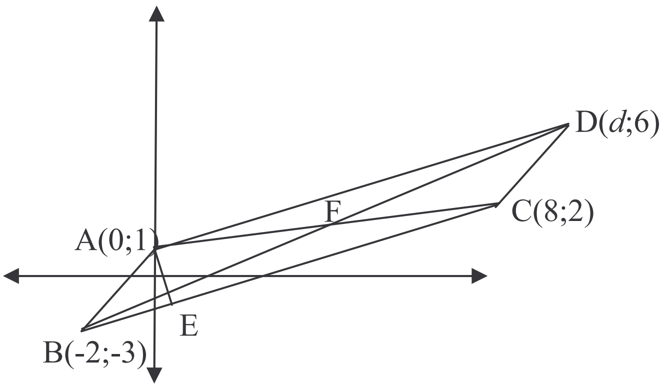
Grade12 Mathematics: Memorandum Paper 2

1.1



1.1.1 $M_{BC} = \frac{2 - (-3)}{8 - (-2)} = \frac{5}{10} = \frac{1}{2}$ ✓ ✓ 2

1.1.2 $M_{AD} = \frac{1}{2}$ ✓ (lines parallel)

AD: $y = \frac{1}{2}x + c$

Sub in (0;1)

$1 = c$ ✓

AD: $y = \frac{1}{2}x + 1$

Sub(d;6): $6 = \frac{1}{2}d + 1$ ✓

$d = 10$ ✓ 4

1.1.3 $M_{BC} = \frac{1}{2}$

$\therefore$ $M_{AE} = -2$ ✓ (lines $\perp$)

A = (0;1) which is the y-intercept of AE.

AE: $y = -2x + 1$ ✓

2

1.1.4 $F = \left(\frac{0 \quad 8}{2};\frac{1+2}{2}\right) = (4;1\frac{1}{2})$ ✓ ✓ 2

1.2.1 $4\cos\theta - 7\sin\theta = -8$ ✓ ($\times$ 7)

$4\sin\theta + 7\cos\theta = 1$ ✓ ($\times$ -4)

$28\cos\theta - 49\sin\theta = -56$

$\underline{-28\cos\theta - 16\sin\theta = -4}$

$\therefore$ $-65\sin\theta = -60$ ✓

$\therefore$ $\sin = \theta \frac{60}{65}$ ✓

$\therefore$ $\theta = 67{,}38°$ ✓

or $180° - 67{,}38° = 112{,}62°$ ✓

$\therefore$ $\theta = 112{,}62°$

OR

$OA = \sqrt{4^2 + 7^2} = \sqrt{65}$ ✓

$OA' = \sqrt{(-8)^2 + 1^2} = \sqrt{65}$ ✓

$AA' = \sqrt{(4 - \cancel{(+8)})^2 \quad (7 \quad 1)^2}$

$= \sqrt{180} = 6\sqrt{5}$ ✓

Using the cos rule:

$(6\sqrt{5})^2 = (\sqrt{65})^2 + (\sqrt{65})^2 - 2.$

$\sqrt{65}\,\sqrt{65}\,.\cos\theta$ ✓

$180 = 130(1 - \cos\theta)$

$\therefore \frac{180}{130} - 1 = -\cos\theta$

$\therefore$ $\cos\theta = -\frac{5}{13}$ ✓

$\therefore$ $\theta = 112{,}62°$ ✓

6

1.2.2 B' = ($8\cos 112{,}62° - 14\sin 112{,}62°$; $8\sin 112{,}62° + 14\cos 112{,}62$) ✓ ✓

= (-16;2) ✓ ✓ 4

1.3.1 Tan$x$ = -0,3421 ✓

$\therefore x = -18{,}89°$ ✓ 2

1.3.2 Sin$x$ = 0,500 ✓

$\therefore x = 30°$ 2

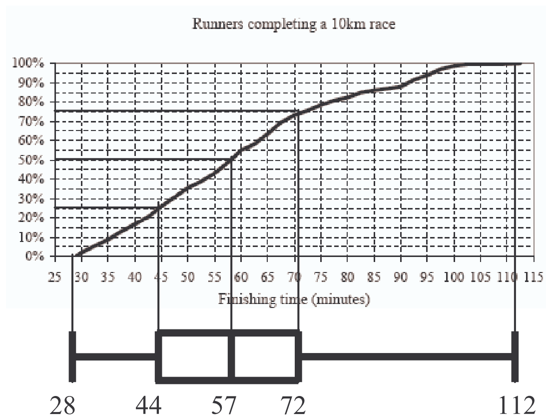
1.4 $y = 4\sin 5x$ ✓ ✓ 2

1.5.1 < 39minutes ✓ ✓ 2

1.5.2 16% $\times$ 590 runners = 94 runners ✓ ✓ ✓

(can accept answers in the range of 94 to 106 runners) 3

1.5.3



Appropriate scale ✓
Correct drawing ✓
Correct min and max values for the range ✓
Correct values for the median etc ✓ ✓
The values can be out by 1unit on either side. 5

2.1.1 Let the centre = $(a; b)$

$a = 2$ ✓

$(x)()\ 2^2 \quad y \quad b^2 = \cancel{r^2}$

Sub in (2;0)

$(2)()\ 2^2 \quad 0 \quad b^2 = \cancel{r^2}$ ✓

$\therefore\ b^2 = r^2$ ✓

Sub in (4; -6)

$(4)()\ 2^2 \quad 6 \quad b^2 = \cancel{b^2}$ ✓

$\therefore 4 + 36 + 12b + \ b^2 = b^2$ ✓

$\therefore 40 + 12b = 0$

$\therefore\ b = -\frac{10}{3}$ ✓

Centre = $\left(2; -\frac{10}{3}\right)$

$(x \quad 2^2 \quad \left(y + \frac{10}{3}\right)^2 = 4$ ✓ 7

2.1.2 $m_{MB} = \dfrac{3\frac{1}{3} \quad () \ 6}{2-4} = -\frac{4}{3}$ ✓

$\therefore M_{tangent} = \frac{3}{4}$ ✓

Tangent: $y = \frac{3}{4}x + c$

Sub in (4;-6)

$-6 = \frac{3}{4}(4) + c$ ✓

$-9 = c$

Tangent: $y = \frac{3}{4}x - 9$ ✓ 4

2.2.1 $y = 5 - 2x$

$x^2 \quad (5)()\ 2x^2 \quad 12x \quad 6\ 5 \quad 2x \quad 20 = \cancel{0}$ ✓

$x^2 \quad 25 \quad 20x \quad 4x^2 \quad 12x \quad 30 \quad 12x \quad 20 = \cancel{0}$

$5x^2 \quad 20x \quad 15 = \cancel{0}$ ✓

$x^2 \quad 4x \quad 3 = \cancel{0}$

$\therefore\ (x(\quad 3 \quad x - 1) = 0$ ✓

$\therefore\ x = 3 \qquad x = 1$ ✓

$y \quad 5 - 2(3 \qquad y \quad 5 - 2(1)$ ✓

$\therefore\ y \quad -1 \qquad y = 3$ ✓

$\therefore$ Points of intersection are:
(3;-1) and (1:3) 6

2.2.2 AB = $\sqrt{(3)()\ 1^2 \quad 1 - 3^2}$ ✓ ✓ 3

AB = $\sqrt{4+16}$

AB = $\sqrt{20}$

AB = $2\sqrt{5}$ ✓

2.2.3 $m_{BC} = \frac{-1-3}{3-1} = \frac{-4}{2} = -2$ ✓

$m_{perp} = \frac{1}{2}$ ✓

Midpoint of AB = $\left(\frac{1+3}{2}; \frac{3-1}{2}\right)$ = (2;1) ✓

Perpendicular bisector: $y = \frac{1}{2}x + c$

Sub (2;1): $1 = \frac{1}{2}(2) + c$ ✓

$\therefore c = 0$

$\therefore\ y = \frac{1}{2}x$ ✓ 5

2.2.4 The x-intercepts of the circle are found by:

$x^2 \quad 12x \quad 20 = \cancel{0}$ ✓

$\therefore\ (x \quad 10)(x - 2) = 0$ ✓

$\therefore x = 10$ or $x = 2$ ✓

$\therefore\ x = 6$ is the perpendicular bisector of the x-intercepts ✓

$\therefore$ the x value of the centre = 6 ✓

$\therefore\ y = 3$ ✓

The centre of the circle = (6;3) 6

3.1.1 (5;1) ✓ ✓ p is 1 unit from C to the line $x = 4$, so the point C' will be 1 unit from $x = 4$ on the other side i.e. 5. The y-value (q) remains the same. ✓ 3

3.1.2 (12;4) ✓ ✓ r is 3 units from C' to the line $x = 9$, so the point C″ will be 3 units from $x = 9$ on the other side i.e. 12. The y-value (s) remains the same. ✓ 3

3.1.3 A translation 10 units right. ✓ Triangle ABC has remained in the same horizontal plane but has moved 10 units along. ✓ 2

3.1.4 If point A (1;3) is reflected about the $x = 9$, it will become A' = (17;3). ✓ If A' is then reflected about the $x = 4$ line, it will become A″ = (-9;3). ✓ This is not the same result as above. ✓ 3

3.2.1 If A = (4;3) then A' = (3;4) ✓ ✓ and A″ = (-3;4) ✓ 3

3.2.2 Rotation of 90° :
A″ = (4cos 90°− 3sin 90°; 4sin 90° + 3cos 90°) ✓ ✓
A″ = (4 × 0 − 3 × 1; 4 × 1 + 3 × 0) ✓ ✓
A″ = (-3;4) 4

4.1.1 $\dfrac{\sin 2 \quad \cos \quad + \cos 2 \quad \sin \theta\theta\theta\theta}{1 + 2(2\cos^2\theta - 1}$ ✓ ✓

$\dfrac{2\sin \quad \cos \quad \cos\theta\theta\theta(2\cos^2 \quad - 1)\sin\theta\theta}{4\cos^2\theta - 1}$ ✓ ✓

$\dfrac{\sin \quad (2\cos^2 \qquad 2\cos^2\theta\theta\theta 1)}{4\cos^2\theta - 1}$ ✓ 6

$= \sin\theta$ ✓

4.1.2 If $\theta = 60°$ then $1 + 2\cos 2\theta = 0$ ✓ and the denominator will be zero which makes the identity undefined. ✓ 2

4.1.3 $120°$ or $240°$ ✓ 1

4.2 In ΔTAB:

$A\hat{T}B = 180° - (\theta + \beta)$ ✓

$\frac{AT}{\sin\theta} = \frac{x}{\sin(180^0 - (\theta + \beta))}$ ✓

$\therefore AT = \frac{x\sin\theta}{\sin(\theta + \beta)}$ ✓

In ΔTAC:

TC = AT $\sin\alpha$ ✓

$\therefore TC = \frac{x\sin\theta \sin\alpha}{\sin(\theta + \beta)}$ ✓ 5

5.1 $f(180) = 1{,}2\cos 0^0 + 6{,}66$ ✓

= 7,86

0,86 × 60minutes = 51,6minutes ✓

Time for sunrise = 7:52 which is the time recorded in the table. ✓ 3

5.2 $f(60) = 1{,}2\cos(60^0 - 180^0) + 6{,}66$ ✓

= 6,06

0,06 × 60minutes = 3,6minutes ✓

Time for sunrise = 06:04.

Actual sunrise is at 06:33.

Difference is about 29minutes. ✓ 3

5.3 Earliest = 17:44 = 17,733 ✓

Latest = 20:01 = 20,016 ✓

$\therefore$ 20,016 − 17,733 = 2,283 ✓ 3

5.4 a is the amplitude of the cos graph which will be half of the time between the earliest and the latest sunset i.e. 2,283 ÷ 2 = 1,142 ✓ ✓

p represents a horizontal shift which has not occurred therefore $p = 0$ ✓

q is the amount that the graph has been shifted upwards and is calculated by : the minimum value + the amplitude of the graph = 17,733 + 1,142 = 18,874. ✓ 4

5.5 $g(285) = 1{,}142\cos(285^0 - 180^0) + 18{,}875$ ✓

= 18,58

0,58 × 60minutes = 34,8minutes ✓

Time for sunset = 18:39

Actual sunset is at 18:57

Difference is about 22minutes. ✓ 3

5.6 $h(x) = 1{,}142\cos x + 18{,}875 - \left(1{,}2\cos\left(x - 180^0\right) + 6{,}66\right)$ ✓

$h(x) = 1{,}142\cos x + 1{,}2\cos x + 12{,}215$

$h(x) = 2{,}342\cos x + 12{,}215$ ✓ 2

5.7 a 1st and 360th day ✓ ✓ 2

b 180th day ✓ ✓ 2

5.8 Predicted:

$h(75) = 2{,}342\cos 75 + 12{,}215$ ✓

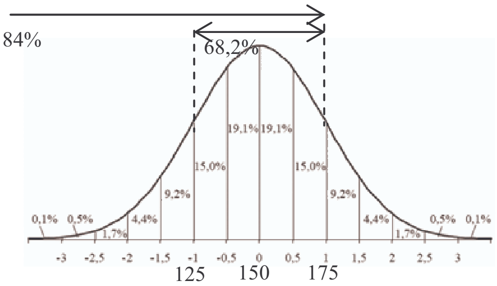
= 12,82 hours ✓

= 12hours 49min 4

Actual:

19:04 − 06:46 = 12hrs18min ✓

Differs by about 31minutes ✓



6.1.1 If the house price was R175 000 then the percentile rank would be = (0,1 + 0,5 + 1,7 + 4,4 + 9,2 + 15 + 19,1 + 19,1 + 15)% ≈ 84% ✓ ✓

This means that 84% of the houses were sold for less than R175 000 and 16% of the houses were sold for more than R175 00. ✓ ✓ 4

6.1.2 The difference between one standard deviations on either side of the mean = (15 + 19,1 + 19,1 + 15)% = 68,2% ✓ ✓

This means that 68,2% of the houses were in the price range of R125 000 and R175 000. ✓ ✓

Mrs Hlope is therefore correct in saying that most of the house were sold between R125 000 and R175 000. ✓ 5

6.2.1 SD = $\sqrt{\frac{\sum(x - \bar{x})^2}{n-1}}$ = 16,08 ✓ ✓ 2

6.2.2 46,62← 62,7→78,78 ✓ ✓

$\therefore \frac{17}{20}$ = 85% scored within one standard deviation. ✓ 3

6.2.3 It is not a normal distribution as we would expect only ≈ 68,2% of the students to fall within one standard deviation. ✓ ✓ 2

7.1 A, the line is above all the points. ✓ ✓ 2

7.2 E ✓ 1

7.3 D, the line goes above the points for lighter eggs and below the point for heavier eggs. ✓ ✓ 2

7.4 B ✓ 1

7.5 C, the line goes through the majority of the points. ✓ ✓ 2