



Provinsie van die
OOS-KAAP
ONDERWYS

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REPUBLIEK VAN SUID-AFRIKA

HOOFDIREKTORAAT – KURRIKULUM BESTUUR

**GRAAD 12 LEERDER
ONDERSTEUNINGSPROGRAM**

**HERSIENING EN REMEDIËRENDE ONDERRIG
INSTRUMENT:
ANTWOORDE**

VAK: WISKUNDE – TWEEDE VRAESTEL

Junie 2009

Hierdie dokument bestaan uit 11 bladsye.

Streng gesproke nie vir toets/eksamen doeleindes nie.

VRAAG 1

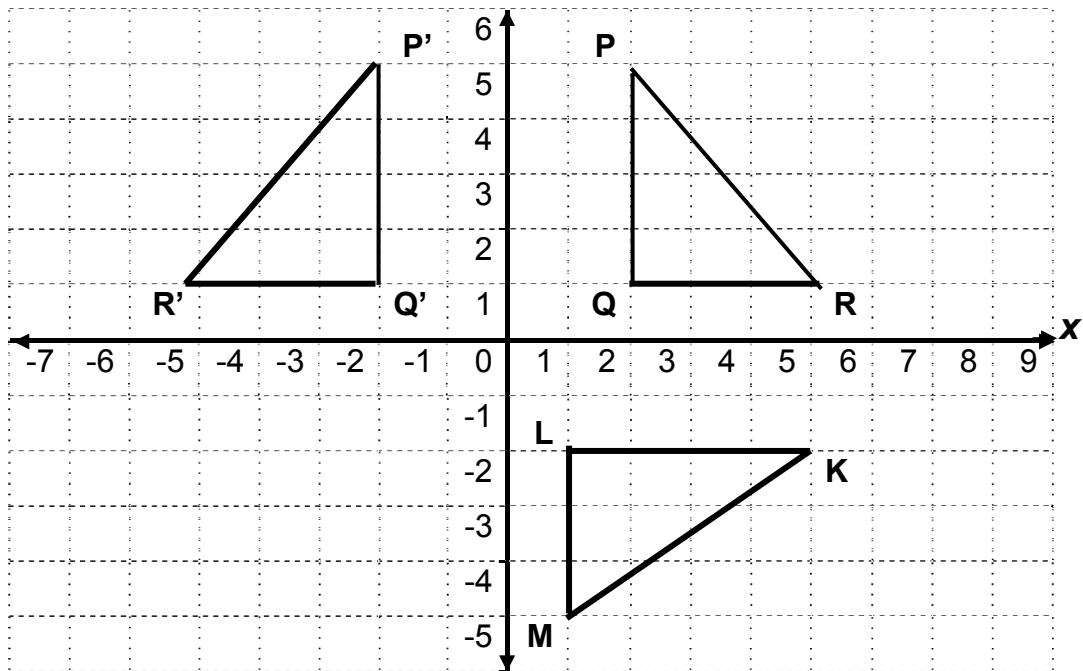
1.1	$P\left(\frac{-5+5}{2}; \frac{-2+4}{2}\right) = P(0; 1)$ $\therefore P \text{ lê op die } y\text{-as omdat } x = 0$	<ul style="list-style-type: none"> ✓ kor. vervanging. midpt. formule ✓ koörd. van P ✓ afleiding 	(3)
1.2	$M_{LM} = \frac{-2 - (-6)}{-5 - (-1)} = \frac{-2 + 6}{-5 + 1} = \frac{4}{-4} = -1$	<ul style="list-style-type: none"> ✓ kor. vervanging. midpt. formule ✓ antwoord 	(2)
1.3	$y = mx + c$ $\therefore y = -1x + 1$	<ul style="list-style-type: none"> ✓ form. reguit lyn ✓ gelyke gradiënte ✓ y-afsnit 	(3)
1.4	<p>Vervang N(2; -1) in $y = -x + 1$</p> <p>LHS = -1</p> <p>RHS = -(2) + 1</p> <p>= -1</p> <p>\therefore LHS = RHS</p> <p>\therefore N(2; -1) lê op $y = -x + 1$</p>	<ul style="list-style-type: none"> ✓ vervanging in vergelyking ✓ resultaat ✓ afleiding 	(3)
1.5	$LM = \sqrt{(-5+1)^2 + (-2+6)^2}$ $= \sqrt{(-4)^2 + (4)^2}$ $= \sqrt{16+16}$ $= \sqrt{32}$ $= 4\sqrt{2}$	<ul style="list-style-type: none"> ✓ afstand formule ✓ vervanging ✓ antwoord 	(3)
1.6	$2PN = 2\sqrt{(2-0)^2 + (1+1)^2}$ $= 2\sqrt{4+4}$ $= 2\sqrt{8}$ $= 2 \cdot 2\sqrt{2}$ $= 4\sqrt{2}$ <p>\therefore LM = 2PN</p>	<ul style="list-style-type: none"> ✓ vervanging ✓ vereenvoudiging ✓ antwoord ✓ eenvoudigste wortel vorm 	(4)

VRAAG 2

2.1	<p>y-afsnit</p> $3x + 2y - 12 = 0$ $\therefore y = -\frac{3}{2}x + 6$ $\therefore y = 6$ $\therefore K(0; 6)$ $RK^2 = KO^2 + OR^2 \text{ (Pyth)}$ $= 6^2 + 4^2$ $RK^2 = 52$ $\therefore RK = \sqrt{52}$ $= 2\sqrt{13}$	<p>✓ standaard vorm van vergelyking</p> <p>✓ waarde van y</p> <p>✓ Pythagoras</p> <p>✓ antwoord</p>	(4)
2.2	$y = -\frac{3}{2}x + 6$ $\therefore M_{KR} = -\frac{3}{2}$ $M_{KR} \cdot M_{NR} = -1 \text{ (radius } \perp \text{ tangent)}$ $\therefore M_{NR} = \frac{2}{3}$ $\therefore y = \frac{2}{3}x + c$ $(4; 0) : 0 = \frac{2}{3}(4) + c \text{ of } y - 0 = \frac{2}{3}(x - 4)$ $\therefore c = -\frac{8}{3} = -2\frac{2}{3} \quad \therefore y = \frac{2}{3}x - \frac{8}{3}$ $\therefore y = \frac{2}{3}x - 2\frac{2}{3}$ $\therefore 3y = 2x - 8$	<p>✓ prod. gradiënte - 1</p> <p>✓ gradiënt MR</p> <p>✓ vergelyking met \perp gradiënt</p> <p>✓ verv. van punt R</p> <p>✓ vergelyking in enige vorm</p>	(5)
2.3	$NR = NP \text{ (radii)}$ $NR^2 = NP^2$ $(x - 4)^2 + (y - 0)^2 = (x - 3)^2 + (y + 5)^2$ $x^2 - 8x + 16 + y^2 = x^2 - 6x + 9 + y^2 + 10y + 25$ $-10y = 2x + 18 \quad (1)$ $\text{Eq NR: } 3y = 2x - 8 \quad (2)$ $(1) - (2) \quad -13y = 26$ $[\div(-13)] \quad y = -2 \quad (3)$ $(3) \text{ in } (1) \quad -10(-2) = 2x + 18$ $\therefore 2x = 20 - 18$ $2x = 2$ $\therefore x = 1$ $\therefore N(1; -2)$	<p>✓ gelyke radiusse</p> <p>✓ vervanging beide kante</p> <p>✓ vereenvoudiging beide kante.</p> <p>✓ vergelyking (1)</p> <p>✓ waarde van y</p> <p>✓ waarde van x</p>	(8)
2.4	$(x - x_N)^2 + (y - y_N)^2 = (x_N - x_P)^2 + (y_N - y_P)^2$ $(x - 1)^2 + (y + 2)^2 = (1 - 4)^2 + (-2 - 0)^2$ $x^2 - 2x + 1 + y^2 + 4y + 4 = 9 + 4$ $x^2 + y^2 - 2x + 4y - 8 = 0$	<p>✓ stel afstande gelyk</p> <p>✓ vereenvoudiging</p> <p>✓ antwoord</p>	(5) [22]

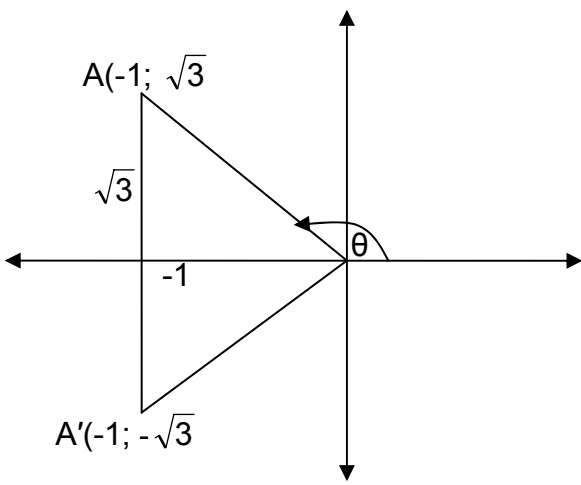
VRAAG 3

3.1.1	Skaal faktor van vergroting is 2	✓✓ antwoord	(2)
3.1.2	Oppervlakte van ABCD = 6 eenhede ² Oppervlakte van A'B'C'D' = 24 eenhede ²	✓✓ antwoord ✓✓ antwoord	(4)
3.1.3	Vier keer	✓ antwoord	(1)
3.1.4	Die vorm is eenders want die vorm word behou.. Die vorm is nie kongruent nie, want die grootte word nie behou in die vergroting nie.	✓ bewering & rede ✓ bewering & rede	(2)
3.2.1	(Sien diagram)	✓✓✓ afsteek van punte	(3)
3.2.2	(Sien diagram) P'(-2 ; 5), Q'(-2 ; 1), R'(-5 ; 1)	✓✓✓ korrekte koördinate van $\Delta P'Q'R'$	(3)
3.2.3	(Sien diagram) K(5 ; -2), L(1 ; -2), M(1 ; -5)	✓✓✓ korrekte koördinate van ΔKLM	(3)



3.2.4	P''(-1 ; 2), Q''(-1 ; -2), R''(-4 ; -2)	✓✓✓ koördinate	(3)
			[21]

VRAAG 4

4.1	 <p> $AO = (-1)^2 + (\sqrt{3})^2$ $= 1 + 3 \Rightarrow AO^2 = 4$ $\therefore AO = 2$ $\therefore \cos \theta = -\frac{1}{2}$ </p>	<p>✓ gebruik van Pythagoras of afstandformule</p> <p>✓ waarde van AO</p> <p>✓ waarde van $\cos \theta$</p>	(3)
4.2	<p> $A'(-1; -\sqrt{3})$ $\therefore \angle AOX = 180^\circ + 60^\circ$ $= 240^\circ$ </p>	<p>✓ beide koördinate</p> <p>✓ $180^\circ + 60^\circ$</p> <p>✓ antwoord</p>	(3) [6]

VRAAG 5

5.1	$-\frac{1}{2} \tan(-225^\circ) \cdot \cos^2 585$ $= -\frac{1}{2} \tan 45^\circ \cdot \cos^2 45^\circ$ $= \frac{1}{2} \cdot 1 \cdot \left(\frac{1}{\sqrt{2}}\right)^2$ $= \frac{1}{2} \cdot \frac{1}{2}$ $= \frac{1}{4}$	<p>✓✓ spesiale hoeke</p> <p>✓✓ 2 verhoudings</p> <p>✓ antwoord</p>	(5)
5.2	$\frac{\sin \theta - \cos \theta}{\tan \theta - 1} - \cos \theta$ $= \frac{\sin \theta - \cos \theta}{\frac{\sin \theta}{\cos \theta} - 1} - \cos \theta$ $= \frac{\sin \theta - \cos \theta}{\frac{\sin \theta - \cos \theta}{\cos \theta}} - \cos \theta$ $= \frac{\sin \theta - \cos \theta}{\sin \theta - \cos \theta} \cdot \cos \theta - \cos \theta$ $= \cos \theta - \cos \theta$ $= 0$	<p>✓ identiteit</p> <p>✓ vereenvoudiging van breuk</p> <p>✓ vereenvoudiging van breuk</p> <p>✓ antwoord</p>	(4) [9]

VRAAG 6

6.1	$\cos(x + 12,4^\circ) = -0,334$ $x + 12,4^\circ = 180^\circ + 70,49^\circ$ $\therefore x = 238,09^\circ$	✓ verwysingshoek ✓ kwadrant ✓ antwoord	(3)
6.2	$2\sqrt{3} \cdot \sin x \cdot \tan x - \sqrt{3} \cdot \tan x - 2 \sin x + 1 = 0$ $\sqrt{3} \tan x(2 \sin x - 1) - 1(2 \sin x - 1) = 0$ $\therefore \tan x = \frac{1}{\sqrt{3}} \text{ of } \sin x = \frac{1}{2}$ $\therefore x = 30^\circ + 180 \cdot k \text{ of } x = 30^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$ $x = 150^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$	✓ gemene faktor ✓ gemene faktor ✓✓ waardes van $\tan x$ en $\sin x$ ✓✓✓ waardes vir x , ✓ k gespesifiseerd	(8) [11]

VRAAG 7

<p>7.</p>			
7.1	f ✓ vorm ✓ y-afsnit ✓ x-afsnit ✓ draaipunte	g ✓ ✓ ✓ ✓	(8)

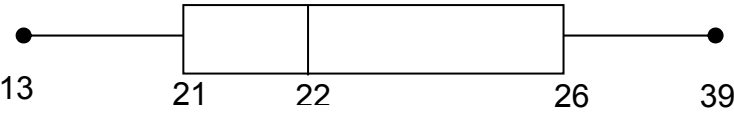
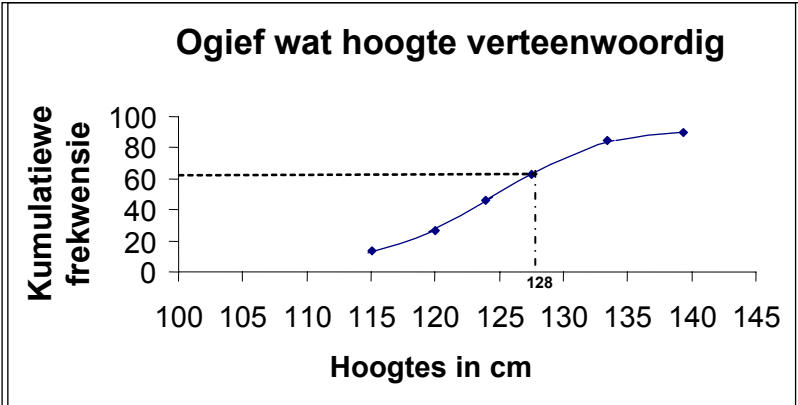
7.2	7.2.1	2✓		(1)
	7.2.2	360°✓		(1)
7.3	$\sin x - \frac{1}{2} + \frac{1}{2} \cos(x + 30^\circ) = \frac{1}{2} \rightarrow -2 \sin x + 1 - \cos(x + 30^\circ) = 1 \checkmark$ $x = 150^\circ$			(2)
7.4	$g: x \rightarrow \cos x + 1$ <ul style="list-style-type: none"> ✓ $\cos x$ ✓ $+ 1$ 			(2)
				[14]

VRAAG 8

8.1.1	<p>In $\triangle ACB$: $\hat{A}CB = 180^\circ - (x + y)$ hoeke van 'n driehoek</p> $\frac{AB}{\sin C} = \frac{BC}{\sin x}$ $\therefore AB = \frac{BC \cdot \sin[180^\circ - (x + y)]}{\sin x}$ $= \frac{BC \cdot \sin(x + y)}{\sin x}$	<ul style="list-style-type: none"> ✓ $\hat{A}CB$ ✓ sinus reël ✓ vervanging 	(3)
8.1.2	<p>$MN = AB$, $AMNB$ is 'n regtehoek</p> $\therefore \frac{h}{BC} = \tan z$ $\therefore BC = \frac{h}{\tan z}$ $MN = \frac{BC \cdot \sin(x + y)}{\sin x} \quad MN = AB$ $= \frac{h}{\tan z} \times \frac{\sin(x + y)}{\sin x}$ $= \frac{h \cdot \sin(x + y)}{\tan z \cdot \sin x}$	<ul style="list-style-type: none"> ✓ eienskappe van regtehoek ✓ tan verhouding ✓ waarde van BC ✓ vervanging van BC 	(4)

8.2	$MN = \frac{h \sin(x + y)}{\tan z \cdot \sin x}$ $70 = \frac{h \sin(52,3^\circ + 27,3^\circ)}{\tan 42^\circ \cdot \sin 52,3^\circ}$ $h = \frac{70 \cdot \tan 42^\circ \cdot \sin 52,3^\circ}{\sin 79,6^\circ}$ $\therefore h = 50,7 \text{ m}$	✓ vervanging ✓ onderwerp van formule ✓ antwoord	(3)
8.3	<p>In $\triangle ABC$: $AC^2 = AB^2 + BC^2 - 2AB \cdot BC \cos \hat{A}BC$</p> $= (70)^2 + (48,3)^2 - 2(70)(48,3)\cos 27,3^\circ$ $= 1224,06 \text{ m}$ $\therefore AC = 34,99 \text{ m}$ <p>\therefore Omtrek van $\triangle ABC$</p> $= 70 \text{ m} + 48,3 \text{ m} + 34,99 \text{ m}$ $= 153,29 \text{ m}$	✓ cos formule ✓ vervanging ✓ AC ✓✓ antwoord	(5)
8.4	$\text{Opp: } \triangle ABC = \frac{1}{2} AB \cdot BC \sin \hat{A}BC$ $= \frac{1}{2} \cdot 70 \cdot 48,3 \cdot \sin 27,3^\circ$ $= 775,3 \text{ m}^2$ $\therefore \text{aantal muise} = \frac{775,3}{10} = 77 \text{ muise}$ <p>(aanvaar 78)</p>	✓ oppervlakte reël ✓ vervanging ✓ oppervlakte ✓✓ antwoord	(5) [20]

VRAAG 9

9.1.1	<p>Laer kwartiel $Q_1 = 21$ Boonste kwartiel $Q_3 = 26$ Interkwartiel wydte = $Q_3 - Q_1 = 26 - 21 = 5$</p>	<p>✓21 ✓26 ✓5</p>	(3)																					
9.1.2		<p>✓ houer en punt ✓ 13 en 39 ✓ 21 en 26 ✓ 22</p>	(4)																					
9.1.3	Die verspreiding van data is skeef na regs.		(2)																					
9.2.1	<table border="1" data-bbox="352 689 1043 994"> <thead> <tr> <th>Hoogte in cm</th> <th>Frekwensie</th> <th>Kumulatiewe Frekwensie</th> </tr> </thead> <tbody> <tr> <td>$110 \leq h < 115$</td> <td>10</td> <td>10</td> </tr> <tr> <td>$115 \leq h < 120$</td> <td>14</td> <td>24</td> </tr> <tr> <td>$120 \leq h < 125$</td> <td>22</td> <td>46</td> </tr> <tr> <td>$125 \leq h < 130$</td> <td>23</td> <td>69</td> </tr> <tr> <td>$130 \leq h < 135$</td> <td>16</td> <td>85</td> </tr> <tr> <td>$135 \leq h < 140$</td> <td>5</td> <td>90</td> </tr> </tbody> </table>	Hoogte in cm	Frekwensie	Kumulatiewe Frekwensie	$110 \leq h < 115$	10	10	$115 \leq h < 120$	14	24	$120 \leq h < 125$	22	46	$125 \leq h < 130$	23	69	$130 \leq h < 135$	16	85	$135 \leq h < 140$	5	90	<p>✓✓ kum. frekwensie</p>	(2)
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9.2.2	<p>Ogief wat hoogte verteenwoordig</p> 	<p>✓ y-koördinate ✓ x-koördinate van punte (boonste limiet van interval) ✓ kurwe</p>	(3)																					
9.2.3	60 meisies is korter as 128 cm (aanvaar 55-65)	✓✓ antwoord	(2)																					

9.3.1	Gemiddelde gewig: $\bar{x} = \frac{304}{8} = 38$ ❖ slegs antwoord d.m.v. sakrekenaar: vol punte	✓ formule vir gem. gewig ✓ antwoord	(2)																																	
9.3.2	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th style="text-align: center;">Gewig in kg</th> <th style="text-align: center;">$x - \bar{x}$</th> <th style="text-align: center;">$(x - \bar{x})^2$</th> </tr> </thead> <tbody> <tr><td style="text-align: center;">26</td><td style="text-align: center;">-12</td><td style="text-align: center;">144</td></tr> <tr><td style="text-align: center;">28</td><td style="text-align: center;">-10</td><td style="text-align: center;">100</td></tr> <tr><td style="text-align: center;">32</td><td style="text-align: center;">-6</td><td style="text-align: center;">36</td></tr> <tr><td style="text-align: center;">35</td><td style="text-align: center;">-3</td><td style="text-align: center;">9</td></tr> <tr><td style="text-align: center;">38</td><td style="text-align: center;">0</td><td style="text-align: center;">0</td></tr> <tr><td style="text-align: center;">43</td><td style="text-align: center;">5</td><td style="text-align: center;">25</td></tr> <tr><td style="text-align: center;">47</td><td style="text-align: center;">9</td><td style="text-align: center;">81</td></tr> <tr><td style="text-align: center;">55</td><td style="text-align: center;">17</td><td style="text-align: center;">289</td></tr> <tr> <td style="text-align: center;">$\Sigma = 304$</td> <td></td> <td style="text-align: center;">$\Sigma = 684$</td> </tr> <tr> <td colspan="2" style="text-align: center;">$\bar{x} = \frac{304}{8} = 38$</td> <td></td> </tr> </tbody> </table>	Gewig in kg	$x - \bar{x}$	$(x - \bar{x})^2$	26	-12	144	28	-10	100	32	-6	36	35	-3	9	38	0	0	43	5	25	47	9	81	55	17	289	$\Sigma = 304$		$\Sigma = 684$	$\bar{x} = \frac{304}{8} = 38$			✓ ✓ $x - \bar{x}$ ✓ ✓ $(x - \bar{x})^2$ ✓ 684	(5)
Gewig in kg	$x - \bar{x}$	$(x - \bar{x})^2$																																		
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9.3.3	$\text{Variasie } \sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n} = \frac{684}{8} = 85,5$ $\therefore S.D. = \sqrt{85,5} = 9,25$ ❖ slegs antwoord d.m.v. sakrekenaar: vol punte	✓ formule vir variasie ✓ variasie ✓ vorm. St. afwyking ✓ standaard afwyk.	(4)																																	
9.3.4	interval 928,75;47,25) dus 5 mense	✓ ✓ antwoord	(2) [29]																																	
TOTAAL:			150																																	