



education

Department:
Education
REPUBLIC OF SOUTH AFRICA

**NASIONALE
SENIOR SERTIFIKAAT**

GRAAD 12

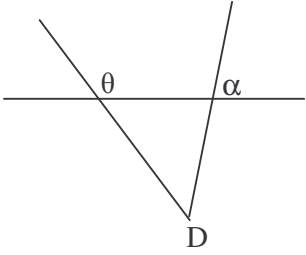
WISKUNDE V2

MODEL 2008

MEMORANDUM

Hierdie memorandum beslaan 12 bladsye

VRAAG 1

1.1	$AC = \sqrt{(2+4)^2 + (5-3)^2}$ $AC = \sqrt{40}$ $AC = 2\sqrt{10}$	✓ substitusie ✓ antwoord (2)
1.2	$M\left(\frac{-4+2}{2}; \frac{3+5}{2}\right) \therefore M(-1;4)$	✓ substitusie ✓ antwoord (2)
1.3	$m_{BD} = \frac{10 - (-2)}{-3 - 1} = \frac{12}{-4} = -3$ $m_{AC} = \frac{5 - 3}{2 - (-4)} = \frac{2}{6} = \frac{1}{3}$ $\therefore m_{BD} \times m_{AC} = -3 \times \frac{1}{3} = -1$ $\therefore BD \perp AC$ $\text{Middelpunt } BD\left(\frac{-3+1}{2}; \frac{10-2}{2}\right) = \text{Middelpunt of } AC$ $= (-1; 4)$ $\therefore \text{halveer by } 90^\circ$	✓ antwoord ✓ antwoord ✓ -1 ✓ koordinate ✓ = Middelpunt AC (5)
1.4	Area ΔABC $= \frac{1}{2} \cdot AC \cdot MB$ $= \frac{1}{2} \cdot \sqrt{40} \cdot \sqrt{(10-4)^2 + (-3+1)^2}$ $= \frac{1}{2} \cdot \sqrt{40} \cdot \sqrt{40}$ $= 20$	✓ formule ✓ substitusie ✓ $MB = \sqrt{40}$ ✓ antwoord (4)
1.5	$m_{DC} = \frac{3+2}{-4-1} = -1$ $y + 2 = -1(x - 1)$ $y = -x - 1$	✓ substitusie ✓ antwoord ✓ antwoord (3)
1.6	$m_{DC} = -1$ $\tan \theta = -1$ $\theta = 135^\circ$	✓ substitusie ✓ antwoord (2)
1.7	$m_{AD} = \frac{5+2}{2-1} = 7$ $\therefore \tan \alpha = 7$ $\alpha = 81,9^\circ$ $\hat{A}DC = \theta - \alpha$ $\hat{A}DC = 135^\circ - 81,9^\circ = 53,1^\circ$ OF $\hat{A}DC = 180^\circ - (45^\circ + 81,9^\circ)$ $\hat{A}DC = 53,1^\circ$	 ✓ 7 ✓ antwoord ✓ substitusie ✓ antwoord (4)

	<p>OF</p> <p>Gebruik Cosinus Reël</p> $AC^2 = DC^2 + AD^2 - 2DC \cdot AD \cos D$ $\therefore 40 = 50 + 50 - 2 \times 50 \cos D$ $\therefore \cos D = 0.6 \quad \therefore \hat{D} = 53,13^\circ$	[22]
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VRAAG 2

2.1	$m_{AB} = \frac{7-5}{12-8}$ $m_{AB} = \frac{1}{2}$ <p>vergelyking van AB is</p> $y - 5 = \frac{1}{2}(x - 8)$ $y = \frac{1}{2}x + 1$ $2y - x = 2$ $x - 2y + 2 = 0$	<p>✓ $m_{AB} = \frac{1}{2}$</p> <p>✓ substitusie</p> <p>✓ antwoord</p> <p style="text-align: right;">(3)</p>
2.2	<p>At D: $x - 2(0) + 2 = 0$</p> $x = -2$ <p>D(-2 ; 0)</p>	<p>✓ substitusie van $y = 0$</p> <p>✓ x antwoord</p> <p style="text-align: right;">(2)</p>
2.3	$C\left(\frac{8-2}{2}; \frac{5+0}{2}\right)$ $C\left(3; \frac{5}{2}\right)$	<p>✓ x - waarde</p> <p>✓ y - waarde</p> <p style="text-align: right;">(2)</p>
2.4	$AC^2 = (8-3)^2 + \left(5 - \frac{5}{2}\right)^2$ $AC^2 = 25 + \frac{25}{4}$ $AC^2 = \frac{125}{4}$ <p>vergelyking van die sirkel is</p> $(x-3)^2 + \left(y - \frac{5}{2}\right)^2 = \frac{125}{4}$	<p>✓ substitusie</p> <p>✓ AC^2</p> <p>✓ $(x - 3)$</p> <p>✓ $\left(y - \frac{5}{2}\right)$</p> <p>✓ antwoord</p> <p style="text-align: right;">(5)</p>
2.5	<p>gradient van raaklyn = -2 (raaklyn \perp radius)</p> <p>vergelyking van raaklyn is</p> $y - 5 = -2(x - 8)$ $y - 5 = -2x + 16$ $y = -2x + 21$	<p>✓ gradient</p> <p>✓ substitusie</p> <p>✓ antwoord</p> <p style="text-align: right;">(3)</p>

2.6	Simmetrie-as $x = 3$ $A'(-2; 5)$	<ul style="list-style-type: none"> ✓ simmetrie-as ✓ x-antwoord ✓ y-antwoord <p style="text-align: right;">(3)</p> <p style="text-align: right;">[18]</p>
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VRAAG 3

3.1.1	$P(2; -\sqrt{3})$	<ul style="list-style-type: none"> ✓✓ koordinate <p style="text-align: right;">(2)</p>
3.1.2	$P(-\sqrt{3}; 2)$	<ul style="list-style-type: none"> ✓✓ koordinate <p style="text-align: right;">(2)</p>
3.2.1		<ul style="list-style-type: none"> ✓ koordinate P ✓ koordinate Q ✓ koordinate R ✓ koordinate S <p style="text-align: right;">(4)</p>
3.2.2	$(x; y) \rightarrow (-x; -y)$ $(-x; -y) \rightarrow (-2x; -2y)$ $\therefore (x; y) \rightarrow (-2x; -2y)$	<ul style="list-style-type: none"> ✓ $(x; y) \rightarrow (-x; -y)$ ✓ $(-x; -y) \rightarrow (-2x; -2y)$ ✓✓ $\therefore (x; y) \rightarrow (-2x; -2y)$ <p style="text-align: right;">(4)</p>
3.2.3	Area ABCD : area PQRS $= 1 : 4$	<ul style="list-style-type: none"> ✓ antwoord <p style="text-align: right;">(1)</p>
3.3.1	Stel $r = OP = OP'$ Die x ko-ordinaat van $P' = r \cos(\alpha + 30^\circ)$ $x' = r \cos(\alpha + 30^\circ)$ $= r(\cos \alpha \cdot \cos 30^\circ - \sin \alpha \cdot \sin 30^\circ)$ $= r \cos \alpha \cdot \cos 30^\circ - r \sin \alpha \cdot \sin 30^\circ$ $= r \cdot \frac{x}{r} \cos 30^\circ - r \cdot \frac{y}{r} \sin 30^\circ$ $= x \cdot \frac{\sqrt{3}}{2} - y \cdot \frac{1}{2}$ Die y ko-ordinaat van P' is $r \sin(\alpha + 30^\circ)$	<ul style="list-style-type: none"> ✓ formule ✓ uitbreiding ✓ vereenvoudiging ✓ substitusie ✓ formule

	Soortgelyk $y' = y \cos 30^\circ + x \sin 30^\circ$ $= y \cdot \frac{\sqrt{3}}{2} + x \cdot \frac{1}{2}$	✓ uitbreiding ✓ vereenvoudiging ✓ substitusie (8)
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3.4	$K' = \left(x \cdot \frac{\sqrt{3}}{2} - y \cdot \frac{1}{2} ; y \cdot \frac{\sqrt{3}}{2} + x \cdot \frac{1}{2} \right)$ $= \left(4 \cdot \frac{\sqrt{3}}{2} - 3 \cdot \frac{1}{2} ; 3 \cdot \frac{\sqrt{3}}{2} + 4 \cdot \frac{1}{2} \right)$ $= (1.96 ; 4.60)$ $L' = \left(x \cdot \frac{\sqrt{3}}{2} - y \cdot \frac{1}{2} ; y \cdot \frac{\sqrt{3}}{2} + x \cdot \frac{1}{2} \right)$ $= \left(3 \cdot \frac{\sqrt{3}}{2} - 6 \cdot \frac{1}{2} ; 6 \cdot \frac{\sqrt{3}}{2} + 3 \cdot \frac{1}{2} \right)$ $= (-0.40 ; 6.70)$	✓ x-koordinaat van K' ✓ y-koordinaat van K' ✓ x-koordinaat van L' ✓ y-koordinaat van L' (4) [25]
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VRAAG 4

<p>4.1</p>	$\frac{\sin 140^\circ \cdot \tan(-315^\circ)}{\cos 230^\circ \cdot \sin 420^\circ}$ $= \frac{\sin 40^\circ \cdot (-\tan 315^\circ)}{(-\cos 50^\circ) \cdot \sin 60^\circ}$ $= \frac{\sin 40^\circ \cdot \tan 45^\circ}{-\sin 40^\circ \cdot \sin 60^\circ}$ $= -\frac{1}{\frac{\sqrt{2}}{\sqrt{3}}}$ $= -\frac{1}{\sqrt{2}} \times \frac{2}{\sqrt{3}}$ $= -\frac{2}{\sqrt{6}} \text{ or } -\sqrt{\frac{2}{3}}$	<ul style="list-style-type: none"> ✓ $\sin 40^\circ$ ✓ $-\cos 50^\circ$ ✓ $\tan 45^\circ$ ✓ $\sin 60^\circ$ ✓ $\sin 40^\circ$ <p>✓ antwoord (6)</p>
<p>4.2</p>	$\tan(180^\circ + x) \cdot \cos(540^\circ + x) \left(\sin(-x) + \frac{\sin^2(90^\circ - x)}{\cos(90^\circ + x)} \right)$ $= \tan x \cdot (-\cos x) \left(-\sin x + \frac{\cos^2 x}{-\sin x} \right)$ $= \frac{\sin x}{\cos x} \cdot (-\cos x) \left(\frac{\sin^2 x + \cos^2 x}{-\sin x} \right)$ $= \sin^2 x + \cos^2 x$ $= 1$	<ul style="list-style-type: none"> ✓ $\tan x$ ✓ $-\cos x$ ✓ $-\sin x$ ✓ $\cos^2 x$ ✓ $-\sin x$ ✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ $\sin^2 x + \cos^2 x$ ✓ antwoord (8)
<p>4.3</p>	$\sin 15^\circ$ $= \sin(45^\circ - 30^\circ)$ $= \sin 45^\circ \cdot \cos 30^\circ - \cos 45^\circ \cdot \sin 30^\circ$ $= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$ $= \frac{\sqrt{2}(\sqrt{3} - 1)}{4}$	<ul style="list-style-type: none"> ✓ $\sin 15^\circ$ ✓ $\sin(45^\circ - 30^\circ)$ ✓ uitbreiding ✓✓ substitusie (5) <p>(Nota: indien 60° en 45° gebruik word, sal dieselfde antwoord verkry word)</p>
<p>4.4.1</p>	$\cos 2\theta + 3 \cos \theta - 1$ $= 2 \cos^2 \theta - 1 + 3 \cos \theta - 1$ $= 2 \cos^2 \theta + 3 \cos \theta - 2$	<ul style="list-style-type: none"> ✓ $\cos 2\theta = 2 \cos^2 \theta - 1$ ✓ vereenvoudiging (2)

4.4.2	$\cos 2\theta + 3\cos \theta - 1 = 0$ $2\cos^2 \theta + 3\cos \theta - 2 = 0$ $(2\cos \theta - 1)(\cos \theta + 2) = 0$ $\cos \theta = \frac{1}{2} \text{ of } \cos \theta = -2 \text{ ongeldig}$ $\theta = \pm 60^\circ + k \cdot 360^\circ \quad k \in Z$	✓ faktore ✓ $\cos \theta = 2$ ongeldig ✓ antwoorde ✓ $k \in Z$ (4) [25]
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VRAAG 5

5.1	Joyce het nie die korrekte uitbreiding vir $\sin(A + B)$ gebruik nie.	✓ antwoord (1)
5.2	$\sin(45^\circ + 21^\circ)$ $= \sin 45^\circ \cdot \cos 21^\circ + \sin 21^\circ \cdot \cos 45^\circ$ $= \frac{\sqrt{2}}{2} \cdot (t) + \sqrt{1-t^2} \left(\frac{\sqrt{2}}{2}\right)$ $= \frac{\sqrt{2}}{2} (t + \sqrt{1-t^2})$	✓ uitbreiding ✓ substitusie ✓ $\sin 21 = \sqrt{1-t^2}$ ✓ antwoord (4) [5]

VRAAG 6

6.1	$\frac{BC}{OB} = \cos \theta$ $BC = 2a \cdot \cos \theta$	✓ $\frac{BC}{OB} = \cos \theta$ (1)
6.2	$\text{area } \Delta OBC = \frac{1}{2} \cdot OB \cdot BC \cdot \sin \theta$ $\text{area } \Delta OBC = \frac{1}{2} \cdot (2a) \cdot (2a \cdot \cos \theta) \cdot \sin \theta$ $\text{area } \Delta OBC = a^2 \cdot 2 \cos \theta \cdot \sin \theta$ $\text{area } \Delta OBC = a^2 \cdot \sin 2\theta$ <p>OF</p> $\text{area } \Delta OBC = \frac{1}{2} (OB) \cdot AC$ $= \frac{1}{2} \cdot (2a) (BC \sin \theta)$ $= a (2a \cdot \cos \theta) (\sin \theta) = a^2 \sin 2\theta$	✓ area reël ✓ substitusie ✓ dubbel hoek (3)

6.3	<p>Area sal 'n maksimum wees indien $\sin 2\theta = 1$. $\theta = 45^\circ$ ΔOBC sal gelykbenig wees $C(a; a)$</p> <p>OF</p> <p>Maximum indien AC 'n maksimum is, as A by D is, dus $AC = a$</p>	<p>✓ antwoord</p> <p>(1)</p> <p>[5]</p>
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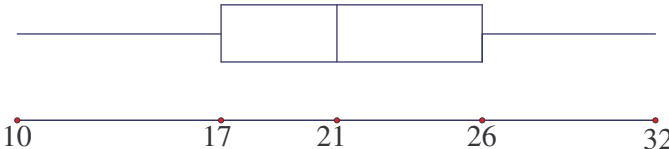
VRAAG 7

7.1	$\frac{20,7}{BD} = \cos 43,6^\circ$ $BD = \frac{20,7}{\cos 43,6}$ $BD = 28,6 \text{ m}$	<p>✓ verhouding ✓ los op vir BD</p> <p>✓ antwoord</p> <p>(3)</p>
7.2	$BE^2 = (28,58)^2 + (28,1)^2 - 2(28,58)(28,1)\cos 35,7^\circ$ $= 302,0610874 \dots$ $BE = 17,4 \text{ m}$	<p>✓✓ cos reël ✓ substitusie ✓ antwoord</p> <p>(4)</p>
7.3	$\text{Area } \Delta BEC = \frac{1}{2}(20,7)(17,38)\sin 63^\circ$ $= 160,4 \text{ vierkante meter}$	<p>✓ substitusie ✓ antwoord</p> <p>(2)</p> <p>[9]</p>

VRAAG 8

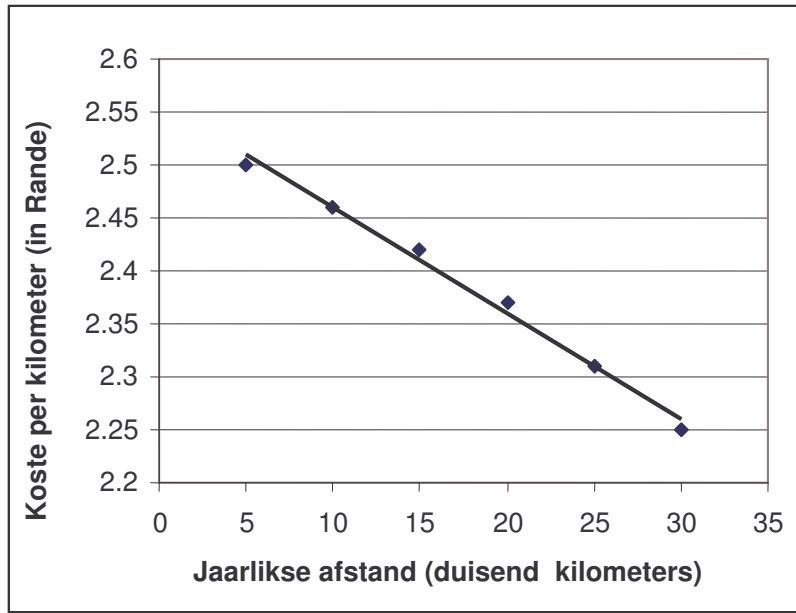
8.1	$\cos \frac{x}{2} = \sin(x - 30^\circ)$ $\cos \frac{x}{2} = \cos(90^\circ - x + 30^\circ)$ $\frac{x}{2} = 120^\circ - x + k.360^\circ \quad \text{or} \quad \frac{x}{2} = -120^\circ + x + k.360^\circ \quad k \in Z$ $\frac{3x}{2} = 120^\circ + k.360^\circ \quad \frac{-x}{2} = -120^\circ + k.360^\circ$ $x = 80 + k.240^\circ \quad x = 240^\circ - k.720^\circ$ $x = 80^\circ; -160^\circ$	$\checkmark \cos(90^\circ - x + 30^\circ)$ $\checkmark \checkmark$ antwoord $\checkmark \checkmark$ antwoorde $\checkmark \checkmark$ antwoord (7)
8.2	$-160^\circ < x < 80^\circ$	\checkmark kritiese waardes \checkmark stelling (2)
[9]		

VRAAG 9

9.1	10, 13, 13, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 30, 32. Mediaan = 21	\checkmark geordende data \checkmark antwoord (2)
9.2	Onderste kwartiel = 17 Boonste kwartiel = 26	\checkmark Q_1 \checkmark Q_3 (2)
9.3		$\checkmark \checkmark$ kwartiele $\checkmark \checkmark$ mond \checkmark snor (5)
9.4	Die range van Geoff en Thabo se reise is dieselfde. Thabo se opsomming neig na regs terwyl Geoff se opsomming is meer eweredig versprei. Hieruit kan afgelei word dat Thabo langer afstande afgelê het as Geoff.	\checkmark range dieselfde \checkmark verduideliking van verspreiding (2)
[11]		

VRAAG 10

10.1
&
10.2



✓✓ plot punte
✓ labels

(3)

✓✓ beste paslyn

(2)

10.3

R 2,47 – R 2,48

✓ antwoorde in
hierdie variasiewyde

(1)

[6]

VRAAG 11

11.1	<p>Gemiddeld</p> <table border="1" data-bbox="414 226 1084 693"> <thead> <tr> <th>Persentasies</th> <th>Middelpunt van interval (x)</th> <th>Frekwensie (f)</th> <th>Totaal ($f \times x$)</th> </tr> </thead> <tbody> <tr> <td>10 – 19</td> <td>14,5</td> <td>6</td> <td>87</td> </tr> <tr> <td>20 – 29</td> <td>24,5</td> <td>14</td> <td>343</td> </tr> <tr> <td>30 – 39</td> <td>34,5</td> <td>16</td> <td>552</td> </tr> <tr> <td>40 – 49</td> <td>44,5</td> <td>11</td> <td>489,5</td> </tr> <tr> <td>50 – 59</td> <td>54,5</td> <td>3</td> <td>163,5</td> </tr> <tr> <td colspan="3">Som</td> <td>1635</td> </tr> </tbody> </table> <p>=</p> $\frac{1635}{50} = 32,7$	Persentasies	Middelpunt van interval (x)	Frekwensie (f)	Totaal ($f \times x$)	10 – 19	14,5	6	87	20 – 29	24,5	14	343	30 – 39	34,5	16	552	40 – 49	44,5	11	489,5	50 – 59	54,5	3	163,5	Som			1635	<p>✓Middelpunte ✓✓✓ totaal) Een punt vir elke twee korrekte getalle in die laaste kolom</p> <p>✓ antwoord (5)</p>
Persentasies	Middelpunt van interval (x)	Frekwensie (f)	Totaal ($f \times x$)																											
10 – 19	14,5	6	87																											
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40 – 49	44,5	11	489,5																											
50 – 59	54,5	3	163,5																											
Som			1635																											
11.2	$\bar{x} = \frac{\sum x}{n} = \frac{7+4+9+4+9+5+4+6}{8} = 6$ $\delta = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$ $= \sqrt{\frac{(7-6)^2 + (4-6)^2 + (9-6)^2 + (4-6)^2 + (9-6)^2 + (5-6)^2 + (4-6)^2 + (6-6)^2}{8}}$ $= \sqrt{\frac{1+4+9+4+9+1+4+0}{8}} = \sqrt{4} = 2$	<p>✓✓ gemiddeld</p> <p>✓ formule</p> <p>✓✓ berekening</p> <p>✓ vereenvoudiging</p> <p>✓ antwoord (7)</p> <p>[12]</p>																												

VRAAG 12

12.1	Verskaffer B	✓ antwoord (1)
12.2	Ek sou verskaffer A kies. Die grafiek vir verskaffer A wys 'n relatief konstante leeftyd vir hul gloeilampe. Die grafiek vir verskaffer B wys gloeilampe met 'n langer leeftyd as verskaffer A, maar ook gloeilampe met 'n korter leeftyd as verskaffer A.	✓ verskaffer A ✓ verduideliking (2) [3]