



CHIEF DIRECTORATE – CURRICULUM MANAGEMENT

**GRADE 12 LEARNER SUPPORT
PROGRAMME**

**REVISION AND REMEDIAL TEACHING
INSTRUMENT:
ANSWERS**

SUBJECT: MATHEMATICS – FIRST PAPER

June 2009

This document consists of 14 pages.

Strictly not for test/examination purposes

QUESTION 1

1.1 1.1.1 $(3 - x)(2x + 3) = 4$
 $6x + 9 - 2x^2 - 3x = 4$ ✓ multiplication
 $2x^2 - 3x - 5 = 0$ ✓ std form
 $(2x - 5)(x + 1) = 0$ ✓ factorise
 $x = \frac{5}{2}$ ✓ or $x = -1$ ✓ answers (5)

1.1.2 $2x^2 + 7x - 5 = 0$ ✓ std form

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(7) \pm \sqrt{(7)^2 - 4(2)(-5)}}{2(2)}$$

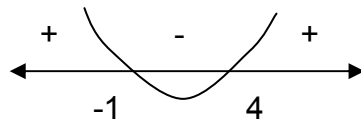
$$= \frac{-7 \pm \sqrt{49 + 40}}{4}$$

$$= \frac{-7 \pm \sqrt{89}}{4}$$

$x = 0,61$ ✓ or $x = -4,11$ ✓ answers (4)
 (-1 for incorrect rounding off)

1.1.3 $x - 3 \leq \frac{4}{x}$; $x > 0$

$$x^2 - 3x \leq 4$$
 ✓ multiplication
 $x^2 - 3x - 4 \leq 0$
 $(x - 4)(x + 1) \leq 0$ ✓ factorise



but $-1 \leq x \leq 4$ ✓✓ answers
 $x > 0$
 $0 < x \leq 4$ ✓ answer (5)

1.2 OPTION 1

$$2x + y = 3 \dots (1)$$

$$\therefore y = 3 - 2x \dots (3)$$

$$x^2 + y + x = y^2 \dots (2)$$

√

y the subject

Subst. (3) into (2):

$$x^2 + (3 - 2x) + x = (3 - 2x)^2$$

√

substitution

$$x^2 + 3 - 2x + x = 9 - 12x + 4x^2$$

$$3x^2 - 11x + 6 = 0$$

√

std form

$$(3x - 2)(x - 3) = 0$$

√

factorise

$$x = \frac{2}{3} \quad \text{or} \quad x = 3$$

√

both answers

Subst. into (3):

$$y = 3 - 2\left(\frac{2}{3}\right) \quad \text{or} \quad y = 3 - 2(3)$$

$$= \frac{5}{3} \quad \checkmark \quad \quad \quad = -3 \quad \checkmark \quad \text{answers}$$

(7)

OPTION 2

$$2x + y = 3 \dots (1)$$

$$x^2 + y + x = y^2 \dots (2)$$

$$\therefore x = \frac{3 - y}{2} \dots (3)$$

√

x the subject

Subst. (3) into (2):

$$\left(\frac{3 - y}{2}\right)^2 + y + \left(\frac{3 - y}{2}\right) = y^2$$

√

substitution

$$\frac{9 - 6y + y^2}{4} + y + \frac{3 - y}{2} = y^2$$

$$9 - 6y + y^2 + 4y + 6 - 2y = 4y^2$$

$$3y^2 + 4y - 15 = 0$$

√

std form

$$(3y - 5)(y + 3) = 0$$

√

factorise

$$y = \frac{5}{3} \quad \text{or} \quad y = -3$$

√

both answers

Subst. into (3):

$$x = \frac{3 - \frac{5}{3}}{2} \quad \text{or} \quad x = \frac{3 - (-3)}{2}$$

$$= \frac{5}{3} \quad \checkmark \quad \quad \quad = 3 \quad \checkmark \quad \text{each answer}$$

(7)

[21]

QUESTION 2

- 2.1 $3x = x\left(1 + \frac{0,12}{4}\right)^{4n}$ ✓ substitution in formula
 $3 = (1,03)^{4n}$ ✓ simplification
 $4n = \frac{\log 3}{\log 1,03}$ ✓ logs
 $= 37,167\dots$
 $n = 9,29$ years ✓ answer (4)
- 2.2 2.2.1 $P = \frac{x[1 - (1+i)^{-n}]}{i}$ ✓ formula
 $= \frac{5000\left[1 - \left(1 + \frac{0,142}{12}\right)^{-60}\right]}{\frac{0,142}{12}}$ ✓✓ value of n / value of i
 $= R\ 213\ 930,57$ ✓ present value
 Deposit = 250 000 - 213 930,57 ✓
 $= R\ 36\ 069,43$ answer (5)
- 2.2.2 $i_{\text{eff}} = \left(1 + \frac{i}{m}\right)^m - 1$ ✓ formula
 $= \left(1 + \frac{0,142}{12}\right)^{12} - 1$ ✓ substitution
 $= 0,151616\dots$
 $r = 15,16\%$ p.a. ✓ answer (3)
- 2.3 $F = \frac{x[(1+i)^n - 1]}{i}$ ✓ formula
 $325\ 000 = \frac{x\left[\left(1 + \frac{0,085}{12}\right)^{48} - 1\right]}{\frac{0,085}{12}}$ ✓ substitution
 $2302,8033\dots = x(0,40326\dots)$ ✓ simplification
 $R5708,62 = x$ ✓ answer (4)

[16]

QUESTION 3

3.1 3.1.1 $T_n = ar^{n-1}$ ✓ formula
 $T_{10} = 32 \cdot \left(-\frac{1}{2}\right)^9$ ✓ substitution
 $= -\frac{1}{16}$ ✓ answer (3)

3.1.2 $S_{10} = \frac{a(r^n - 1)}{r - 1}$ ✓ formula
 $= \frac{32\left[\left(-\frac{1}{2}\right)^{10} - 1\right]}{-\frac{1}{2} - 1}$ ✓ substitution
 $= 21,31 \text{ or } \frac{341}{16}$ ✓ answer (3)

3.2 $\sum_{k=1}^n (19 - 2k) = 0$
 $17 + 15 + 13 + \dots + (19 - 2k) = 0$ ✓ series
 $a = 17 ; d = -2$
 $S_n = \frac{n}{2} [2a + (n - 1)d]$
 $0 = \frac{n}{2} [2(17) + (n - 1)(-2)]$ ✓ substitution
 $0 = \frac{n}{2} [34 - 2n + 2]$
 $0 = \frac{n}{2} [36 - 2n]$
 $0 = 18n - n^2$ ✓ standard form
 $0 = n(18 - n)$
 $n = 0 \text{ or } n = 18$ ✓✓ answers (5)
[11]

QUESTION 4

4.1 30 matchsticks ✓ answer (1)

4.2 $h^2 = 4^2 - 2^2$ ✓ Pythagoras
 $= 16 - 4$
 $= 12$

Height of 1 Δ is $\sqrt{12}$ ✓ solution

Height of 4 storey tower = $4\sqrt{12}$ cm ✓ answer (3)
 $= 8\sqrt{3}$ cm or 13,86 cm

4.3 3 ; 9 ; 18 ; 30
 First difference 6 9 12
 Second difference 3 3 ✓ 2nd difference

$$T_n = an^2 + bn + c$$

OPTION 1 subst n = 1 ; n = 2 ; n = 3

$$T_1 = a + b + c = 3 \dots\dots (1)$$

$$T_2 = a(2)^2 + b(2) + c = 9$$

$$= 4a + 2b + c = 9 \dots\dots (2) \quad \checkmark \quad \text{setting up equation}$$

$$T_3 = a(3)^2 + 3b(3) + c = 18$$

$$= 9a + 3b + c = 18 \dots\dots (3) \quad \checkmark \quad \text{setting up equation}$$

$$T_2 - T_1 = 3a + b = 6 \quad \text{and} \quad T_3 - T_2 = 5a + b = 9$$

$$3a + b = 6 \dots\dots (4)$$

$$5a + b = 9 \dots\dots (5)$$

$$(5) - (4) : \quad 2a = 3 \quad \therefore a = \frac{3}{2} \quad \checkmark \quad \text{value of a}$$

$$\text{Subst. into (4):} \quad 3\left(\frac{3}{2}\right) + b = 4$$

$$b = \frac{3}{2} \quad \checkmark \quad \text{value of b}$$

$$\text{Subst. into (1):} \quad \left(\frac{3}{2}\right) + \left(\frac{3}{2}\right) + c = 3$$

$$c = 0 \quad \checkmark \quad \text{value of c}$$

$$T_n = \frac{3}{2}n^2 + \frac{3}{2}n \quad \checkmark \quad \text{answer} \quad (7)$$

OPTION 2

$$2a = 3 \quad \therefore a = \frac{3}{2}$$

✓ value of a

$$3a + b = 6$$

✓ setting up equation

$$3\left(\frac{3}{2}\right) + b = 6 \quad \therefore b = \frac{3}{2}$$

✓ value of b

$$a + b + c = 3$$

✓ setting up equation

$$\left(\frac{3}{2}\right) + \left(\frac{3}{2}\right) + c = 3$$

$$c = 0$$

✓ value of c

$$T_n = \frac{3}{2} n^2 + \frac{3}{2} n$$

✓ answer

(7)
[11]

QUESTION 5

5.1 $r = 1 + 2x$

✓ value of r

$$-1 < r < 1$$

$$-1 < 1 + 2x < 1$$

✓ substitution

$$-2 < 2x < 0$$

$$-1 < x < 0$$

✓ answer

(3)

5.2 $S_\infty = 1$

$$\frac{a}{1-r} = 1$$

✓ formula

$$\frac{1+2x}{1-(1+2x)} = 1$$

✓ substitution in formula

$$1 + 2x = -2x$$

$$4x = -1$$

✓ simplification

$$x = -\frac{1}{4}$$

✓ answer

(4)
[7]

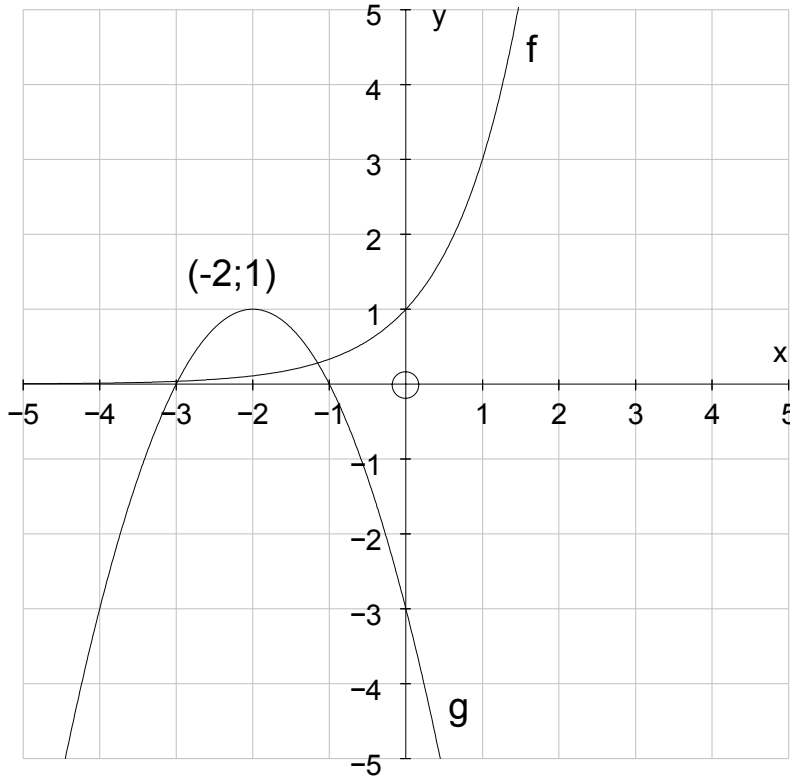
QUESTION 6

- 6.1 $-2 = a(-2)^2$ $(-2; -2)$ \checkmark substitution
 $2 = 4a$
- $-\frac{1}{2} = a$ \checkmark answer
- and $-2 = \frac{k}{-2}$ $(-2; -2)$ \checkmark substitution
 $k = 4$ \checkmark answer (4)
- 6.2 g has been shifted
 1 unit vertically upwards and \checkmark answer
 2 units horizontally to the right. \checkmark answer (2)
- 6.3 $f^{-1}: x = -\frac{1}{2}y^2$ \checkmark swapping x and y
 $-2x = y^2$
 $\therefore y = \pm\sqrt{-2x}$ \checkmark answer (2)
- 6.4 $x \geq 0$ or $x \leq 0$ $\checkmark\checkmark$ answers (2)
[10]

QUESTION 7

- 7.1 $g(x) = -x^2 - 4x - 3$
 $= -[x^2 + 4x + 3]$ \checkmark common factor
 $= -[x^2 + 4x + 4 - 4 + 3]$ \checkmark add & subtract
 $= -[(x + 2)^2 - 1]$ \checkmark factorise
 $= -(x + 2)^2 + 1$ \checkmark answer (4)
- 7.2 T.P. $(-2; 1)$ $\checkmark\checkmark$ answers (2)
- 7.3 y-int: $(0; -3)$ \checkmark answer
 x-int: $x^2 + 4x + 3 = 0$
 $(x + 1)(x + 3) = 0$ \checkmark factorise
 $x = -1$ or $x = -3$ $\checkmark\checkmark$ answers
 $(-1; 0)$ or $(-3; 0)$ (4)

7.4



Parabola:

- ✓ x-ints
- ✓ y-int
- ✓ turning point
- ✓ shape

Exponential:

- ✓ y-int
- ✓ shape

(6)

7.5 $f^{-1}: x = 3^y$
 $y = \log_3 x$

- ✓ swapping x and y
- ✓ answer

(2)

7.6 $h(x) = 3^{-x}$ or $(\frac{1}{3})^x$

- ✓ answer

(1)

[19]

QUESTION 8

8.1 $a = 1$
 $b = 2$
 $c = 3$

- ✓ answer
- ✓ answer
- ✓ answer

(3)

8.2 120°

- ✓ answer

(1)

8.3 $x = -90^\circ$

- ✓ answer

(1)

[5]

QUESTION 9

- 9.1 9.1.1 $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$
- $$= \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \quad \checkmark \quad \text{substitution in formula}$$
- $$= \lim_{h \rightarrow 0} \frac{\frac{x - (x+h)}{x(x+h)}}{h} \quad \checkmark \quad \text{common denominator}$$
- $$= \lim_{h \rightarrow 0} \frac{-h}{x(x+h)} \times \frac{1}{h} \quad \checkmark \quad \text{simplification}$$
- $$= \lim_{h \rightarrow 0} \frac{-1}{x^2 + xh} \quad \checkmark \quad \text{cancelling}$$
- $$= \frac{-1}{x^2} \quad \checkmark \quad \text{answer} \quad (5)$$
- 9.1.2 $f'(-2) = -\frac{1}{(-2)^2}$
- $$= -\frac{1}{4} \quad \checkmark \quad \text{substitution}$$
- $$= -\frac{1}{4} \quad \checkmark \quad \text{answer} \quad (2)$$
- 9.1.3 gradient of tangent = $-\frac{1}{4}$ \checkmark gradient
- $$y = -\frac{1}{4}x + c \quad (-2 ; -\frac{1}{2})$$
- $$-\frac{1}{2} = -\frac{1}{4}(-2) + c \quad \checkmark \quad \text{substitution}$$
- $$c = -1$$
- $$\therefore y = -\frac{1}{4}x - 1 \quad \checkmark \quad \text{answer} \quad (3)$$
- 9.2 9.2.1 $y = 3x^4 - 2x^3 + x - 1$
- $$\frac{dy}{dx} = 12x^3 - 6x^2 + 1 \quad \checkmark\checkmark\checkmark \quad \text{each answer} \quad (3)$$
- 9.2.2 $y = 2\sqrt{x} + \frac{x}{2}$
- $$= 2x^{\frac{1}{2}} + \frac{x}{2} \quad \checkmark \quad \text{changing square root}$$
- $$\frac{dy}{dx} = x^{-\frac{1}{2}} + \frac{1}{2} \quad \checkmark\checkmark \quad \text{each answer} \quad (3)$$

(3)
[16]

QUESTION 10

- 10.1 $d = -3$ \checkmark answer (1)
- 10.2 x-intercepts: $x = 1$ or $x = 3$ $\checkmark\checkmark$ answers (2)
- 10.3 $3x^2 - 10x + 7 = 0$ \checkmark $f'(x) = 0$
 $(3x - 7)(x - 1) = 0$ \checkmark factors
 $x = \frac{7}{3}$ or $x \neq 1$
 $\therefore p = \frac{7}{3}$ \checkmark answer (3)
- 10.4 $y = \left(\frac{7}{3}\right)^3 - 5\left(\frac{7}{3}\right)^2 + 7\left(\frac{7}{3}\right) - 3$ \checkmark substitution
 $= -\frac{32}{27}$
 $\therefore q = -\frac{32}{27}$ \checkmark answer (2)
- 10.5 $-\frac{32}{27} < k < 0$ $\checkmark\checkmark$ answer (2)

[10]

QUESTION 11

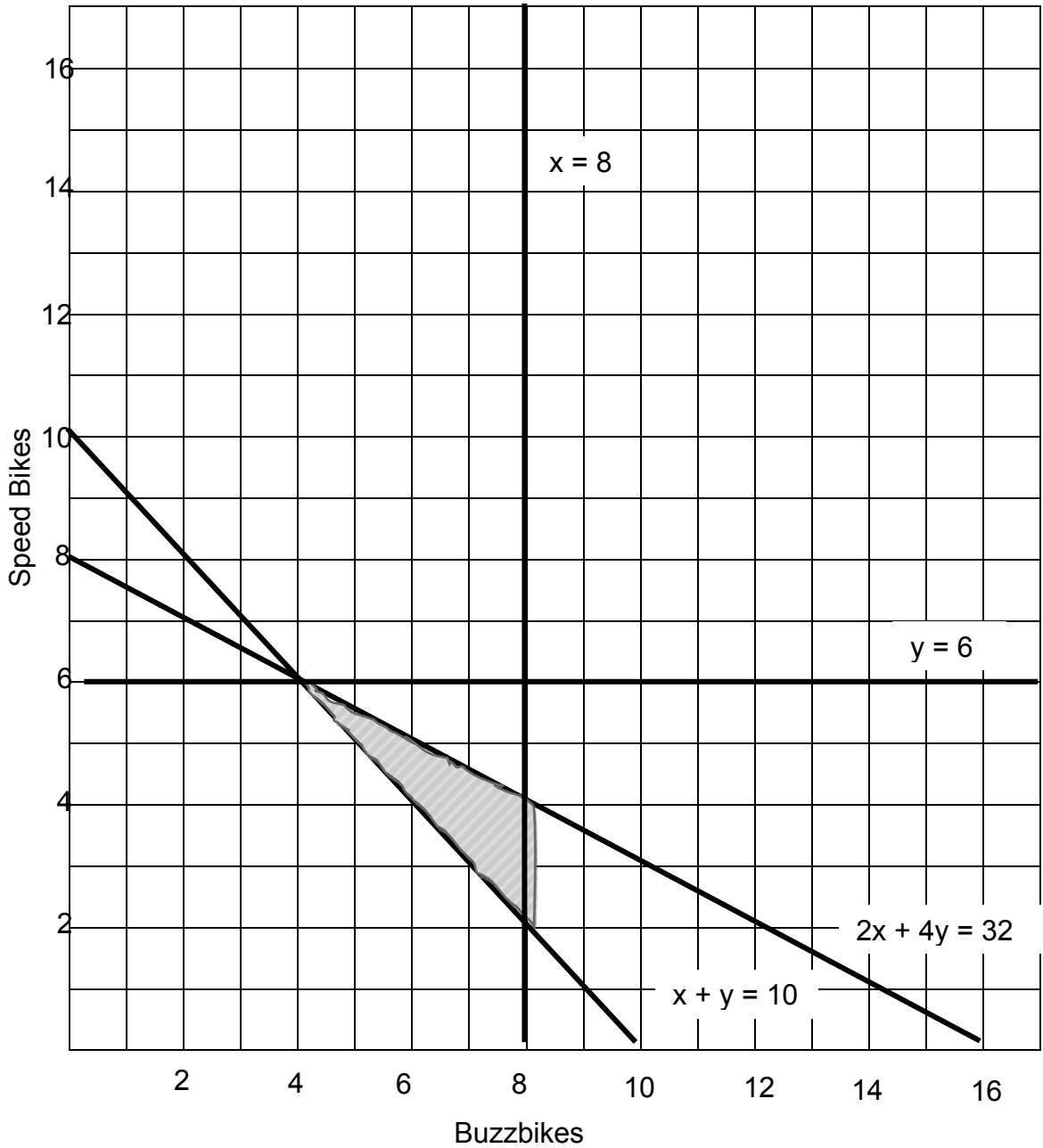
- 11.1
- | | | | |
|-----------------------------------|---|----------------|-----|
| Area = $2rd + \frac{1}{2}\pi r^2$ | √ | area | |
| $20 = 2rd + \frac{1}{2}\pi r^2$ | √ | substitution | |
| $2rd = 20 - \frac{1}{2}\pi r^2$ | √ | simplify | |
| $4rd = 40 - \pi r^2$ | | | |
| $d = \frac{40 - \pi r^2}{4r}$ | √ | d, the subject | (4) |
- 11.2
- | | | | |
|--|---|--------------|-----|
| $P = 2d + 2r + \pi r$ | √ | perimeter | |
| $= 2\left(\frac{40 - \pi r^2}{4r}\right) + 2r + \pi r$ | √ | substitute d | |
| $= \frac{20}{r} - \frac{\pi}{2}r + 2r + \pi r$ | | | |
| $= \frac{20}{r} + \frac{\pi}{2}r + 2r$ | | | (2) |
- 11.3
- | | | | |
|--------------------------------------|---|----------------|--|
| Min perimeter: $\frac{dP}{dr} = 0$ | √ | derivative = 0 | |
| $-20r^{-2} + \frac{\pi}{2} + 2 = 0$ | √ | derivative | |
| $\frac{\pi}{2} + 2 = \frac{20}{r^2}$ | | | |
| $r^2 = \frac{20}{\frac{\pi}{2} + 2}$ | √ | simplification | |
| $r^2 = 5,6\dots$ | | | |
| $r = 2,37\dots$ | √ | answer | |

(4)
[10]

QUESTION 12

- 12.1 $0 \leq x \leq 8$ ✓ constraint
- $0 \leq y \leq 6$ ✓ constraint
- $x + y \geq 10$ ✓ constraint
- $2x + 4y \leq 32$ ✓ constraint (4)

12.2



- ✓ $y = 6$
- ✓ $x = 8$
- ✓ $x + y = 10$
- ✓ $2x + 4y = 32$
- ✓ feasible region (5)

12.3 $P = 900x + 1200y$ ✓ answer (1)

12.4 OPTION 1

$P = 900(4) + 1200(6)$ (4 ; 6) ✓ substitution
 $= R10\ 800$

$P = 900(8) + 1200(4)$ (8 ; 4) ✓ substitution
 $= R12\ 000$

$P = 900(8) + 1200(2)$ (8 ; 2) ✓ substitution
 $= R9\ 600$

Maximum profit is R12 000 ✓ answer

OPTION 2

Objective function: $y = -\frac{3}{4}x + \frac{P}{1200}$

$m = -\frac{3}{4}$ ✓ search line

Optimum point: (8 ; 4) ✓ finding optimum point

Maximum profit: $P = 900(8) + 1200(2)$ ✓ substitution
 $= R12\ 000$ ✓ answer (4)
[14]

TOTAL: 150