Province of the
EASTERN CAPE
EDUCATION

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REPUBLIC OF SOUTH AFRICA

CHIEF DIRECTORATE – CURRICULUM MANAGEMENT

GRADE 12 LEARNER SUPPORT PROGRAMME

REVISION AND REMEDIAL TEACHING INSTRUMENT: QUESTIONS AND ANSWERS

SUBJECT: MATHEMATICS – FIRST PAPER

June 2009

This document consists of 12 pages.

Strictly not for test/examination purposes

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions:

1. This question paper consists of TWELVE questions. Answer ALL the questions.

2. Show clearly ALL calculations, diagrams, graphs, etcetera, which you have used in determining the answers.

3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

5. Number the answers correctly according to the numbering system used in this question paper.

6. Diagrams are NOT necessarily drawn to scale.

7. It is in your own interest to write legibly and to present your work neatly.

8. An information sheet with formulae is attached.

9. A diagram sheet is supplied for QUESTION 7.4 and QUESTION 12.2. Write your name in the space provided and then hand the diagram sheet in with your answer book.

QUESTION 1

1.1 Solve for x:

1.1.1 $(3 - x)(2x + 3) = 4$ (5)

1.1.2 $2x^2 + 7x = 5$ (4)

1.1.3 $x - 3 \leq \frac{4}{x}; \quad x > 0$ (5)

1.2 Solve for x and y simultaneously:
$2x + y = 3$ and $x^2 + y + x = y^2$. (7)
[21]

QUESTION 2

2.1 How long will it take a sum of money to be three times as big if the interest rate is 12% p.a. compounded quarterly. (4)

2.2 A caravan costs R250 000. Sarah wants to pay R5 000 per month for five years. The interest rate on a dealer financed loan, compounded monthly, is 14,2% p.a.

2.2.1 How much will she have to give the dealer as a deposit? (5)

2.2.2 Calculate the effective interest rate that she will pay the dealer per annum. (3)

2.3 Sarah's brother, Mark, decides to buy a caravan in 4 years time. He sets up a sinking fund to make provision for the purchase price of R325 000. The account used offers 8,5% interest p.a. compounded monthly. Calculate his monthly payments into the sinking fund. (4)
[16]

QUESTION 3

3.1 Consider the following sequence:

32 ; -16 ; 8 …

If this pattern continues consistently, determine:

3.1.1 The 10^{th} term of the sequence. (3)

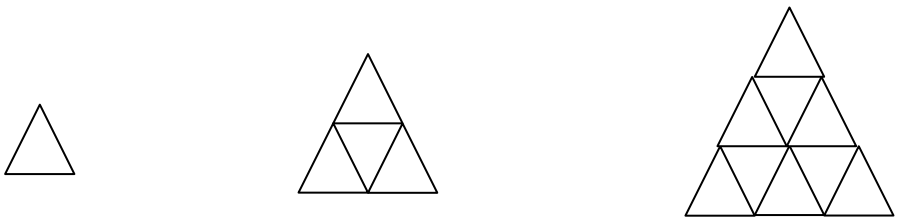
3.1.2 The sum of the first 10 terms. (3)

3.2 Find the value of n if $\sum_{k=1}^{n}(19-2k)=0$ (5)

[11]

QUESTION 4

During my summer vacation, Unathi enjoyed building towers out of matchsticks.
The picture below shows 1, 2 and 3 storey towers respectively.



1 storey tower	2 storey tower	3 storey tower
3 matchsticks	9 matchsticks	18 matchsticks

4.1 How many matchsticks are needed to build a 4 storey tower? (1)

4.2 Given that a match is 4 cm long and that we can consider each triangle to be equilateral, determine the approximate height of the 4 storey tower. (3)

4.3 Determine the formula for the number of matchsticks in the n^{th} storey tower. (7)

[11]

QUESTION 5

Given the series: $(1 + 2x) + (1 + 2x)^2 + (1 + 2x)^3 + \ldots$

5.1 For which values of x will the series converge? (3)

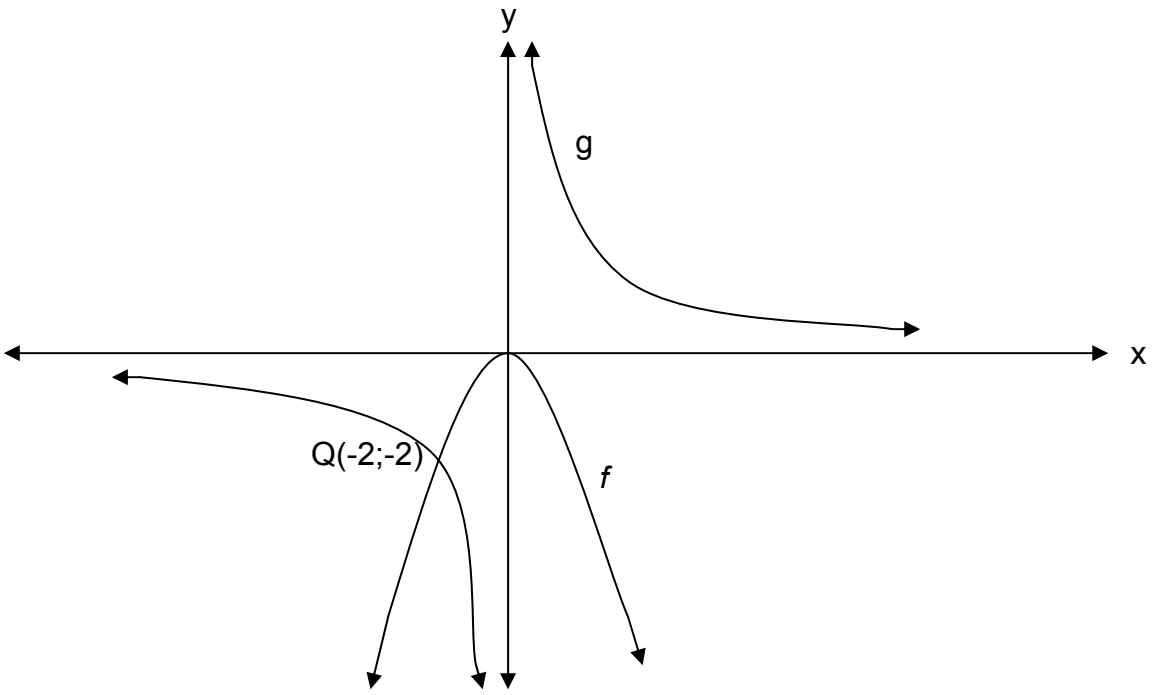
5.2 If $(1 + 2x) + (1 + 2x)^2 + (1 + 2x)^3 + \ldots = 1$, determine the value of x. (4)

[7]

QUESTION 6

The diagram below shows the graph of $f(x) = ax^2$ and $g(x) = \frac{k}{x}$.

The point Q (-2;-2) is the point of intersection of f and g.



6.1 Determine the value(s) of a and k. (4)

6.2 If g(x) is transformed to give $h(x) = \frac{4}{x-2} + 1$, describe the transformation that occurred to get h. (2)

6.3 Determine the equation of the inverse of f(x) in the form y = … (2)

6.4 How must the domain of f(x) be restricted so that $f^{-1}(x)$ will be a function? (2)

[10]

QUESTION 7

Given: $f(x) = 3^x$ and $g(x) = -x^2 - 4x - 3$

7.1 Write the equation of g(x) in the form $y = a(x - p)^2 + q$ (4)

7.2 Write down the coordinates of the turning point of g. (2)

7.3 Determine the x and y intercepts of g(x). (4)

7.4 On the system of axes provided on the diagram sheet, draw neat sketch graphs of f and g indicating all the intercepts, asymptotes and turning points. (6)

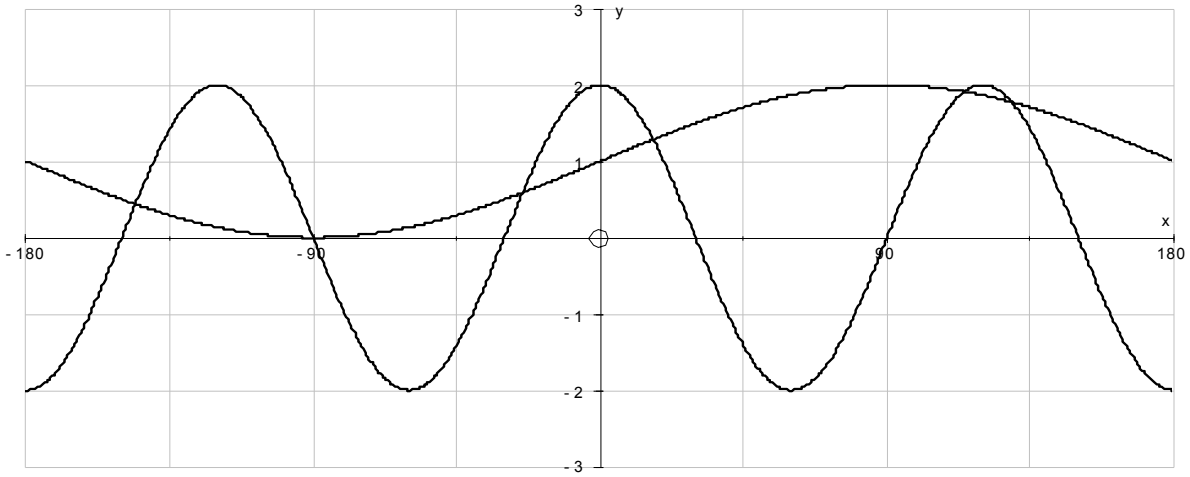
7.5 Determine $f^{-1}(x)$ in the form $y = \ldots$ (2)

7.6 f(x) is reflected about the y-axis. Determine the equation of the resultant function h(x). (1)

[19]

QUESTION 8

The graphs represent the following functions: $f(x) = \sin x + a$ and $g(x) = b \cos cx$ for $x \in [-180° ; 180°]$



8.1 Write down the values of a, b and c. (3)

8.2 What is the period of g? (1)

8.3 For which value(s) of x is f(x) = 0? (1)

[5]

QUESTION 9

9.1 Given: $f(x) = \frac{1}{x}$

9.1.1 Differentiate $f(x) = \frac{1}{x}$ from first principles. (5)

9.1.2 Hence, determine the gradient of f at the point with x = -2. (2)

9.1.3 Determine the equation of the tangent to the curve of f at $(-2 ; -\frac{1}{2})$. (3)

9.2 Determine $\frac{dy}{dx}$:

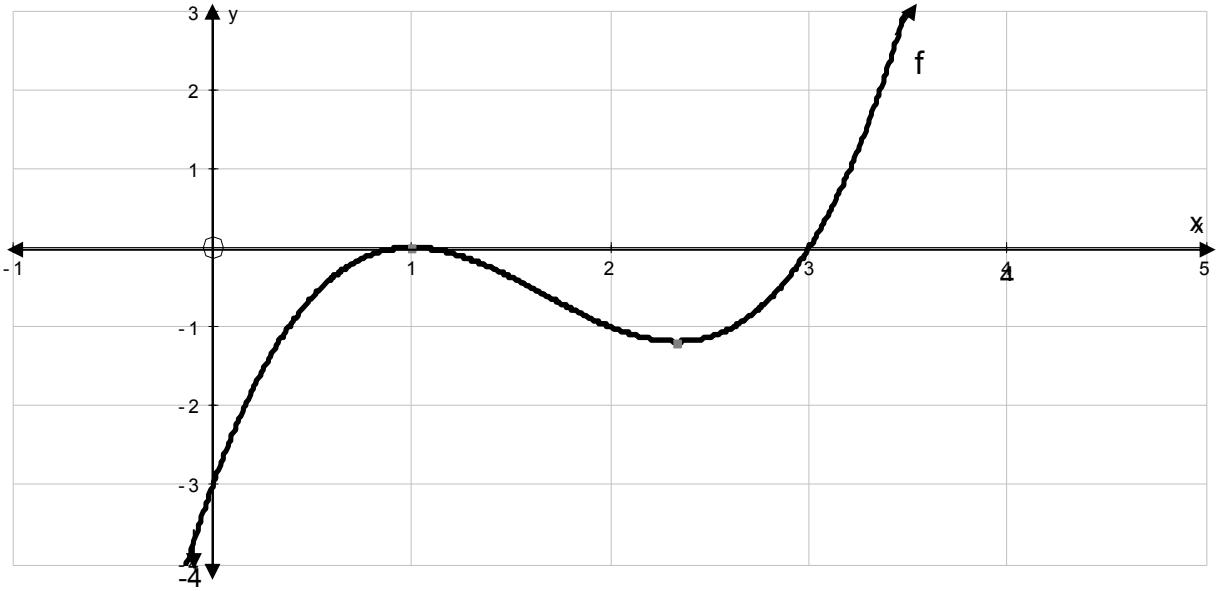
9.2.1 $y = 3x^4 - 2x^3 + x - 1$ (3)

9.2.2 $y = 2\sqrt{x} + \frac{x}{2}$ (3)

[16]

QUESTION 10

The sketch represents the function $f(x) = ax^3 + bx^2 + cx + d$, which touches the x-axis at x = 1 and has another of its turning points at (p ; q).



10.1 What is the value of d? (1)

10.2 Using the sketch above, write down the roots of the equation $f(x) = ax^3 + bx^2 + cx + d = 0$. (2)

10.3 If, for this graph, $f'(x) = 3x^2 - 10x + 7$, use this information to calculate the value of p , the x-coordinate of the turning point. (3)

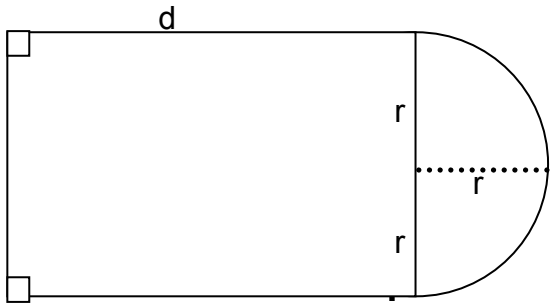
10.4 If $f(x) = x^3 - 5x^2 + 7x - 3$, determine the value of q, the y-coordinate of the turning point. (2)

10.5 Determine the values of k for which the equation f(x) = k will have three real roots. (2)

[10]

QUESTION 11

The figure below consists of a rectangle with length d and width 2r joined to a semi-circle with radius r. The area of the figure given below is 20 m^2.



Area of circle = πr^2
Circumference of circle $2\pi r$

11.1 Prove that the distance d = $\frac{40 - \pi r^2}{4r}$ (4)

11.2 Show that the perimeter P of the figure is given by:
$P = \frac{20}{r} + 2r + \frac{\pi . r}{2}$ (2)

11.3 Calculate the value of r when P is a minimum. Round your answer off to one decimal place. (4)
[10]

QUESTION 12

A factory produces two types of motorbikes, namely Buzzbikes and Speedbikes.

A maximum of 8 Buzzbikes and 6 Speedbikes can be produced per day, but management demands that in total at least 10 bikes must be manufactured daily.

There are 32 labourers available per day. It requires 2 labourers to produce one Buzzbike and 4 labourers to produce one Speedbike, per day.

The profit per Buzzbike is R900 and R1 200 per Speedbike.

Let the number of Buzzbikes be x and the number of Speedbikes be y.

12.1 Write down the constraints that satisfy the above information. (4)

12.2 Represent the constraints graphically on the graph paper provided, clearly indicating the feasible region. (5)

12.3 Write down an equation to determine the profit, P. (1)

12.4 Determine the maximum daily profit. (4)
[14]

TOTAL: 150

FORMULA SHEET: MATHEMATICS
FORMULEBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\qquad$ $A = P(1-ni)$

$A = P(1-i)^n$ $\qquad$ $A = P(1+i)^n$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\qquad$ $P = \frac{x[1-(1+i)^{-n}}{i}$

$\sum_{i=1}^{n} 1 = n$ $\qquad$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$$\sum_{i=1}^{n} (a + (i-1)d) = \frac{n}{2}(2a + (n-1)d)$$

$\sum_{i=1}^{n} ar^{i-1} = \frac{a(r^n - 1)}{r-1}$; r ≠ 1 $\qquad$ $\sum_{i=1}^{n} ar^{i-1} = \frac{a}{r-1}$; -1 < r < 1

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\qquad$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\qquad$ $y - y_1 = m(x - x_1)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\qquad$ $A = P(1+ni)$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\qquad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\qquad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\qquad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

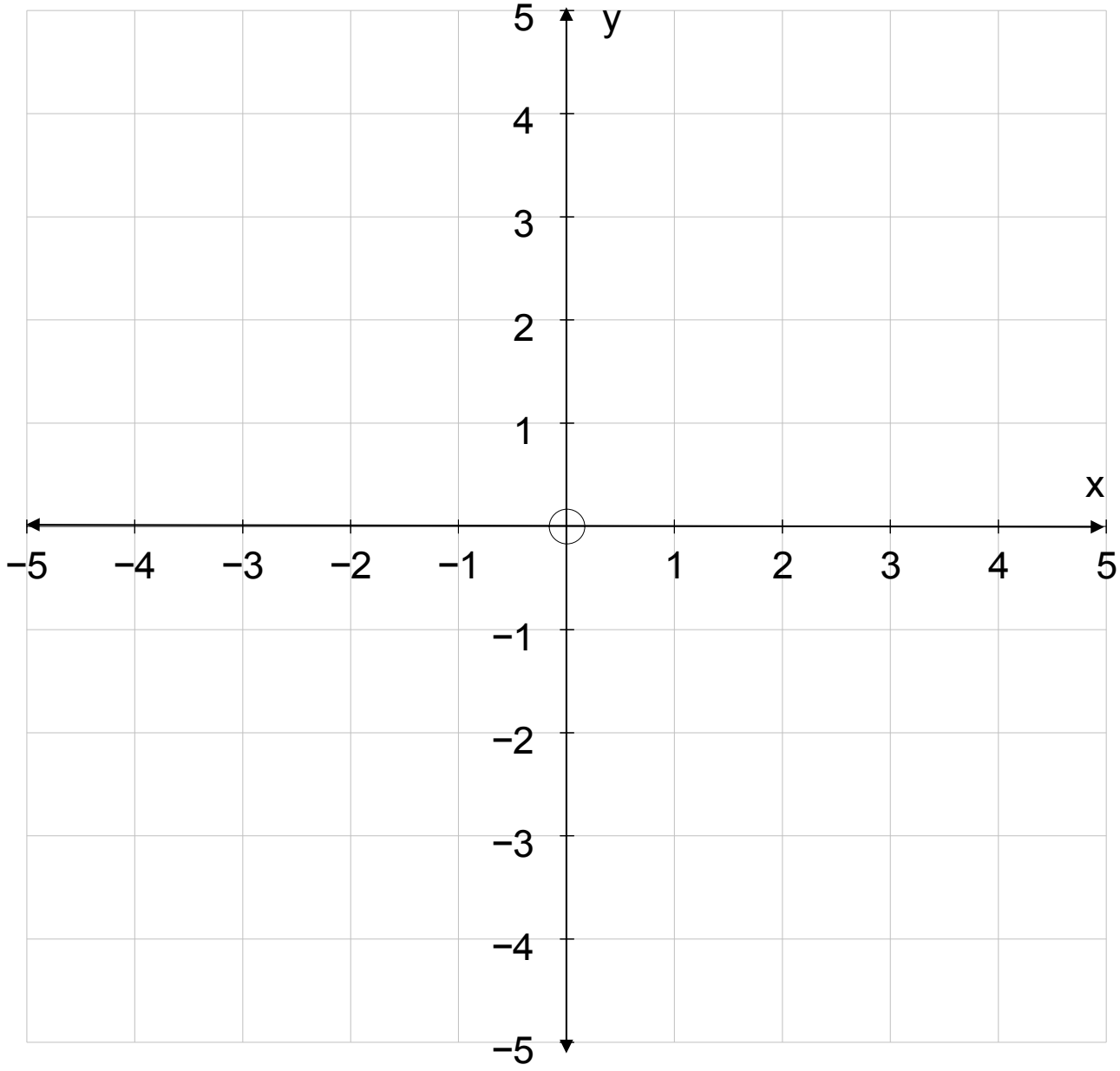
$P(A) = \frac{n(A)}{n(S)}$ $\qquad$ $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$

NAME/EXAMINATION NUMBER:

DIAGRAM SHEET 1

QUESTION 7

7.4



NAME/EXAMINATION NUMBER:

DIAGRAM SHEET 2

QUESTION 12

12.2