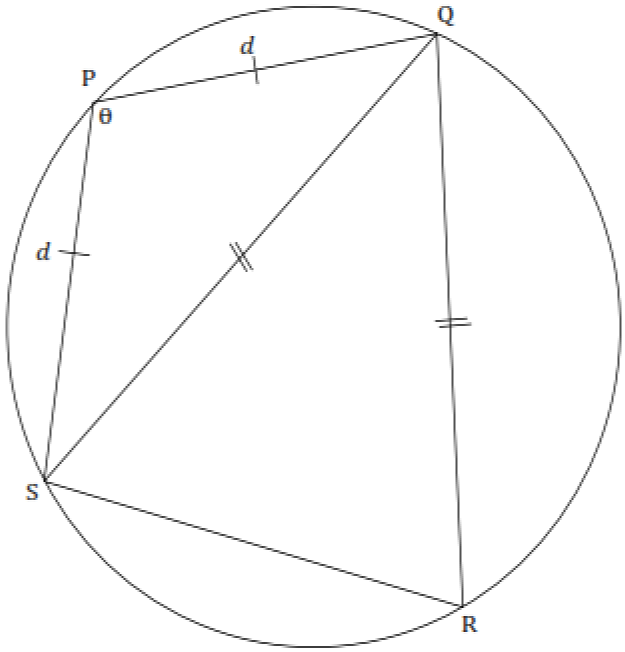
6.3.2 $f(x).\, g(x) > 0$? (2)

6.4 If the graph of g is moved 60° to the left, determine the equation of the new graph which is formed, in its simplest form. (2)

[11]

QUESTION 7

In the diagram, PQRS is a cyclic quadrilateral with QS = QR and PQ = PS = d units. $Q\hat{P}S = \theta$.



Use the diagram to prove that:

7.1. $QS = d\sqrt{2(1 - \cos\theta)}$ (2)

7.2 The area of $\Delta QRS = -d^2 \sin 2\theta\,(1 - \cos\theta)$ (3)

[5]

PREP 2019_MPUMALANGA

QUESTION 5

5.1 Simplify to one trigonometric ratio:

$$\frac{\sin(A-180°).\tan(180°-A).\cos A}{\cos(90°+A)}$$ (5)

5.2 If $\sin 28° = a$ and $\cos 32° = b$, determine the following in terms a and b.

5.2.1 $\cos 28°$ (2)

5.2.2 $\cos 64°$ (2)

5.2.3 $\sin 4°$ (4)

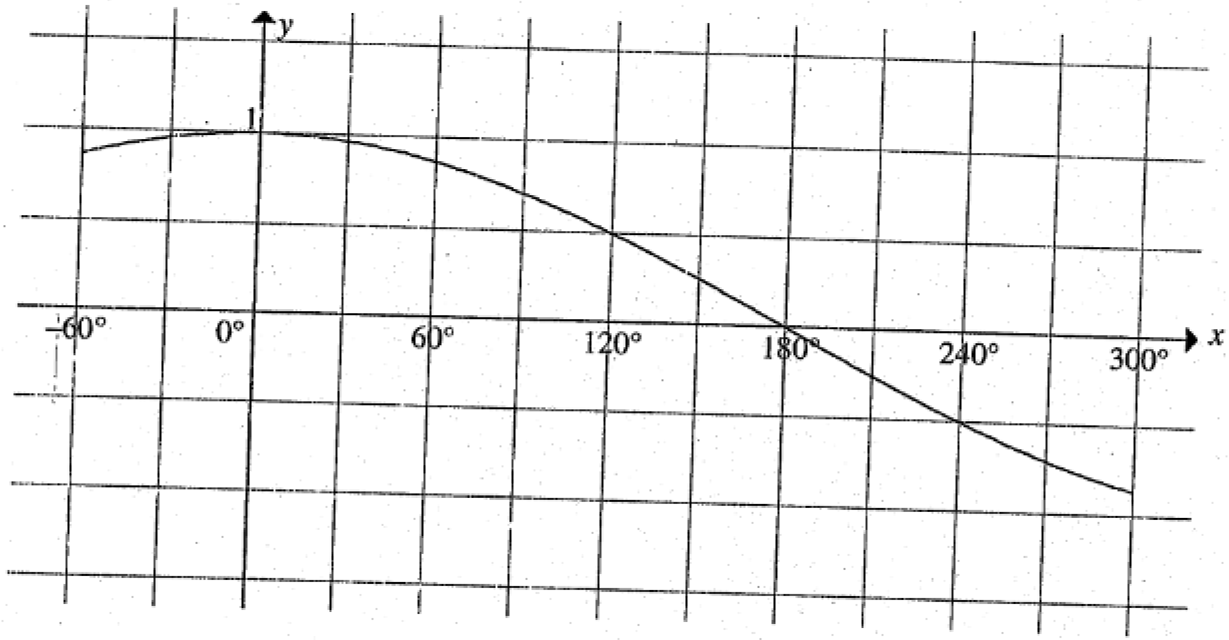
5.3 Prove the identity: $\frac{1}{1-\sin x}-\frac{1}{1+\sin x}=\frac{2\tan x}{\cos x}$ (5)

[18]

QUESTION 6

6.1 Solve for x if $\sin(x+60°)=\cos\frac{1}{2}x$ and $x\in[-60°;300°]$. (7)

6.2 In the diagram below, the graph of $f(x)=\cos\frac{1}{2}x$ is drawn for $x\in[-60°;300°]$.



6.2.1 On the grid provided IN THE ANSWER BOOK, draw the graph of $g(x)=\sin(x+60°)$ for $x\in[-60°;300°]$. Clearly show all maximum and minimum points, the intercepts with the axes and the endpoints. (3)

6.2.2 Use the solutions obtained in 6.1 as well as the graphs drawn in 6.2.1 to determine the value(s) of $x\in[-60°;300°]$ for which:

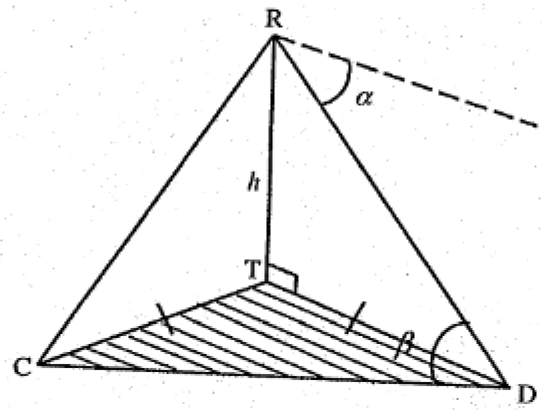
6.3.1 $f(x)<g(x)$ (3)

6.3.2 $f(x).g(x)\leq 0$ (3)

[16]

QUESTION 7

In the diagram, RT represents the height of a vertical tower. C and D represent two point equidistant from T and which lie on the same horizontal plane as T. The height of the tower i h. The angle of depression of D from R is α. $R\hat{D}C = \beta$.



7.1 Determine the size of $C\hat{R}D$ in terms of β. (2)

7.2 Prove that $CD = \dfrac{2h.\cos\beta}{\sin\alpha}$. (5)

7.3 Calculate the height of the tower, rounded off to nearest unit, if

$CD = 5{,}4$ units, $\alpha = 51°$ and $\beta = 65°$. (2)

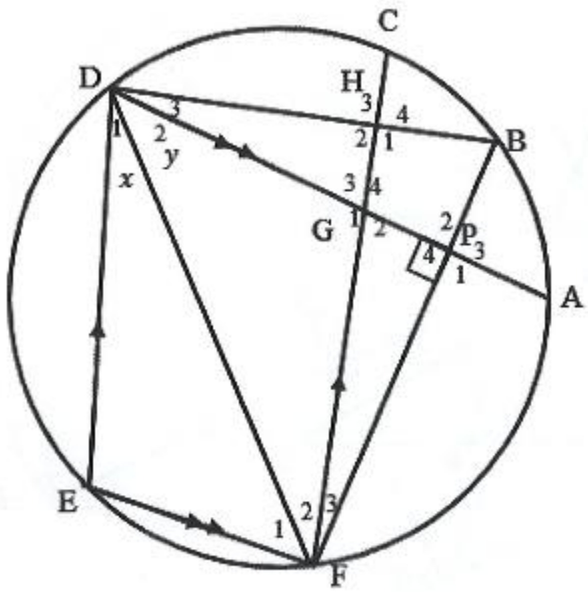
[9]

EUCLIDEAN GEOMETRY

PREP 2019_LIMPOPO

QUESTION 9

9.1 A, B, C, D, E and F are points on the circumference of the circle. ED ∥ FC and AD ∥ FE. DA ⊥ FB. FC intersects DA at G and DB at H.



Let $\hat{EDF} = x$ and $\hat{ADF} = y$.

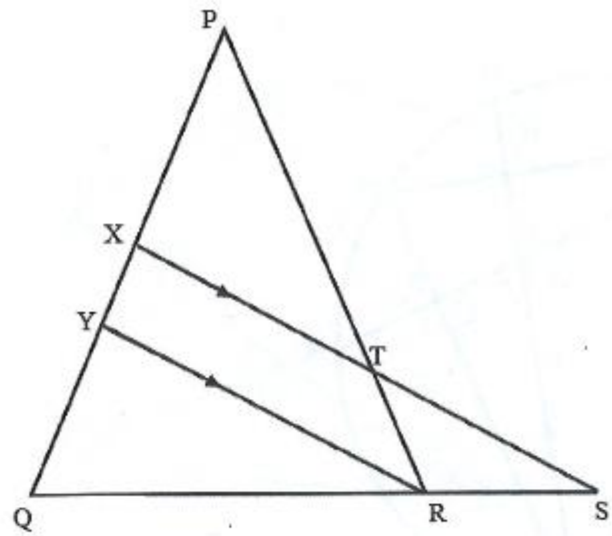
Prove that:

9.1.1 $\hat{B} = x + y$ (4)

9.1.2 $\hat{H}_1 = 90°$ (6)

9.2 In the diagram QR of Δ PQR is produced such that XS $\parallel$ YR.

$\frac{PX}{XQ} = \frac{2}{5}$ and $\frac{PT}{TR} = \frac{3}{2}$.

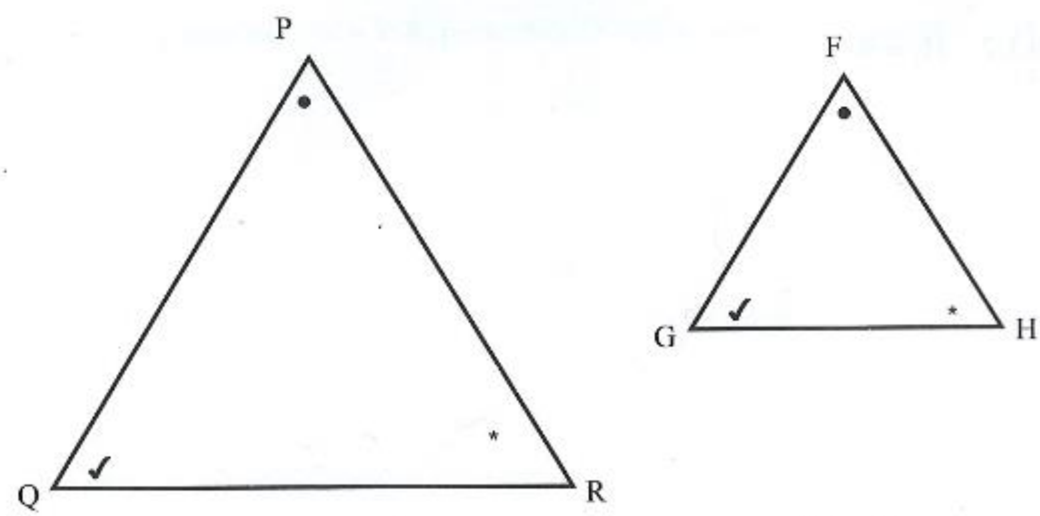


Calculate, giving reasons, the value of $\frac{QR}{RS}$. (6)

[16]

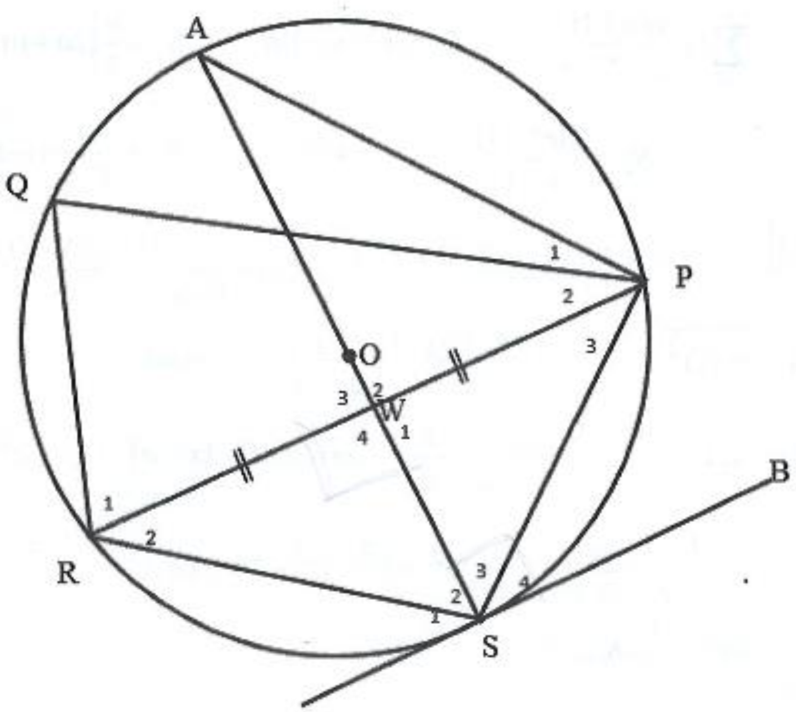
QUESTION 10

10.1 ΔPQR and Δ FGH are given. $\hat{P} = \hat{F}, \hat{Q} = \hat{G}$ and $\hat{R} = \hat{H}$.



Prove the theorem which states that if $\Delta PQR \parallel\!\mid \Delta\ FGH$, then $\frac{PQ}{FG} = \frac{PR}{FH}$. (6)

10.2 P, A, Q, R and S lie on the circle with centre O. SB touches the circle at S and RW = WP. AS and RP are straight lines.



Prove that:

10.2.1 $SB \parallel RP$ (5)

10.2.2 $\Delta\, APS\ |||\ \Delta\, RWS$ (4)

10.2.3 $RS^2 = WS.AS$ (4)

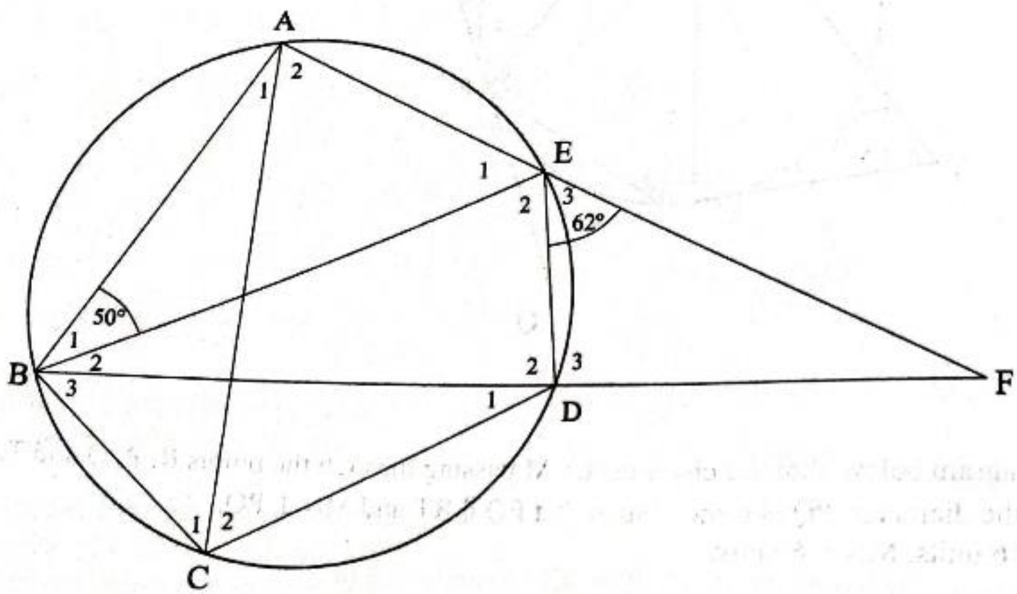
10.2.4 $AS = \dfrac{RW^2}{WS} + WS$ (4)

[23]

PREP 2019_EASTERN CAPE

QUESTION 8

8.1 In the given diagram, A, B, C, D and E are points on the circle.
BE is a diameter. $\hat{E}_3 = 62°$ and $\hat{B}_1 = 50°$.
BD produced meets AE produced at F.



Determine, with reasons:

8.1.1 $B\hat{A}E$ (2)

8.1.2 $\hat{E}_1$ (2)

8.1.3 $\hat{C}_1$ (2)

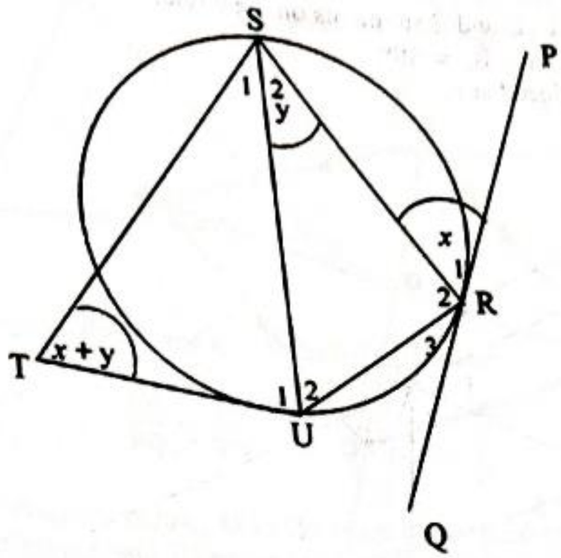
8.1.4 $\hat{C}_2$ (2)

8.1.5 $A\hat{B}D$ (2)

8.2 Complete the following theorem statement:

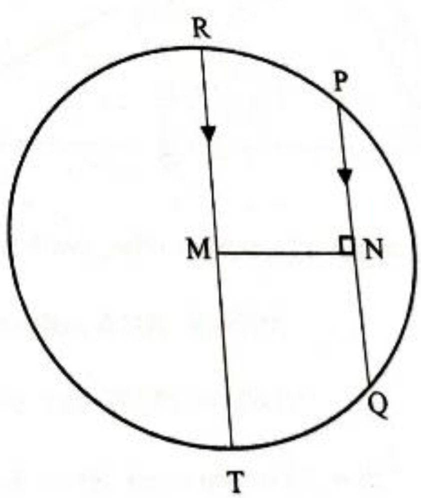
The angle between the tangent to a circle and the chord drawn from the point of contact is … (1)

8.3 In the diagram below, PRQ is a tangent to the circle SUR at R. SU, SR and UR are drawn. Lines from S and U produced meet at T outside the circle.
$\hat{R}_1 = x$; $\hat{S}_2 = y$ and $S\hat{T}U = x + y$



Prove that STUR is a cyclic quadrilateral. (5)

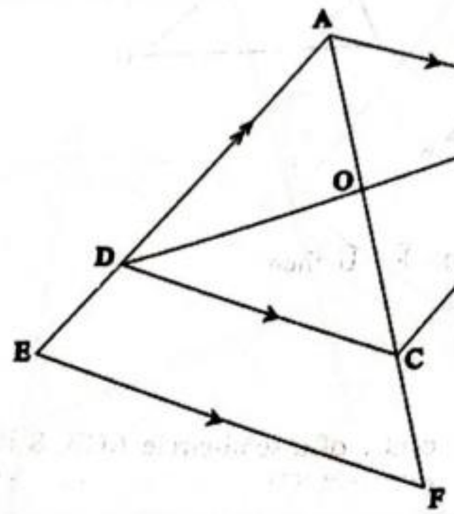
8.4 The diagram below shows a circle centre M passing through the points R, P, Q and T. RT is the diameter. PQ is a chord such that PQ ∥ RT and MN ⊥ PQ.
PQ = 16 units, MN = 6 units.



Determine the length of RT.

QUESTION 9

In the diagram below, ABCD is a parallelogram. AD and AC are produced to E and F respectively so that EF || DC. AF and DB intersect at O.
AD = 12 units; DE = 3 units; DC = 14 units; CF = 5 units.



9.1 Calculate, giving reasons, the length of:

9.1.1 AC (3)

9.1.2 AO (1)

9.1.3 EF (3)

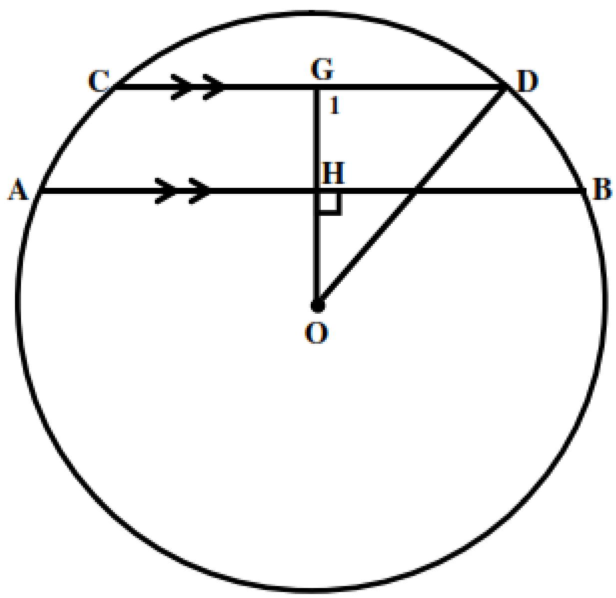
9.2 Prove that $\frac{\text{area } \Delta ADO}{\text{area } \Delta AEF} = \frac{8}{25}$ (3)

[10]

PREP 2019_GAUTE

QUESTION 9

9.1 In the diagram, O is the centre of the circle with AB || CD and OH ⊥ AB, AB = 24 cm, CD = 10 cm and OD = 13 cm.



9.1.1 Give reasons for the statements below.

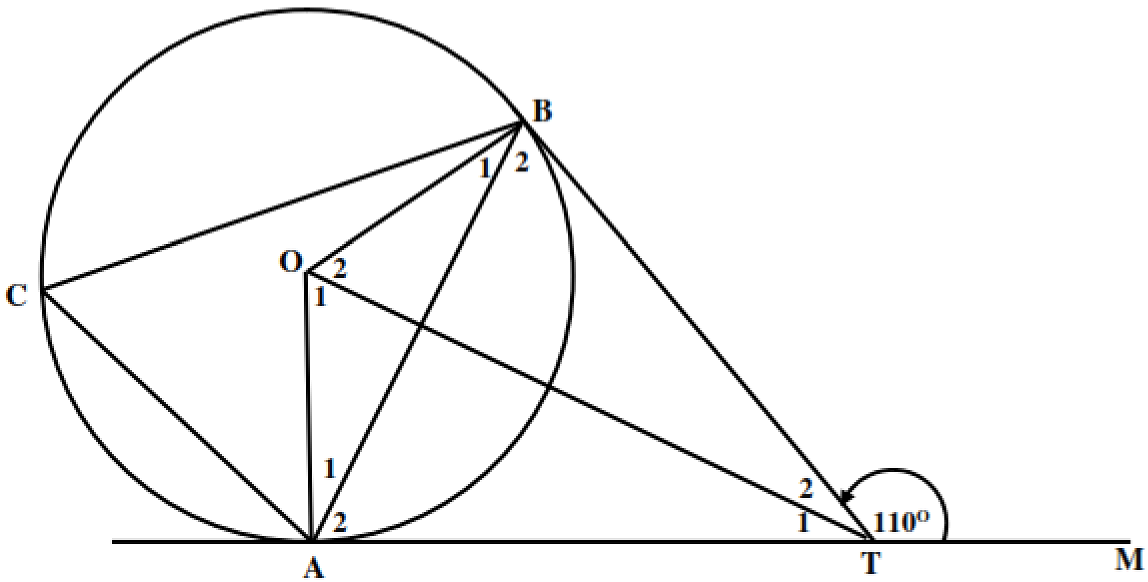
(a) $\hat{G}_1 = 90°$

(b) AH = 12

(c) OB = 13 (3)

9.1.2 Calculate the length of GH. (5)

9.2 In the diagram, O is the centre of circle ACB. TA and TB are tangents to the circle at A and B respectively. O and T are joined. $B\hat{T}M = 110^\circ$.



9.2.1 Prove that AOBT is a cyclic quadrilateral. (3)

9.2.2 Give a reason why $\hat{T}_1 = \hat{T}_2$. (2)

9.2.3 Calculate $\hat{C}$. (4)

[17]

Let $\hat{N}_2 = x$, $\hat{P}_2 = y$ and $\hat{M}_2 = z$.

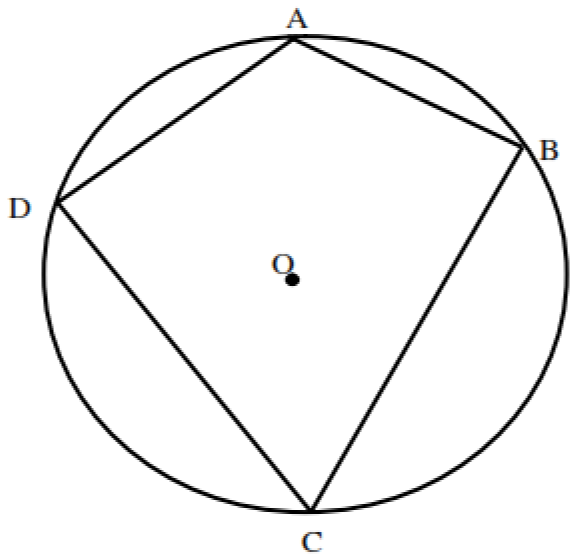
10.2.1 Write down the sizes of $\hat{A}$, $\hat{B}$ and $\hat{C}$ in terms of x, y and z. (2)

10.2.2 Hence, calculate the value of $\hat{A} + \hat{B} + \hat{C}$. (4)

[11]

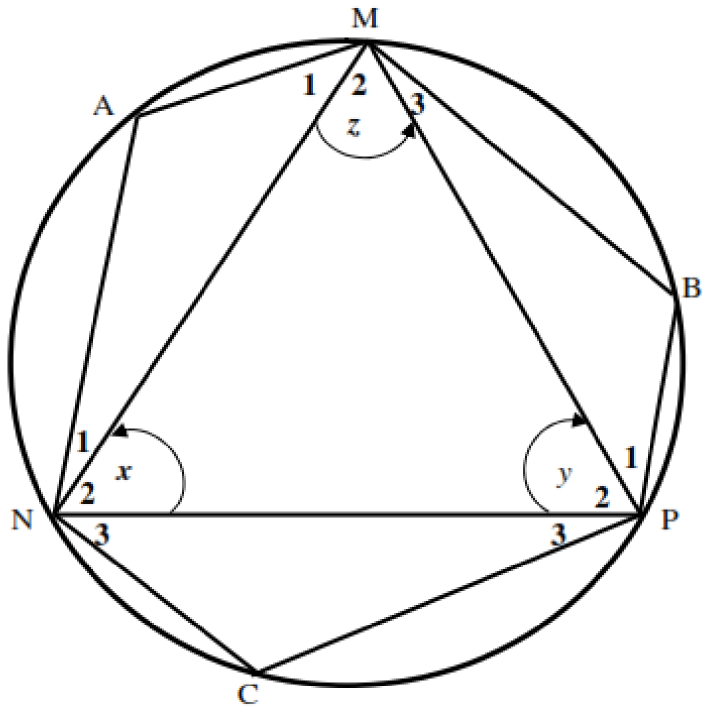
QUESTION 10

10.1 In the diagram, O is the centre of the circle. A, B, C and D are points on the circumference of the circle. Use Euclidean Geometry methods to prove the theorem which states that $\hat{A} + \hat{C} = 180°$. (5)



10.2 In the diagram, the circle passes through A, M, B, P, C and N.

10.2 In the diagram, the circle passes through A, M, B, P, C and N.



Let $\hat{N}_2 = x$, $\hat{P}_2 = y$ and $\hat{M}_2 = z$.

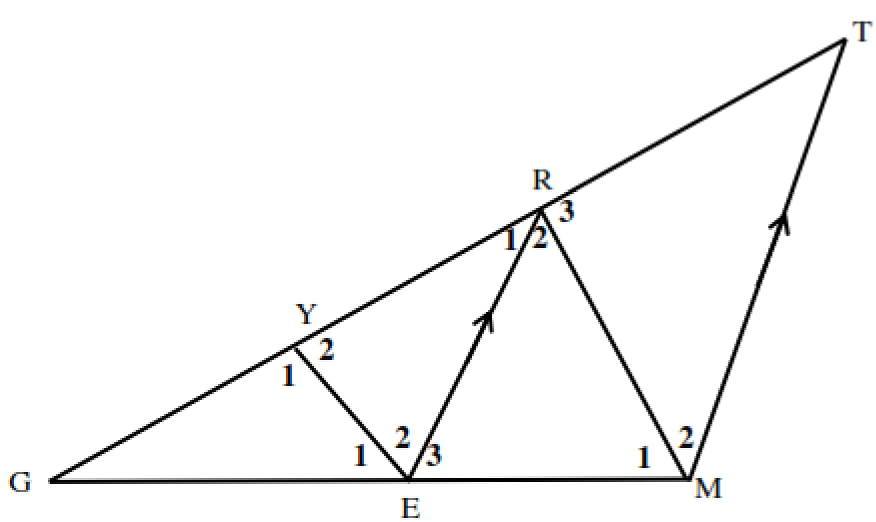
10.2.1 Write down the sizes of $\hat{A}$, $\hat{B}$ and $\hat{C}$ in terms of x, y and z. (2)

10.2.2 Hence, calculate the value of $\hat{A} + \hat{B} + \hat{C}$. (4)

[11]

QUESTION 11

In $\triangle RGM$, $\hat{E}_1 = \hat{R}_1 = \hat{R}_2$. T lies on GR produced so that TM || RE.

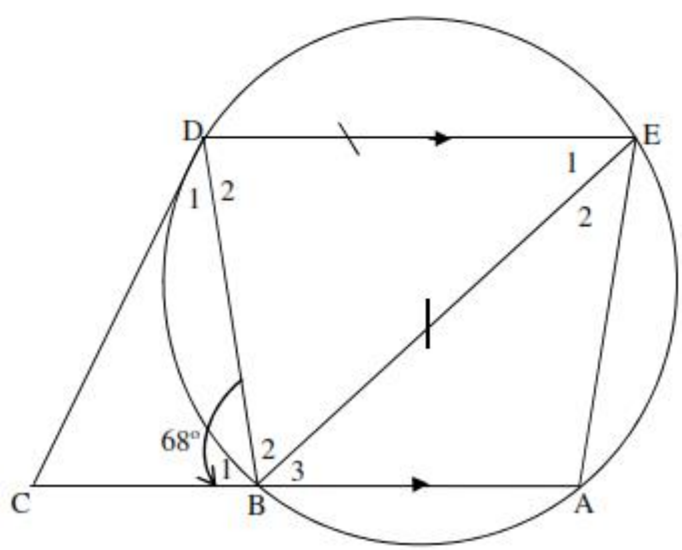


11.1	Give with reasons, TWO **other** angles which are equal to $\hat{R}_1$.	(4)
11.2	Prove that $\frac{EM}{EG} = \frac{RM}{RG}$.	(4)
11.3	Prove that $\Delta GYE \parallel\!\mid \Delta GER$.	(4)
11.4	Hence, prove that $\frac{EG}{EY} = \frac{RG}{RE}$.	(1)
11.5	If it is further given that $G\hat{R}M = 90^\circ$, RM = 6 and GM = 10. Calculate the lengths of:	
	11.5.1 RG	(2)
	11.5.2 GE	(4) **[19]**

PREP 2019_NORTH WEST

QUESTION 8

In the diagram below, BAED is a cyclic quadrilateral withBA || DE. BE = DE and $D\hat{B}C = 68^\circ$. The tangent to the circle at D meets AB produced to C.



8.1 Calculate, with reasons, the size of:

8.1.1 $D\hat{E}A$ (2)

8.1.2 $\hat{A}$ (1)

8.1.3 $\hat{D}_2$ (2)

8.1.4 $\hat{B}_2$ (1)

8.1.5 $\hat{D}_1$ (3)

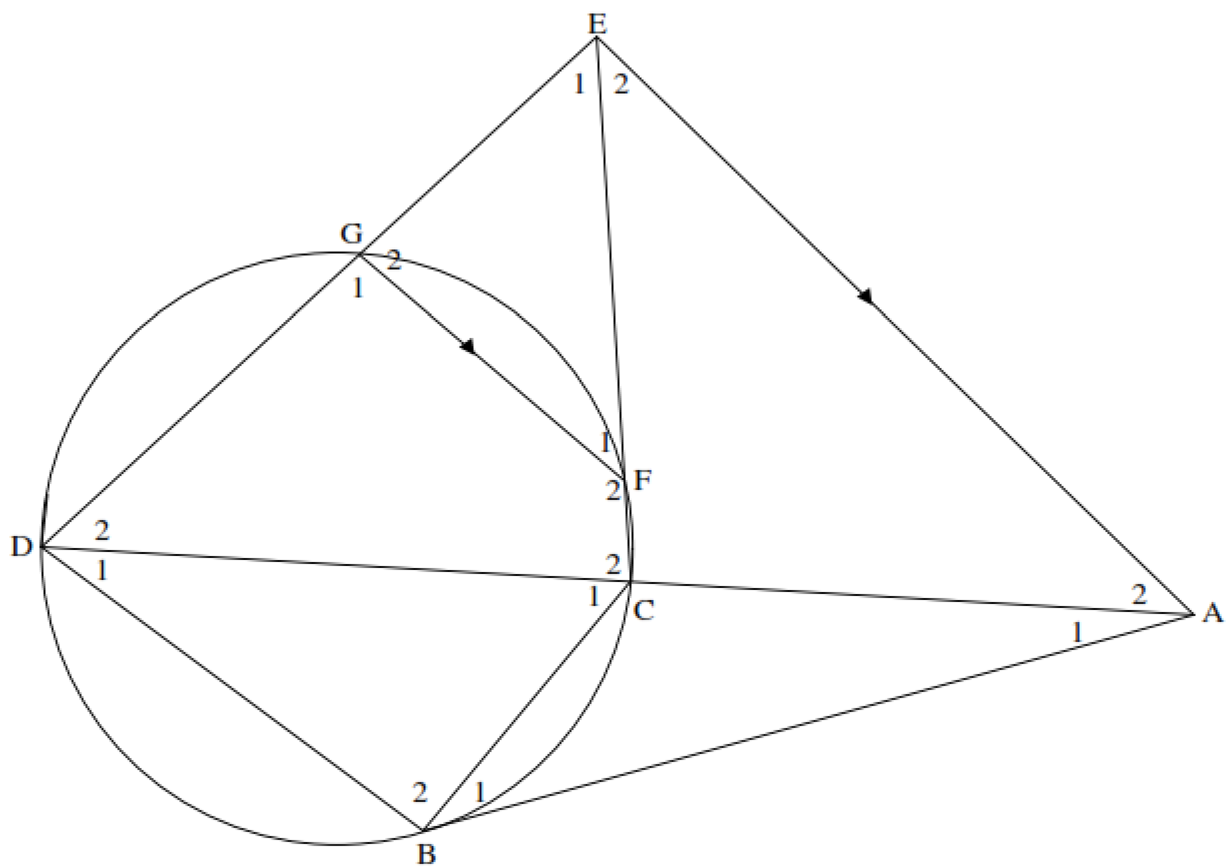
8.2 Prove that ΔBDC is isosceles. (2)

8.3 Prove that DE is a tangent to the circle that passes through the points C, B and D at D. (2)

[13]

QUESTION 9

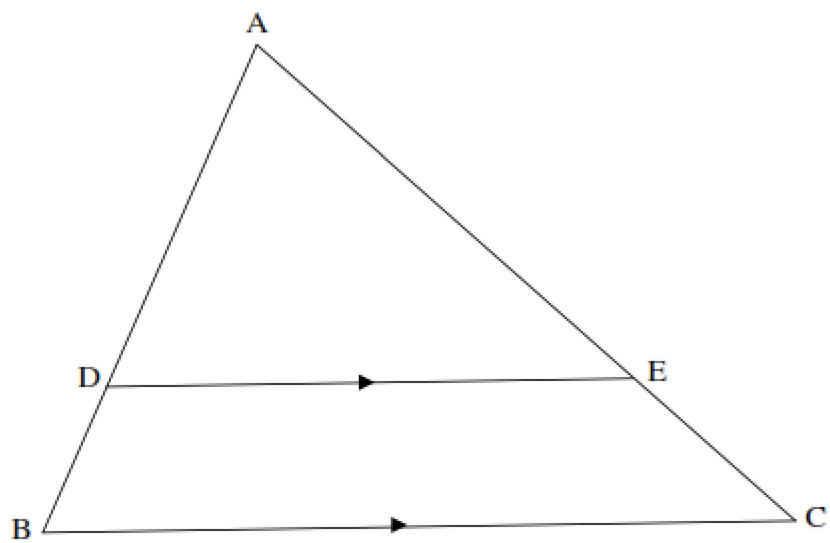
In the diagram,DGFC is a cyclic quadrilateral and AB is a tangent to the circle at B. Chords DB and BC are drawn. DG producedand CF produced meetin E and DC is produced to A. EA | | GF



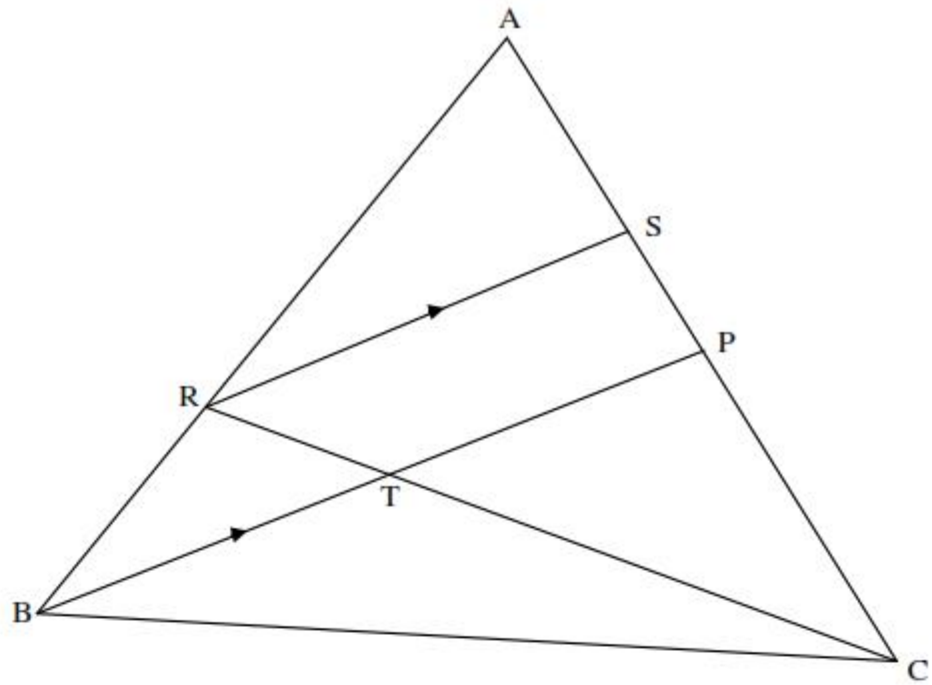
9.1	Give a reason why $\hat{B}_1 = \hat{D}_1$.	(1)
9.2	Prove $\Delta ABC \mid\mid\mid \Delta ADB$.	(3)
9.3	Prove $\hat{E}_2 = \hat{D}_2$	(4)
9.4	Prove AE= $\sqrt{AD \times AC}$.	(5)
9.5	Hence, show that AE = AB.	(3)
		[16]

QUESTION 10

10.1 In ΔABC below, D and E are points on AB and AC respectivelysuch thatDE | | BC. Prove the theorem which states that $\frac{AD}{DB} = \frac{AE}{EC}$. (6)



10.2 In the diagram below, P is the midpoint of AC in ΔABC. R is a point on AB such that RS || BP and $\frac{AR}{AB}=\frac{3}{5}$. RC intersects BP in T.



Determine, with reasons, the following ratios:

10.2.1 $\frac{AS}{SC}$ (4)

10.2.2 $\frac{RT}{TC}$ (3)

10.2.3 $\frac{\text{Area of } \Delta RAS}{\text{Area of } \Delta RSC}$ (2)

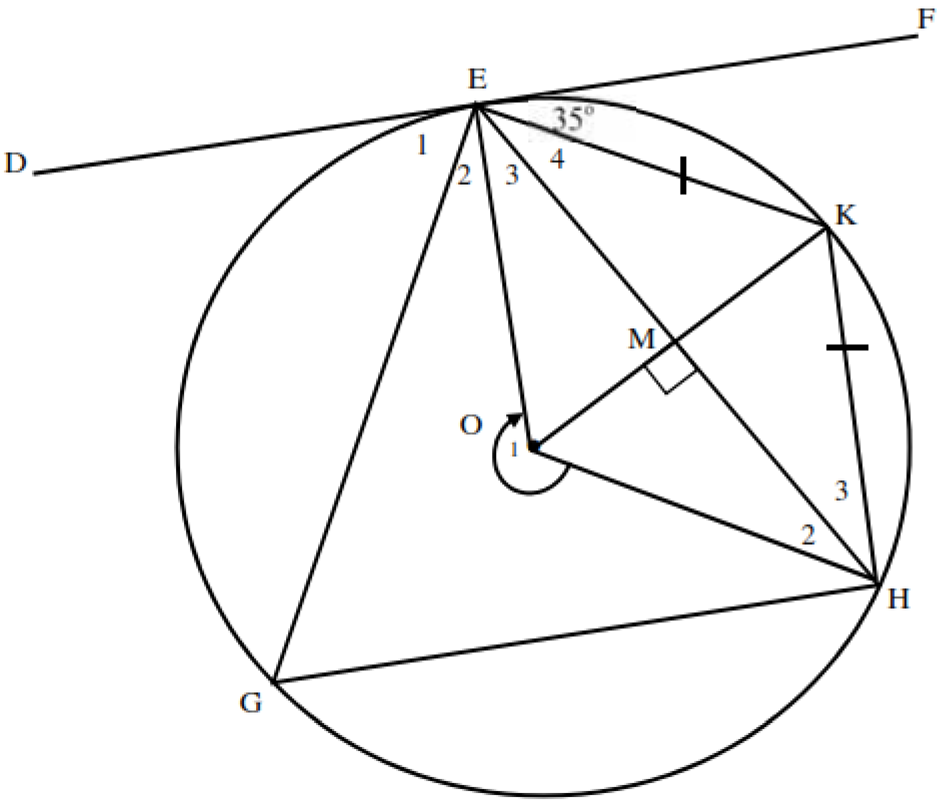
10.2.4 $\frac{\text{Area of } \Delta TPC}{\text{Area of } \Delta RSC}$ (3)

[18]

PREP 2019_FREE STATE

QUESTION 8

DF is a tangent to the circle at E. EKHG is a cyclic quadrilateral. $K\hat{E}F = 35^\circ$.
O is the centre of the circle. $OK \perp EH$ and $EK = HK$.



8.1 Determine, with reasons, the size of each of the following:

8.1.1 $\hat{E}_4$ (3)

8.1.2 $E\hat{K}H$ (2)

8.1.3 $\hat{G}$ (2)

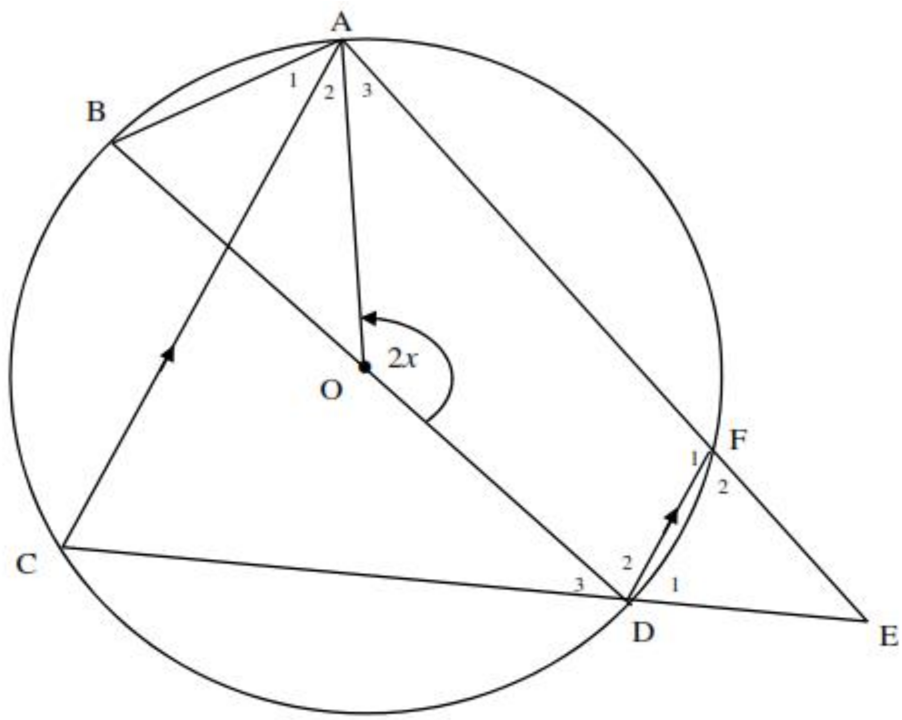
8.1.4 $\hat{O}_1$ (2)

8.2 It is further given that EH = 24 units. KM = 4 units and the radius of the circle EKHG is x. Determine the value of x. (4)

[13]

QUESTION 9

9.1 A circle with centre O is given below. Lines CD and AF are produced to E.
$A\hat{O}D = 2x$ and BD is the diameter. AC||FD.



9.1.1 Determine, with reasons, four other angles that are each equal to x. (6)

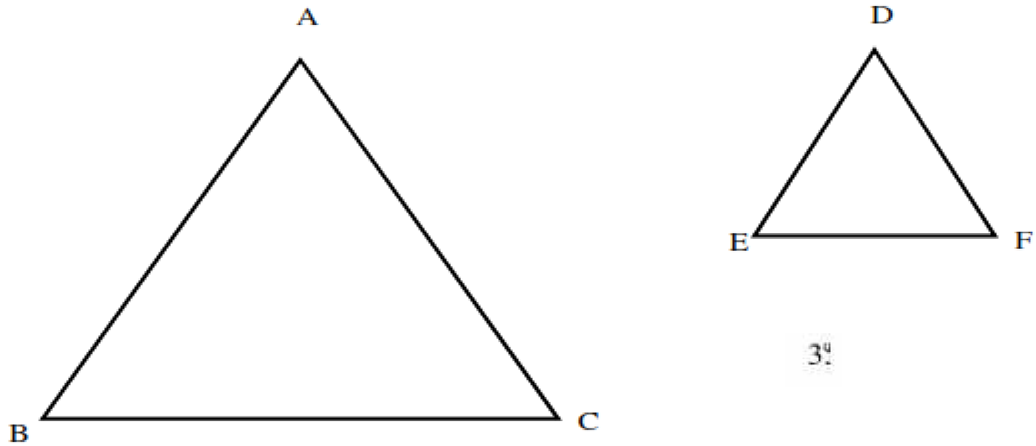
9.1.2 Express $\hat{E}$ in terms of x. (2)

9.1.3 Prove that AODE is a cyclic quadrilateral. (2)

9.2 It is further given that ED : DC = 8 : 12 and FE = 10. Calculate the length of AF. (3)
[13]

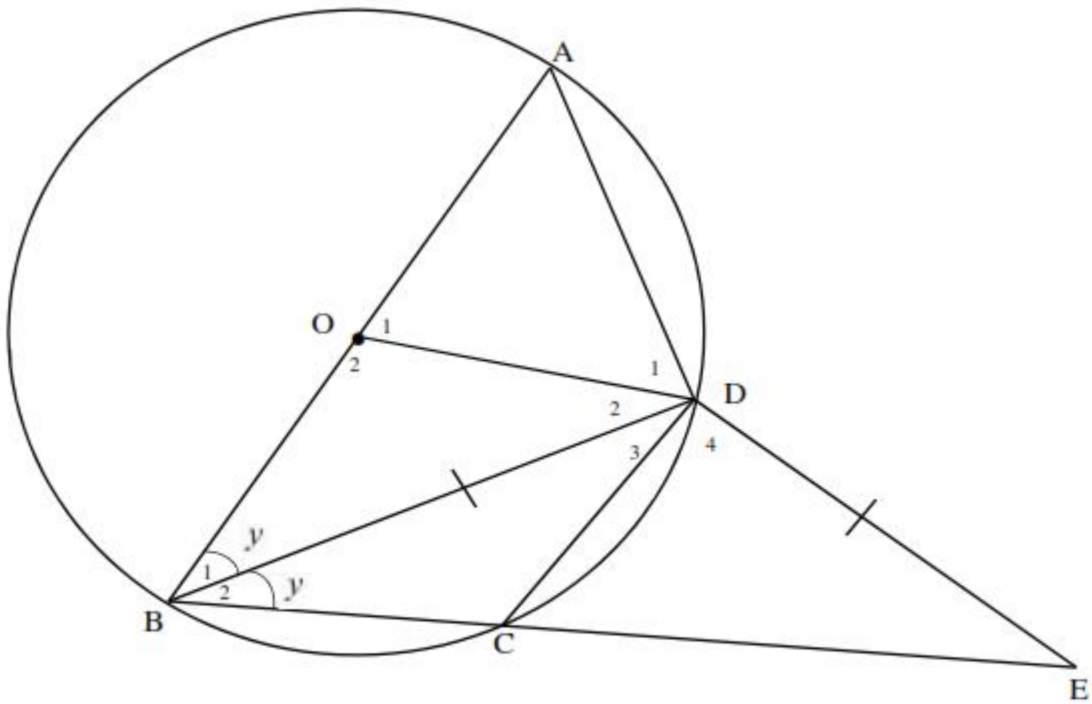
QUESTION 10

10.1 In the diagram below, ΔABC and ΔDEF are drawn with $\hat{A} = \hat{D}$; $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.



Prove the theorem that states that if two triangles are similar, then the sides are proportional, i.e. $\frac{DE}{AB} = \frac{EF}{BC} = \frac{DF}{AC}$. (5)

10.2 In the diagram below, AB is the diameter of a circle with centre O. BD and BC are chords. BD = DE. BCE is a line. $\hat{B}_1 = \hat{B}_2 = y$.



Prove that:

10.2.1 $\hat{D}_4 = 90^\circ$ (5)

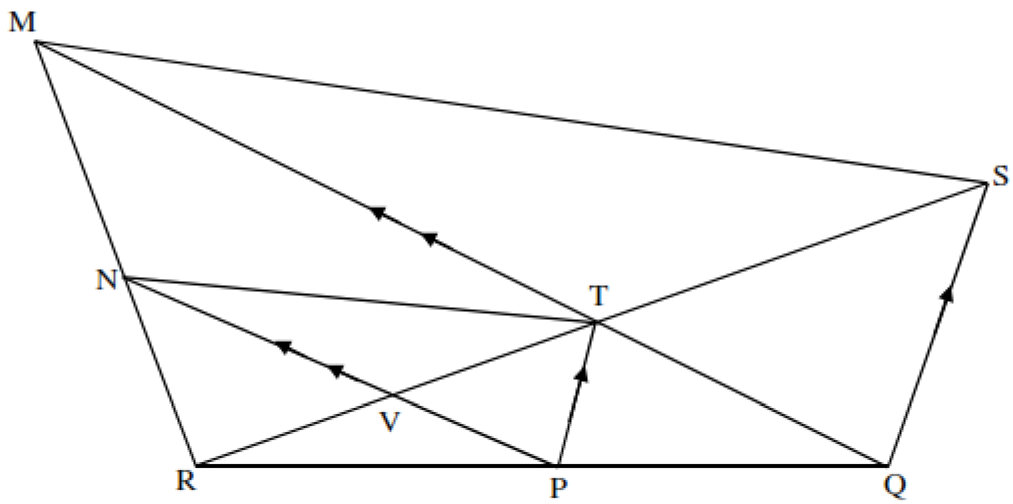
10.2.2 $\Delta BOD \,|||\, \Delta BDE$ (3)

10.2.3 $DE^2 = BE.OD$ (4)

[17]

QUESTION 11

In the diagram below, RQSM is a quadrilateral. N and P are points on MR and RQ respectively such that MQ || NP. The diagonals intersect at T. P is a point on RQ such that TP || SQ. TR and NP intersect at V.



11.1 Prove that NT || MS. (4)

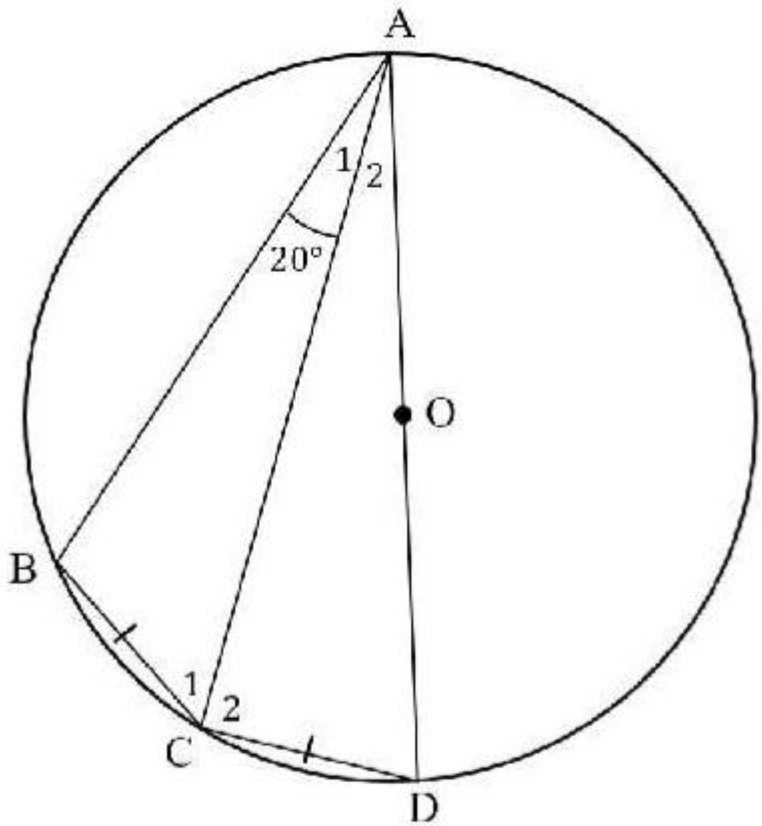
11.2 If $RN = \frac{3}{5}NM$ and RS = 32, determine VT. (4)

[8]

PREP 2019_WESTERN CAPE

QUESTION 8

8.1 In the diagram, AOD is a diameter of the circle with centre at O.
BC = CD and $\hat{A}_1 = 20°$.



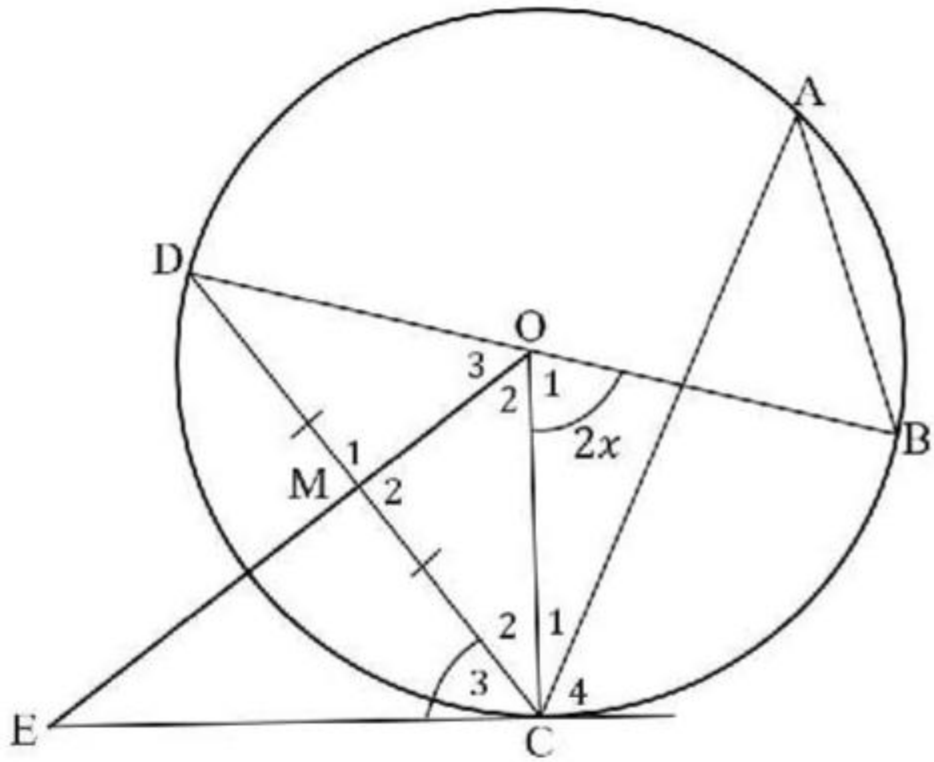
Determine, with reasons, the size of each of the following angles:

8.1.1 $\hat{A}_2$ (2)

8.1.2 $\hat{C}_2$ (2)

8.1.3 $A\hat{B}C$ (3)

8.2 O is the centre of the circle in the diagram and EC is a tangent to the circle at C.
$DM = MC$ and OME is a straight line.
Let $\hat{O}_1 = 2x$.



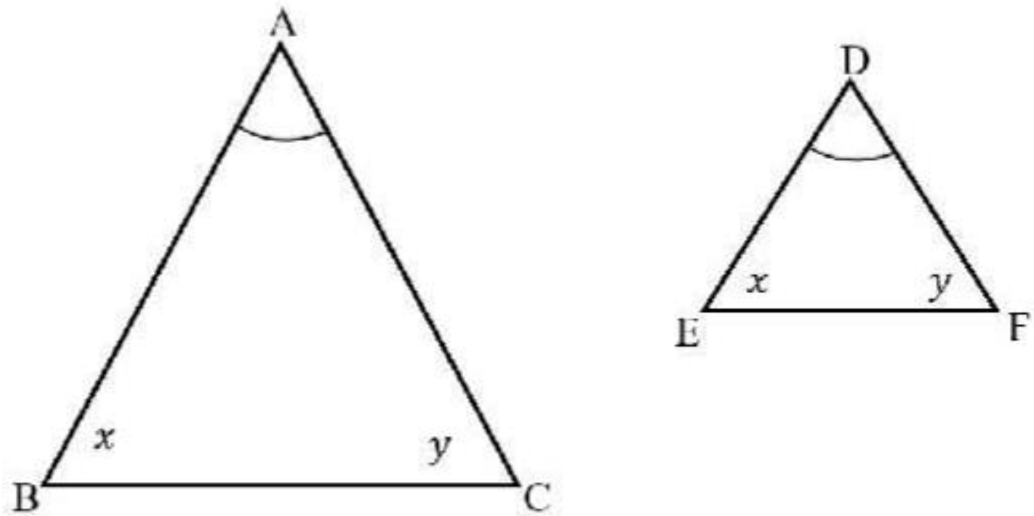
8.2.1 Give, with reasons, THREE angles equal to x. (6)

8.2.2 Prove that $\hat{O}_2 = 90° - x$ (3)

8.2.3 Prove that DOCE is a cyclic quadrilateral. (4)
[20]

QUESTION 9

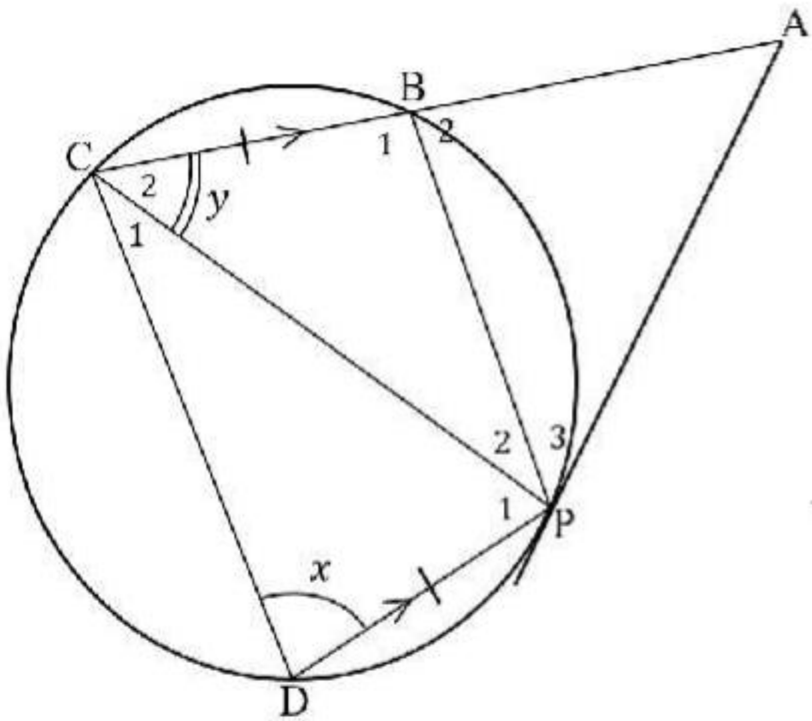
9.1 In the diagram, ΔABC and ΔDEF are drawn such that $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.



Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, that is $\frac{D}{A} = \frac{D}{A}$

(6)

9.2 In the diagram is AP a tangent to the circle at P. CB ∥ DP and CB = DP. CBA is a straight line.
Let $\hat{D} = x$ and $\hat{C}_2 = y$.



Prove, with reasons that:

9.2.1 ΔAPC ||| Δ ABP (4)

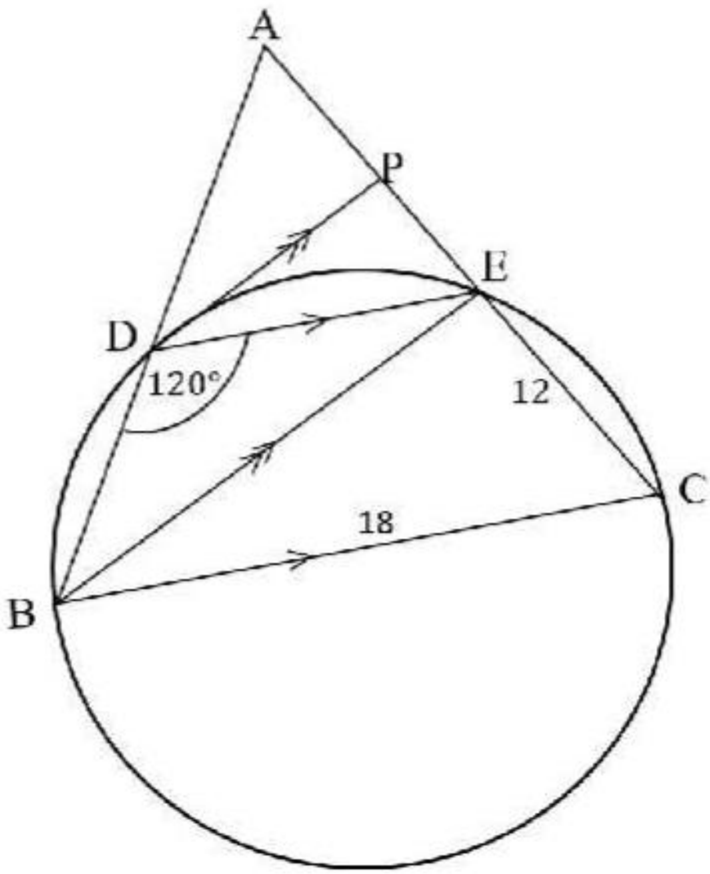
9.2.2 $AP^2 = AB.AC$ (1)

9.2.3 ΔAPC ||| Δ CDP (4)

9.2.4 $AP^2 + PC^2 = AC^2$ (4)
[19]

QUESTION 10

In ΔA in the diagram, D is a point on AB such that AD : DB = 5 : 4.
P and E are points on AC such that DE ∥ BC and DP ∥ BE.
BC is NOT a diameter of the circle.
Given: $B\hat{D}E = 120°$, $EC = 12$ units and $BC = 18$ units.



10.1 Determine, with reasons:

10.1.1 The length of AE (3)

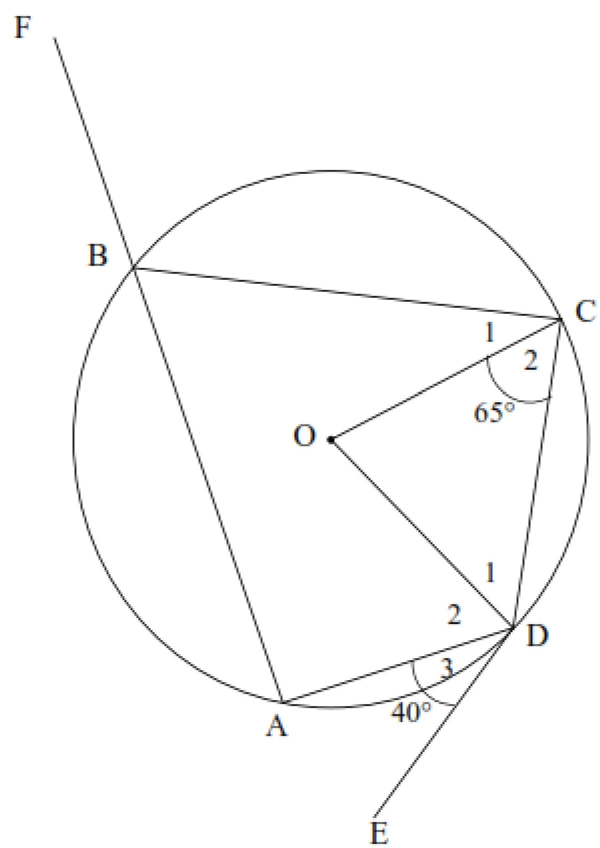
10.1.2 $\dfrac{\text{area of } \Delta\ AEB}{\text{area of } \Delta\ ECB}$ (2)

10.2 Hence, determine the length of DP if ΔADP ||| ΔABE (6)
[11]

PREP 2019_KZN

QUESTION 8

8.1 In the diagram, ABCD is a cyclic quadrilateral in the circle centered at O. ED is a tangent to the circle at D. Chord AB is produced to F. Radii OC and OD are drawn. $A\hat{D}E = 40°$ and $\hat{C}_2 = 65°$,

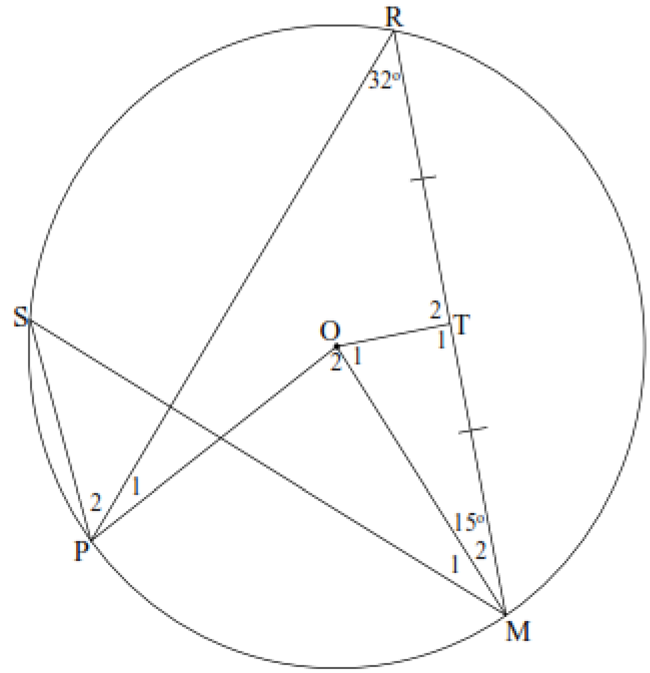


Determine, giving reasons, the size of each of the following angles:

8.1.1 $\hat{D}_2$ (3)

8.1.2 $F\hat{B}C$ (4)

8.2 In the diagram, O is the centre of the circle RMPS. OT bisects RM with T a point on RM. $P\hat{R}M = 32°$. SP, SM and radii OP and OM are drawn. $O\hat{M}T = 15°$.



Calculate, with reasons, the size of the angles:

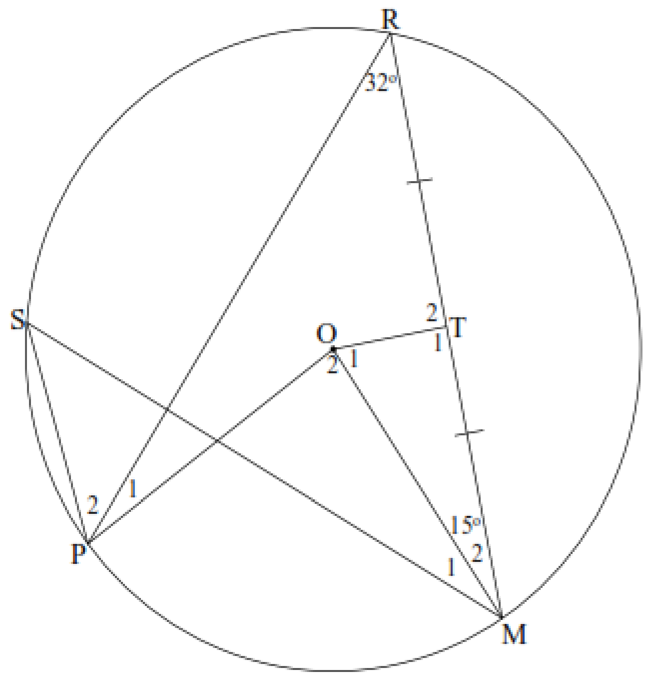
8.2.1 $\hat{S}$ (2)

8.2.2 $\hat{O}_2$ (2)

8.2.3 $\hat{O}_1$ (3)

[14]

8.2 In the diagram, O is the centre of the circle RMPS. OT bisects RM with T a point on RM. $P\hat{R}M = 32°$. SP, SM and radii OP and OM are drawn. $O\hat{M}T = 15°$.



Calculate, with reasons, the size of the angles:

8.2.1 $\hat{S}$ (2)

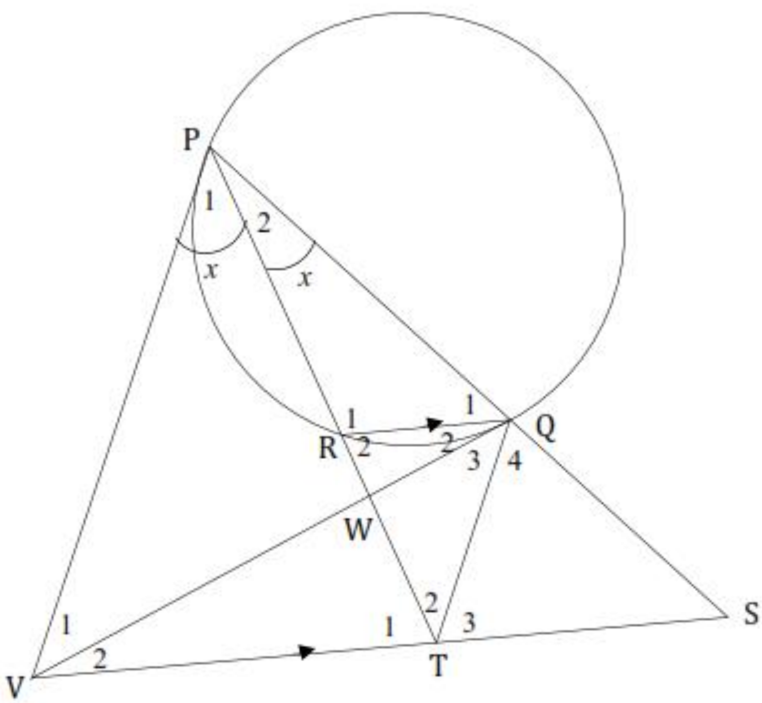
8.2.2 $\hat{O}_2$ (2)

8.2.3 $\hat{O}_1$ (3)

[14]

QUESTION 9

In the diagram, PV and VQ are tangents to the circle at P and Q. PQ is produced to S and chord PR is produced to T such that VTS || RQ. VQ and RT intersect at W. $\hat{P}_1 = \hat{P}_2 = x$.



Prove that:

9.1 $\hat{S} = x$ (4)

9.2 PQTV is a cyclic quadrilateral (5)

9.3 TQ is a tangent to the circle passing through Q, W and P (3)

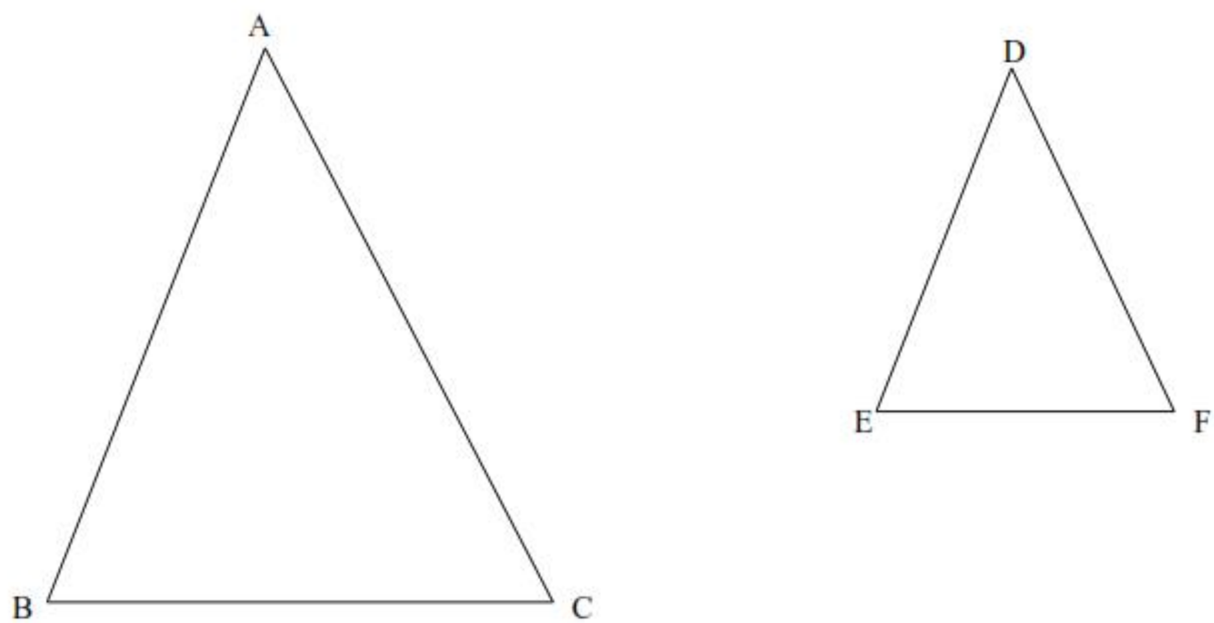
[12]

QUESTION 10

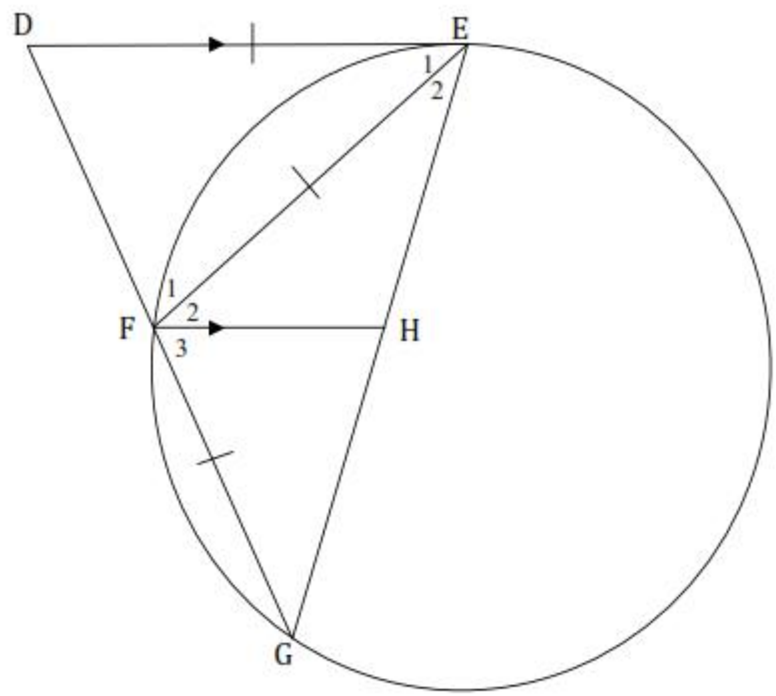
10.1 In the diagram below, ΔABC and ΔDEF are drawn with $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.

Prove the theorem that states that equiangular triangles are similar and therefore

$\frac{AB}{DE} = \frac{AC}{DF}$. (7)



10.2 In the diagram, DE is a tangent to the circle at E and DFG is a secant intersecting the circle at F and G. DE = EF = FG. H is a point on EG such that FH || DE.



10.2.1 Determine, giving reasons, 3 angles each equal to $D\hat{E}F$. (4)

10.2.2 Prove that:

a) $\Delta DEF \mid\mid\mid \Delta DGE$ (3)

b) $\hat{D} = 72^0$. (5)

10.2.3 If it is further given that DF = k units and FG = 2 units, prove that $k^2 + 2k = 4$. (3)

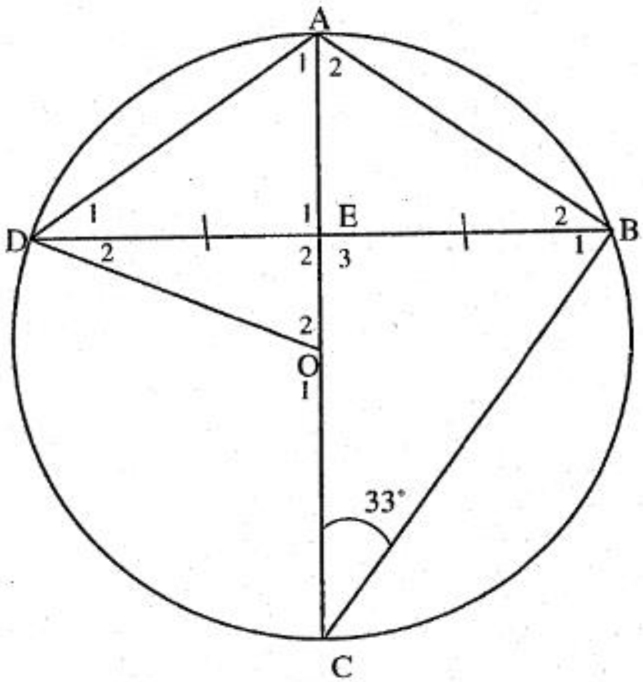
10.2.4 Determine, giving reasons, the ratio of $\frac{GH}{GE}$ in terms of k. (2)

[24]

PREP 2019_MPUMALANGA

QUESTION 8

In the diagram, AC is the diameter of the circle with centre O. AC and chord DB intersect at E such that DE = EB. Chords AB, BC and AD and radius OD are drawn. $A\hat{C}B = 33°$.



Determine the size of:

8.1 $\hat{D}_1$ (2)

8.2 $\hat{A}_1$ (3)

8.3 $\hat{O}_1$ (2)

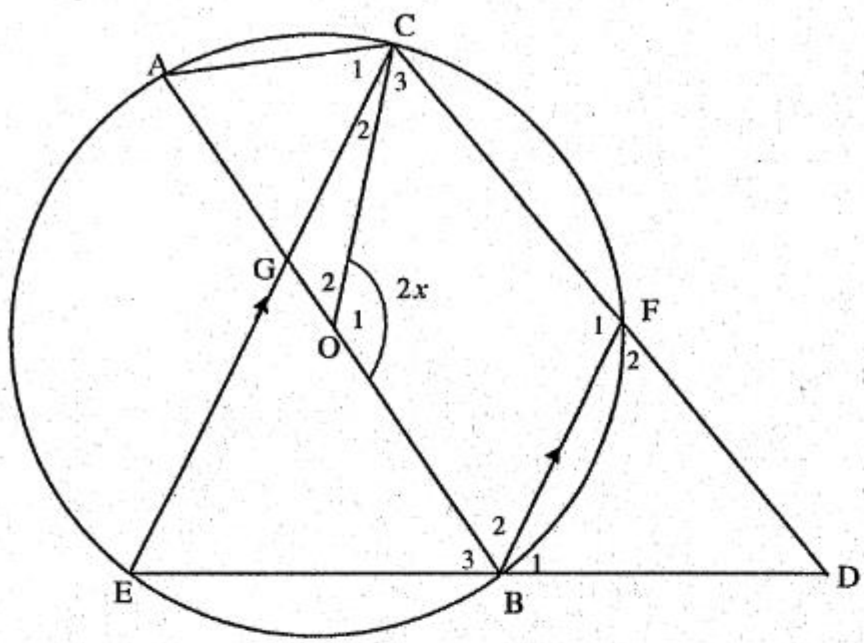
8.4 $\hat{D}_2$ (2)

8.5 $\hat{A}_2$ (3)

[12]

QUESTION 9

In the diagram below, AB passes through the centre O of the circle. Chords CF and EB are produced to meet at D. $EC \parallel BF$. Chord AC and radius OC are drawn. Let $\hat{O}_1 = 2x$.



9.1 Determine, in terms of x, the size of $\hat{F}_2$. (3)

9.2 Prove that DF = BD. (4)

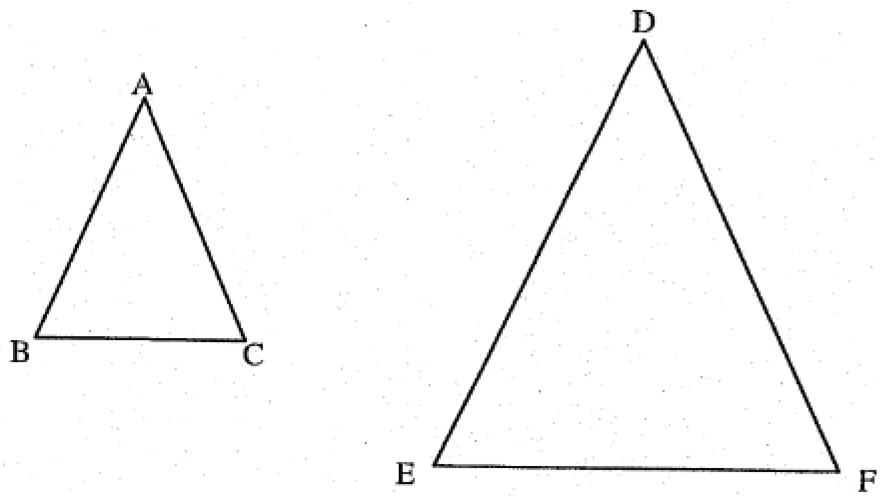
9.3 Show that $\hat{C}_1 = \hat{C}_3$. (4)

9.4 If it is further given that CG = FB, FD = 1 and BG = 2, determine the value of $\frac{BD}{ED}$. (5)

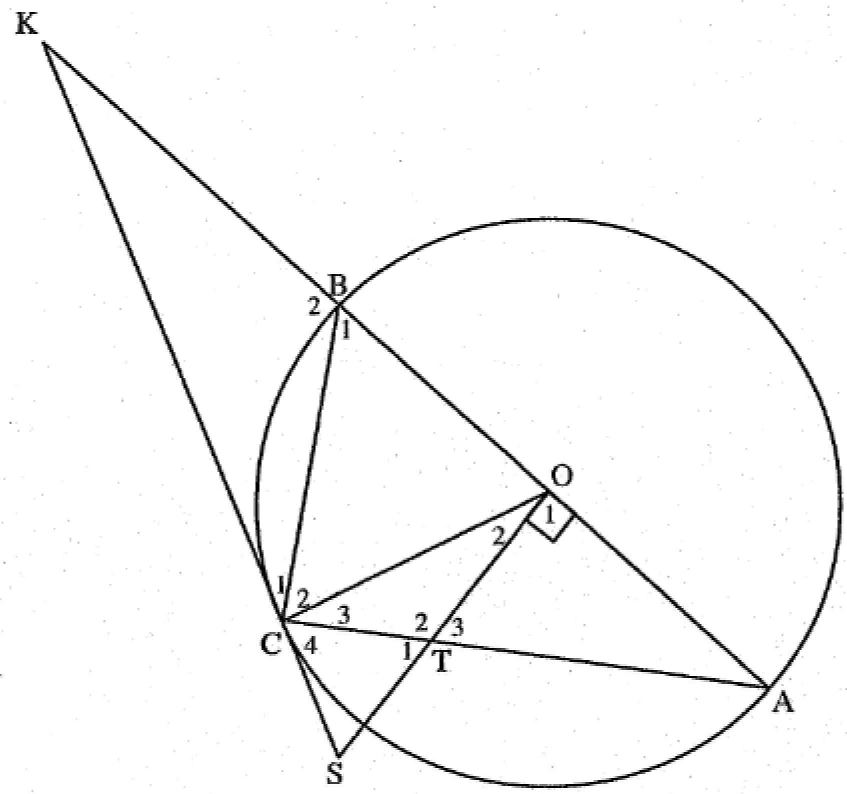
[16]

QUESTION 10

10.1 In the diagram below, ΔABC and ΔDEF are given with $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$. Proof the theorem that states that $\frac{AB}{DE} = \frac{AC}{DF}$. (6)



10.2 In the diagram, AB is the diameter of the circle with centre O. AB is produced to K such that SK is a tangent to the circle at C. $SO \perp AB$. CA and SO intersect at T.



Prove that:

10.2.1 $\Delta CKB \parallel\!\mid \Delta AKC$ (3)

10.2.2 $K\hat{C}T = \hat{T}_2$ (4)

10.2.3 $\Delta COT \parallel\!\mid \Delta AKC$ (3)

10.2.4 $BK.AK = \dfrac{OT^2.CA^2}{CT^2}$ (4)

[20]