



education

Department:
Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

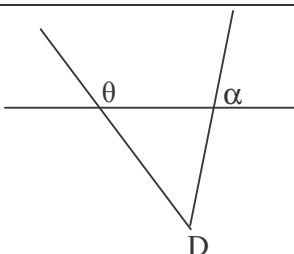
MATHEMATICS P2

EXEMPLAR 2008

MEMORANDUM

This memorandum consists of 12 pages.

QUESTION 1

1.1	$AC = \sqrt{(2+4)^2 + (5-3)^2}$ $AC = \sqrt{40}$ $AC = 2\sqrt{10}$	✓ substitution ✓ answer (2)
1.2	$M\left(\frac{-4+2}{2}; \frac{3+5}{2}\right) \therefore M(-1;4)$	✓ substitution ✓ answer (2)
1.3	$m_{BD} = \frac{10 - (-2)}{-3 - 1} = \frac{12}{-4} = -3$ $m_{AC} = \frac{5 - 3}{2 - (-4)} = \frac{2}{6} = \frac{1}{3}$ $\therefore m_{BD} \times m_{AC} = -3 \times \frac{1}{3} = -1$ $\therefore BD \perp AC$ $\text{Midpoint } BD\left(\frac{-3+1}{2}; \frac{10-2}{2}\right) = \text{Midpoint of } AC$ $= (-1; 4)$ $\therefore \text{bisect at } 90^\circ$	✓ answer ✓ answer ✓ -1 ✓ coordinates ✓ = Midpoint AC (5)
1.4	$\text{Area } \Delta ABC$ $= \frac{1}{2} \cdot AC \cdot MB$ $= \frac{1}{2} \cdot \sqrt{40} \cdot \sqrt{(10-4)^2 + (-3+1)^2}$ $= \frac{1}{2} \cdot \sqrt{40} \cdot \sqrt{40}$ $= 20$	✓ formula ✓ substitution ✓ $MB = \sqrt{40}$ ✓ answer (4)
1.5	$m_{DC} = \frac{3+2}{-4-1} = -1$ $y+2 = -1(x-1)$ $y = -x-1$	✓ substitution ✓ answer ✓ answer (3)
1.6	$m_{DC} = -1$ $\tan \theta = -1$ $\theta = 135^\circ$	✓ substitution ✓ answer (2)
1.7	$m_{AD} = \frac{5+2}{2-1} = 7$ $\therefore \tan \alpha = 7$ $\alpha = 81,9^\circ$ $\hat{A}DC = \theta - \alpha$ $\hat{A}DC = 135^\circ - 81,9^\circ = 53,1^\circ$ OR	 ✓ 7 ✓ 81,9° ✓ $\hat{A}DC = \theta - \alpha$ ✓ 53,1° (4)

	$\hat{A}DC = 180^\circ - (45^\circ + 81,9^\circ)$ $\hat{A}DC = 53,1^\circ$ <p>OR999 Use Cosine Rule</p> $AC^2 = DC^2 + AD^2 - 2DC \cdot AD \cos D$ $\therefore 40 = 50 + 50 - 2 \times 50 \cos D$ $\therefore \cos D = 0.6 \quad \therefore \hat{D} = 53,13^\circ$	<ul style="list-style-type: none"> ✓ Use Cosine Rule ✓✓ Substitution ✓ answer <p style="text-align: right;">[22]</p>
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QUESTION 2

2.1	$m_{AB} = \frac{7-5}{12-8}$ $m_{AB} = \frac{1}{2}$ <p>equation of AB is</p> $y - 5 = \frac{1}{2}(x - 8)$ $y = \frac{1}{2}x + 1$ $2y - x = 2$ $x - 2y + 2 = 0$	<ul style="list-style-type: none"> ✓ $m_{AB} = \frac{1}{2}$ ✓ substitution ✓ answer <p style="text-align: right;">(3)</p>
2.2	<p>At D: $x - 2(0) + 2 = 0$</p> $x = -2$ <p>D(-2 ; 0)</p>	<ul style="list-style-type: none"> ✓ substitution of $y = 0$ ✓ x answer <p style="text-align: right;">(2)</p>
2.3	$C\left(\frac{8-2}{2}; \frac{5+0}{2}\right)$ $C\left(3; \frac{5}{2}\right)$	<ul style="list-style-type: none"> ✓ x-value ✓ y-value <p style="text-align: right;">(2)</p>
2.4	$AC^2 = (8-3)^2 + \left(5 - \frac{5}{2}\right)^2$ $AC^2 = 25 + \frac{25}{4}$ $AC^2 = \frac{125}{4}$ <p>equation of the circle is</p> $(x-3)^2 + \left(y - \frac{5}{2}\right)^2 = \frac{125}{4}$	<ul style="list-style-type: none"> ✓ substitution ✓ AC^2 ✓ $(x-3)$ ✓ $\left(y - \frac{5}{2}\right)$ ✓ answer <p style="text-align: right;">(5)</p>
2.5	<p>gradient of tangent = -2 (tangent \perp radius)</p> <p>equation of tangent is</p>	<ul style="list-style-type: none"> ✓ gradient ✓ substitution ✓ answer

2.6	$y - 5 = -2(x - 8)$ $y - 5 = -2x + 16$ $y = -2x + 21$ Axis of symmetry $x = 3$ $A'(-2; 5)$	(3) ✓ axis of symmetry ✓ x-answer ✓ y-answer (3) [18]
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QUESTION 3

3.1.1	$P(2; -\sqrt{3})$	✓✓ coordinates (2)
3.1.2	$P(-\sqrt{3}; 2)$	✓✓ coordinates (2)
3.2.1		✓ coordinates P ✓ coordinates Q ✓ coordinates R ✓ coordinates S (4)
3.2.2	$(x; y) \rightarrow (-x; -y)$ $(-x; -y) \rightarrow (-2x; -2y)$ $\therefore (x; y) \rightarrow (-2x; -2y)$	✓ $(x; y) \rightarrow (-x; -y)$ ✓ $(-x; -y) \rightarrow (-2x; -2y)$ ✓✓ $\therefore (x; y) \rightarrow (-2x; -2y)$ (4)
3.2.3	Area ABCD : area PQRS = 1 : 4	✓ answer (1)
3.3.1	Let $r = OP = OP'$ The x co-ordinate of $P' = r \cos(\alpha + 30^\circ)$ $x' = r \cos(\alpha + 30^\circ)$ $= r(\cos \alpha \cdot \cos 30^\circ - \sin \alpha \cdot \sin 30^\circ)$ $= r \cos \alpha \cdot \cos 30^\circ - r \sin \alpha \cdot \sin 30^\circ$	✓ formula ✓ expansion ✓ simplification ✓ substitution

	$= r \cdot \frac{x}{r} \cos 30^\circ - r \cdot \frac{y}{r} \sin 30^\circ$ $= x \cdot \frac{\sqrt{3}}{2} - y \cdot \frac{1}{2}$ <p>The y co-ordinate of P' is $r \sin(\alpha + 30^\circ)$</p> <p>Similarly</p> $y' = y \cos 30^\circ + x \sin 30^\circ$ $= y \cdot \frac{\sqrt{3}}{2} + x \cdot \frac{1}{2}$	<p>✓ formula</p> <p>✓ expansion ✓ simplification ✓ substitution</p> <p>(8)</p>
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3.4	$K' = \left(x \cdot \frac{\sqrt{3}}{2} - y \cdot \frac{1}{2} ; y \cdot \frac{\sqrt{3}}{2} + x \cdot \frac{1}{2} \right)$ $= \left(4 \cdot \frac{\sqrt{3}}{2} - 3 \cdot \frac{1}{2} ; 3 \cdot \frac{\sqrt{3}}{2} + 4 \cdot \frac{1}{2} \right)$ $= (1.96 ; 4.60)$ $L' = \left(x \cdot \frac{\sqrt{3}}{2} - y \cdot \frac{1}{2} ; y \cdot \frac{\sqrt{3}}{2} + x \cdot \frac{1}{2} \right)$ $= \left(3 \cdot \frac{\sqrt{3}}{2} - 6 \cdot \frac{1}{2} ; 6 \cdot \frac{\sqrt{3}}{2} + 3 \cdot \frac{1}{2} \right)$ $= (-0.40 ; 6.70)$	<p>✓ x-coordinate of K' ✓ y-coordinate of K'</p> <p>✓ x-coordinate of L' ✓ y-coordinate of L'</p> <p>(4) [25]</p>
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QUESTION 4

4.1	$\frac{\sin 140^\circ \cdot \tan(-315^\circ)}{\cos 230^\circ \cdot \sin 420^\circ}$ $= \frac{\sin 40^\circ \cdot (\tan 315^\circ)}{(\cos 50^\circ) \cdot \sin 60^\circ}$ $= \frac{\sin 40^\circ \cdot \tan 45^\circ}{\sin 40^\circ \cdot \sin 60^\circ}$ $= \frac{-1}{\frac{\sqrt{3}}{2}}$ $= \frac{-2}{\sqrt{3}}$	<ul style="list-style-type: none"> ✓ $\sin 40^\circ$ ✓ $-\cos 50^\circ$ ✓ $\tan 45^\circ$ ✓ $\sin 60^\circ$ ✓ $\sin 40^\circ$ <p>✓ answer (6)</p>
4.2	$\tan(180^\circ + x) \cdot \cos(540^\circ + x) \left(\sin(-x) + \frac{\sin^2(90^\circ - x)}{\cos(90^\circ + x)} \right)$ $= \tan x \cdot (-\cos x) \left(-\sin x + \frac{\cos^2 x}{-\sin x} \right)$ $= \frac{\sin x}{\cos x} \cdot (-\cos x) \left(\frac{\sin^2 x + \cos^2 x}{-\sin x} \right)$ $= \sin^2 x + \cos^2 x$ $= 1$	<ul style="list-style-type: none"> ✓ $\tan x$ ✓ $-\cos x$ ✓ $-\sin x$ ✓ $\cos^2 x$ ✓ $-\sin x$ ✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ $\sin^2 x + \cos^2 x$ ✓ answer (8)
4.3	$\sin 15^\circ$ $= \sin(45^\circ - 30^\circ)$ $= \sin 45^\circ \cdot \cos 30^\circ - \cos 45^\circ \cdot \sin 30^\circ$ $= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$ $= \frac{\sqrt{2}(\sqrt{3} - 1)}{4}$	<ul style="list-style-type: none"> ✓ $\sin 15^\circ$ ✓ $\sin(45^\circ - 30^\circ)$ ✓ expansion ✓✓ substitution (5) <p>(note using 60° and 45° will also give same answer)</p>
4.4.1	$\cos 2\theta + 3\cos \theta - 1$ $= 2\cos^2 \theta - 1 + 3\cos \theta - 1$ $= 2\cos^2 \theta + 3\cos \theta - 2$	<ul style="list-style-type: none"> ✓ $\cos 2\theta = 2\cos^2 \theta - 1$ ✓ simplification (2)

4.4.2	$\cos 2\theta + 3\cos \theta - 1 = 0$ $2\cos^2 \theta + 3\cos \theta - 2 = 0$ $(2\cos \theta - 1)(\cos \theta + 2) = 0$ $\cos \theta = \frac{1}{2} \text{ or } \cos \theta = -2 \text{ invalid}$ $\theta = \pm 60^\circ + k.360^\circ \quad k \in Z$	✓ factors ✓ $\cos \theta = -2$ invalid ✓ answers ✓ $k \in Z$
		(4) [25]

QUESTION 5

5.1	Joyce did not use the correct expansion for $\sin(A + B)$.	✓ answer (1)
5.2	$\sin(45^\circ + 21^\circ)$ $= \sin 45^\circ \cdot \cos 21^\circ + \sin 21^\circ \cdot \cos 45^\circ$ $= \frac{\sqrt{2}}{2} \cdot (t) + \sqrt{1-t^2} \left(\frac{\sqrt{2}}{2}\right)$ $= \frac{\sqrt{2}}{2} (t + \sqrt{1-t^2})$	✓ expansion ✓ substitution ✓ $\sin 21 = \sqrt{1-t^2}$ ✓ answer (4)
		[5]

QUESTION 6

6.1	$\frac{BC}{OB} = \cos \theta$ $BC = 2a \cdot \cos \theta$	✓ $\frac{BC}{OB} = \cos \theta$ (1)
6.2	$\text{area } \triangle OBC = \frac{1}{2} \cdot OB \cdot BC \cdot \sin \theta$ $\text{area } \triangle OBC = \frac{1}{2} \cdot (2a) \cdot (2a \cdot \cos \theta) \cdot \sin \theta$ $\text{area } \triangle OBC = a^2 \cdot 2 \cos \theta \cdot \sin \theta$ $\text{area } \triangle OBC = a^2 \cdot \sin 2\theta$	✓ area rule ✓ substitution ✓ double angle (3)
6.3	Area will be a maximum when $\sin 2\theta = 1$. $\theta = 45^\circ$	✓ answer

	ΔOBC will be isosceles $C(a; a)$	(1)
		[5]

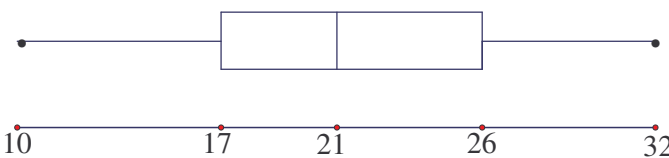
QUESTION 7

7.1	$\frac{20,7}{BD} = \cos 43,6^\circ$ $BD = \frac{20,7}{\cos 43,6}$ $BD = 28,6 \text{ m}$	✓ ratio ✓ solving for BD ✓ answer (3)
7.2	$BE^2 = (28,58)^2 + (28,1)^2 - 2(28,58)(28,1)\cos 35,7^\circ$ $= 302,0610874 \dots$ $BE = 17,4 \text{ m}$	✓✓ cos rule ✓ substitution ✓ answer (4)
7.3	$\text{Area } \Delta BEC = \frac{1}{2}(20,7)(17,4)\sin 63^\circ$ $= 160,4 \text{ square metres}$	✓ substitution ✓ answer (2)
		[9]

QUESTION 8

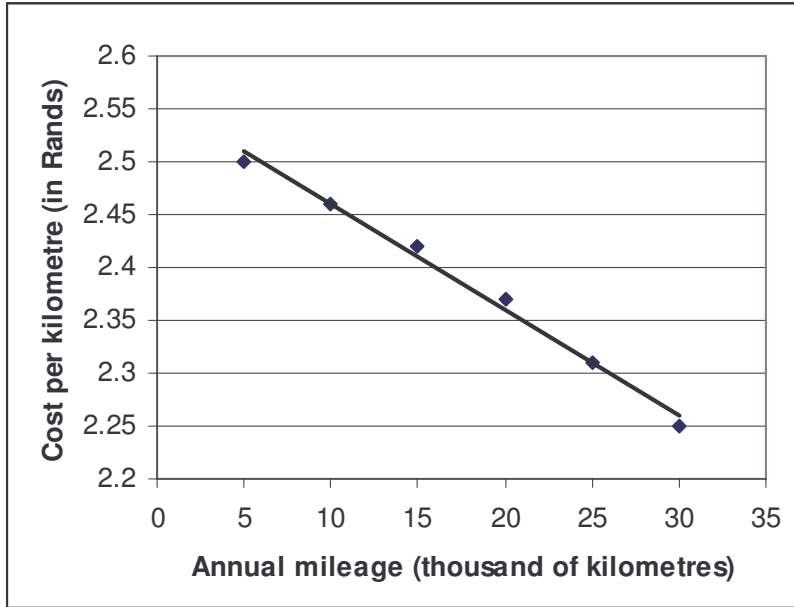
<p>8.1</p>	$\cos \frac{x}{2} = \sin(x - 30^\circ)$ $\cos \frac{x}{2} = \cos(90^\circ - x + 30^\circ)$ $\frac{x}{2} = 120^\circ - x + k.360^\circ \quad \text{or} \quad \frac{x}{2} = -120^\circ + x + k.360^\circ \quad k \in Z$ $\frac{3x}{2} = 120^\circ + k.360^\circ \quad \frac{-x}{2} = -120^\circ + k.360^\circ$ $x = 80 + k.240^\circ \quad x = 240^\circ - k.720^\circ$ <p>$x = 80^\circ; -160^\circ$</p> <p>A(-160° ; 0,174) and B(80° ; 0,766)</p>	<p>✓ cos(90° - x + 30°)</p> <p>✓✓ answer</p> <p>✓✓ answers</p> <p>✓✓ answer</p> <p>(7)</p>
<p>8.2</p>	<p>$-160^\circ < x < 80^\circ$</p>	<p>✓ critical values</p> <p>✓ statement</p> <p>(2)</p> <p>[9]</p>

QUESTION 9

<p>9.1</p>	<p>10, 13, 13, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 30, 32.</p> <p>Median = 21</p>	<p>✓ ordered data</p> <p>✓ answer</p> <p>(2)</p>
<p>9.2</p>	<p>lower quartile = 17</p> <p>upper quartile = 26</p>	<p>✓ Q₁</p> <p>✓ Q₃</p> <p>(2)</p>
<p>9.3</p>		<p>✓✓ quartiles</p> <p>✓✓ box</p> <p>✓ whiskers</p> <p>(5)</p>
<p>9.4</p>	<p>The range of distances that Geoff and Thabo's travelled is the same.</p> <p>Thabo's summary is skewed to the right while Geoff's summary is more evenly distributed.</p> <p>This suggest that Thabo had covered greater distances than Geoff. Also Thabo's median(viz. 25) is larger than Geoff's median(21).</p>	<p>✓ range the same</p> <p>✓ explanation of skewed data.</p> <p>(2)</p> <p>[11]</p>

QUESTION 10

10.1
&
10.2



✓✓ plotting points
✓ labels

(3)

✓✓ line of best fit

(2)

10.3

R 2,47 or R 2,48

✓ answer in this
range

(1)
[6]

QUESTION 11

11.1	<table border="1" style="margin: auto;"> <thead> <tr> <th style="padding: 5px;">Percentages</th> <th style="padding: 5px;">Midpoint of interval (x)</th> <th style="padding: 5px;">Frequency (f)</th> <th style="padding: 5px;">Total ($f \times x$)</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">10 – 19</td> <td style="text-align: center;">14,5</td> <td style="text-align: center;">6</td> <td style="text-align: center;">87</td> </tr> <tr> <td style="text-align: center;">20 – 29</td> <td style="text-align: center;">24,5</td> <td style="text-align: center;">14</td> <td style="text-align: center;">343</td> </tr> <tr> <td style="text-align: center;">30 – 39</td> <td style="text-align: center;">34,5</td> <td style="text-align: center;">16</td> <td style="text-align: center;">552</td> </tr> <tr> <td style="text-align: center;">40 – 49</td> <td style="text-align: center;">44,5</td> <td style="text-align: center;">11</td> <td style="text-align: center;">489,5</td> </tr> <tr> <td style="text-align: center;">50 – 59</td> <td style="text-align: center;">54,5</td> <td style="text-align: center;">3</td> <td style="text-align: center;">163,5</td> </tr> <tr> <td colspan="3" style="text-align: center;">Sum</td> <td style="text-align: center;">1635</td> </tr> </tbody> </table>	Percentages	Midpoint of interval (x)	Frequency (f)	Total ($f \times x$)	10 – 19	14,5	6	87	20 – 29	24,5	14	343	30 – 39	34,5	16	552	40 – 49	44,5	11	489,5	50 – 59	54,5	3	163,5	Sum			1635	<p>✓ midpoints ✓✓✓ total) one mark for every two numbers correct in the last column)</p>
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Sum			1635																											
	<p>Mean = $\frac{1635}{50} = 32,7$</p>	<p>✓ answer (5)</p>																												
11.2	<p>$\bar{x} = \frac{\sum x}{n} = \frac{7+4+9+4+9+5+4+6}{8} = 6$</p> <p>$\delta = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$</p> <p>$= \sqrt{\frac{(7-6)^2 + (4-6)^2 + (9-6)^2 + (4-6)^2 + (9-6)^2 + (5-6)^2 + (4-6)^2 + (6-6)^2}{8}}$</p> <p>$= \sqrt{\frac{1+4+9+4+9+1+4+0}{8}} = \sqrt{4} = 2$</p>	<p>✓✓ mean</p> <p>✓ formula</p> <p>✓✓ calculation</p> <p>✓ simplification</p> <p>✓ answer (7)</p>																												
		[12]																												

QUESTION 12

12.1	Supplier B	✓ answer (1)
12.2	I would select supplier A. The graph for supplier A shows a fairly consistent lifetime of their bulbs. Whilst the graph for supplier B shows that their bulbs last longer than supplier A, we also have a situation where their bulbs have a shorter lifetime than supplier A.	✓ supplier A ✓ explanation (2) [3]