Grade 12 Mathematics: Question Paper 1

MARKS: 150 **TIME: 3 hours**

QUESTION 1

1.1 Solve for x:

1.1.1 $\log_3 x = 2$ (1)

1.1.2 $10^{\log 27} = x$ (1)

1.1.3 $3^{2x-1} = 27^{2x-1}$ (2)

1.2 Determine the value of the following expression: $\sum_{i=3}^{7} 2i$ (2)

1.3 The sum of n terms is given by $S_n \quad \frac{n}{2}(1 + n)$ find T_5. (3)

1.4 Determine the 7$^{\text{th}}$ term of the following sequence: $64 ; \frac{3}{32} ; \frac{9}{16} ; \frac{27}{8}$ (3)

1.5 If inflation is expected to be 8.7% per annum for the next 10 years. During which year will prices be double what they are today? (3)

1.6 Given that $f(1) = 0$; solve for $f(x) \quad x^3 \quad x^2 \quad 4x - 4 =$ (4)

1.7 Given : $f(x) = \frac{1}{x-5}$

1.7.1 Determine the equation of the vertical asymptote of $f(x)$ (1)

1.7.2 Determine the y-intercept of $f(x)$ (1)

1.7.3 Determine x if $f(x) = -1$ (2)

1.7.4 Determine the equation of one of the axes of symmetry of $f(x)$. (2)

1.8 The inverse of a function is $f^{-1}(x) \quad 2x - 4$, what is the function $f(x)$? (3)

1.9 Which of the following functions does no increase over the interval (0;10)?

A) $y = \log x$ B) $y = 10^x$ C) $y = \frac{10}{x}$ (2)

1.10 Determine a function f(x) such that $f'(x) = 3x^2$ (2)

1.11 A car travelled for 1 hour. The average speed for the first 15 minutes was 60 km/h and for the remaining 45 minutes the average speed was 80km/h. How far did the car travel? (3)

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QUESTION 2

2.1 The population of a certain bacteria in a body is expected to grow exponentially at a rate of 15 % every hour. If the initial population is 5 000. How long will it take for the population to reach 100 000? (4)

2.2 If the first term a geometric series is 10 and the common ratio is 0,5:

2.2.1 Find the sum of the first 8 terms. (3)

2.2.2 For what value of *n* is $\left|S_\infty - S_n\right| < 0{,}01$? (4)

2.3 The first, second and third terms and an arithmetic series are a; b and $a - b$ respectively ($a > 0$).
The first, second and third terms and a geometric series are a; $a - b$ and 1 respectively.
Show that $a = 9$ and determine the value of b. (6)

2.4 $n!$ is defined as $n! = n(n-1)\times(n-2)\times\ldots\times 2\times 1$ e.g. 4!=4 x 3 x 2 x 1 = 24

Evaluate the following: $\sum_{i=3}^{5} i!$ (3)

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QUESTION 3

3.1 You wish to purchase your first home. The bank will only allow bond repayments that are no greater than 30 % of your net monthly salary. Your gross salary is R 8 250 per month and you have deductions of 25 % per month from your salary.

3.1.1 What is your net salary? (how much do you take home after deductions) (1)

3.1.2 What is the maximum bond repayment you can afford? (1)

3.1.3 The bank offers a fixed bond rate of 13,5% per annum compounded monthly, over a 20 year period. There is a flat that costs R 150 000. Can you afford the flat? (Show all working) (6)

3.2 A bank is offering a saving account with an interest rate of 10% per annum compounded monthly. You can afford to save R 300 per month. How long will it take you to save up R 20 000? (to the nearest month) (5)

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QUESTION 4

4.1 The following data were collected. From the graph of this data, is would appear as if the output is an exponential function of the input: $f(x) = a \times b^x$

Input	-1	0	1	2	2,3	6
Output	0,67	2	6	17	24,9	1465

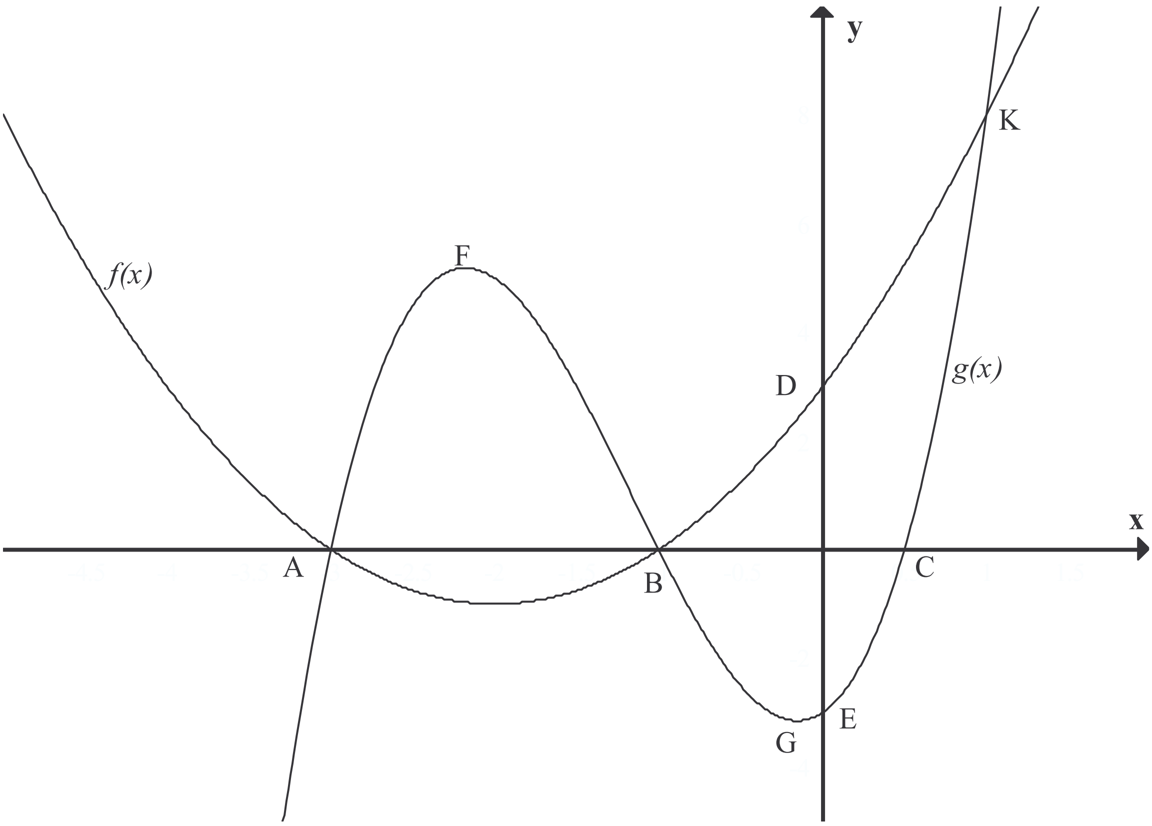
4.1.1 Kate used the input values of 0 and 1 and the corresponding output values to determine the function variables *a* and *b*. Write the function that Kate determined in the form $f(x) = ...$ (3)

4.1.2 Dolly used the input values of 0 and 2 and the corresponding output values to determine the function variables *a* and *b*. Write the function that Dolly determined in the form $g(x) = ...$ (3)

4.1.3 Determine $f(2,3)$; $f(6)$; $g(2,3)$; and $g(6)$ (2)

4.1.4 State, with reasons, which of the two functions is the better approximation of the relationship between *input* and *output*? (2)

4.2 Below are the graphs of $f(x) = x^2 \quad 4x + 3$ and $g(x)$ a cubic function. The two functions have the roots at A and B and $g(x)$ has another root at $x = \frac{1}{2}$. The length of DE = 6 units.



4.2.1 Find the roots at A and B. (3)

4.2.2 Give the co-ordinates of E. (1)

4.2.3 Find the equation of the function $g(x)$. (3)

4.2.4 Determine the co-ordinates of K, where the two functions intersect. (4)

4.2.5 Does F, the turning point of $g(x)$ lie on the axis of symmetry of $f(x)$? Show all working. (5)

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4.2.6 There are two x values where the two functions are increasing at the same rate. Find these values correct to two decimal places. (6)

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QUESTION 5

5.1 The following seems to show that 2 = 1. Explain where and why the error occurred.

line 1	a	$=$	b	
line 2	a^2	$=$	ab	multiply by a
line 3	$a^2 \quad b^2$		$ab - b^2$	subtract b^2
line 4	$(a \quad b)(a \quad b)$		$b(a - b) -$	factorise
line 5	$(a \quad b)$	$=+$	b	divide by a - b
line 6	$b \quad b$	$=+$	b	a = b so substitute b for a
line 7	$2b$	$=$	b	
line 8	2	$=$	1	divide by b

(3)

5.2 Given $f(x) \quad 2x^3 \quad x^2 \quad 7x - 6 + =$

5.2.1 Determine all values of x such that $f(x) = 0$. (5)

5.2.2 Hence of otherwise solve: $2(x \quad 2)^3 \quad (x \quad 2)^2 \quad 7(x \quad 2) = 6 - + -$ (3)

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QUESTION 6

6.1 Determine the derivative of $f(x) = \dfrac{1}{x-2}$ using first principles (5)

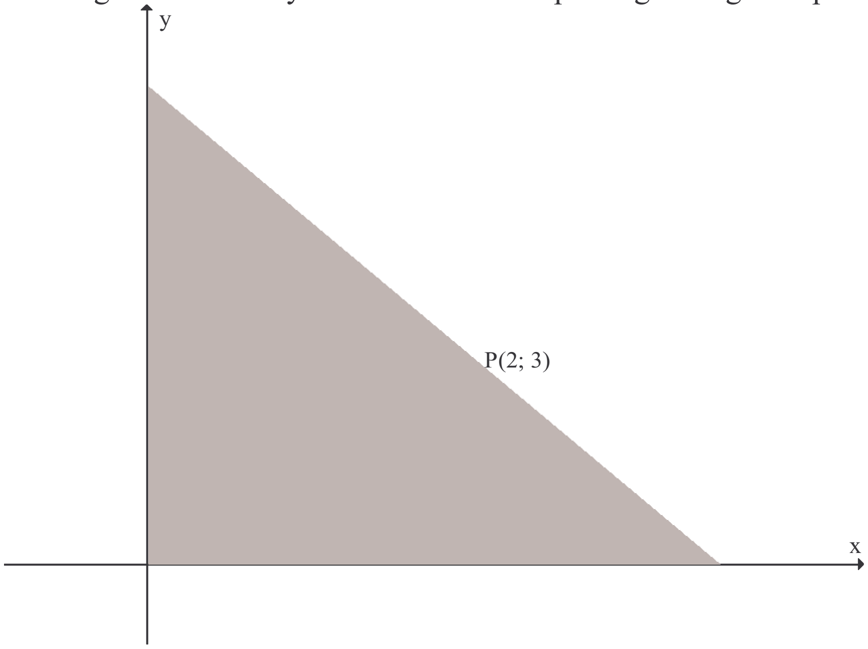
6.2 Determine $\dfrac{dy}{dx}$ if $y = \dfrac{x^3 \quad 2\sqrt{x} - 3}{x}$ (5)

6.3 For a given function $f(x)$ the derivative is $f'(x) \quad x^2 \quad x + 2 =$

6.3.1 What is the gradient of the tangent to the function $f(x)$ at $x = 0$? (1)

6.3.2 Where is $f(x)$ increasing? (4)

6.4 A triangle is formed by the axes and a line passing through the point P(2; 3).



6.4.1 If $y = mx + c$ is the equation of the line find c in terms of m. (2)

6.4.2 Find the x-intercept in terms of m. (2)

6.4.3 Give an expression for the area of the triangle in terms of m. (2)

6.4.4 Hence, or otherwise, find for what value of m, the triangle will have a minimum area. (5)

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QUESTION 7

A company produces two types of jeans, straight-leg or bootleg. The straight-leg jeans requires twice as much labour time as the bootleg jeans. If all the jeans were bootleg jeans, then the company could produce a total of 500 jeans per day. The market limits the daily sales of straight-leg jeans to 150 and bootleg jeans to 250 per day. The profits for straight-leg jeans are R 8 and for bootleg jeans R 5.

7.1 If all the jeans were straight-leg jeans how many could be produced in a day? (1)

7.2 Sketch a graph of the feasible region. (5)

7.3 Determine the maximum profit the company could make on the production of jeans. (5)

7.4 If the profit on the straight-leg jeans increased to R 11, how many of each type of jeans should be produce? (2)

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