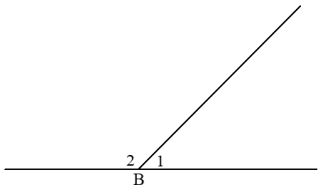
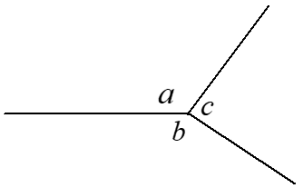
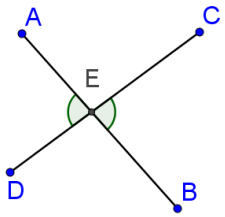
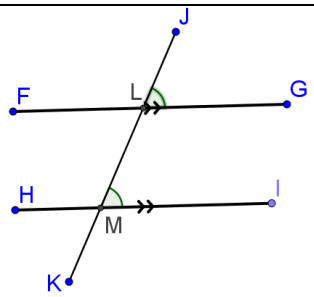
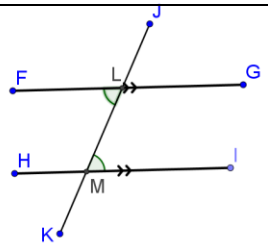
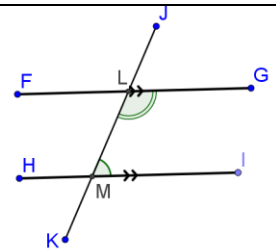
Province of the
EASTERN CAPE
EDUCATION

**HAND OUT TO CANDIDATES: SUMMARY OF IMPORTANT FACTS:**

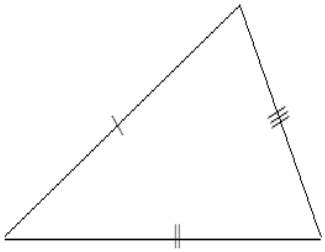
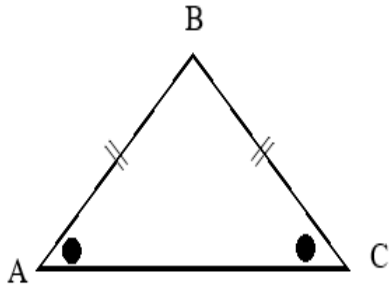
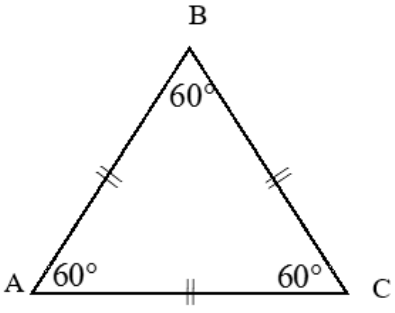
**TECHNICAL MATHEMATICS PROGRAMME FOR GRADE 12 LEARNERS FROM 11 – 29 MAY 2020**

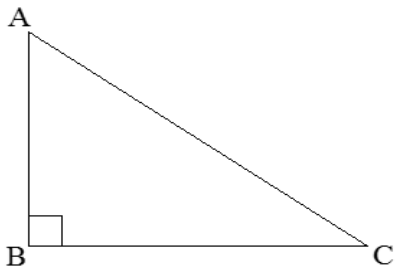
**TOPIC: EUCLIDEAN GEAOMETRY**

| **SUMMARY** | | |
|---|---|---|
| **Diagram** | **Explanantion** | **Mathematical statement** |
|  | Adjacent supplementaryangles | $\hat{B}_1 + \hat{B}_2 = 180°\ (\angle s \text{ on str line})$ |
|  | Angles round a point | $a+b+c = 360°\ (\angle s \text{ around pt})$ |
|  | Vertically opposite angles | $A\hat{E}D = B\hat{E}C\ (\text{vert opp } \angle s =)$<br>$A\hat{E}C = B\hat{E}D\ (\text{vert opp } \angle s =)$ |
|  | Corresponding angles<br>*"F-form"*<br>  | $J\hat{L}G = L\hat{M}I\ \ (\text{corresp } \angle s;\ FG \parallel HI)$ |
|  | Alternate angles<br>"Z-form"<br>  | $F\hat{L}M = L\hat{M}I\ \ (\text{alt } \angle s;\ FG \parallel HI)$ |

| | | |
|---|---|---|
|  | Co-interior angles<br><br>"U-shape"<br><br>U ⊂<br>⊃ ∩ | $G\hat{L}M + L\hat{M}I = 180°$ (co-int ∠s; FG \|\| HI) |

**SUMMARY FOR TRIANGLES**

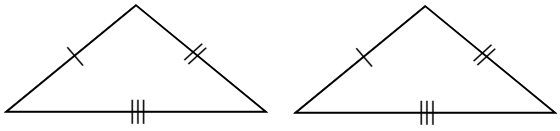
| | | |
|---|---|---|
|  | Scalene Triangle | No sides are equal in length |
|  | Isosceles Triangle, AB=BC | $\hat{A} = \hat{C}$ ($\angle s$ opp equal sides are =) |
|  | Equilateral Triangle,<br><br>AB=AC=BC | $\hat{A} = \hat{C} = \hat{B}$ (All $\angle s$ are =) |

| | | |
|---|---|---|
|  | The Theorem of Pythagoras<br>$\hat{A} = 90^{\circ}$ | $AC^2 = AB^2 + BC^2$<br>or<br>$AB^2 = AC^2 - BC^2$<br>or<br>$BC^2 = AC^2 - AB^2$<br>(sqaure on hyp=sum of sqaures on other 2 sides) |
| | | |

## CONGRUENCY OF TRIANGLES (four conditions)

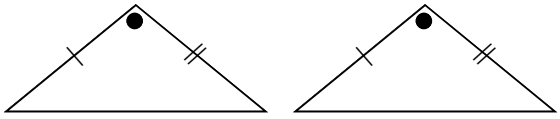
**Condition 1**

Two triangles are congruent if three sides of one triangle are equal in length to the three sides of the other triangle (SSS).



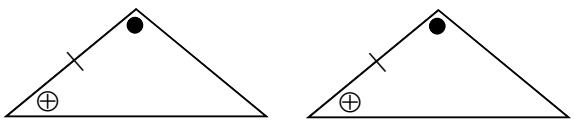
**Condition 2**

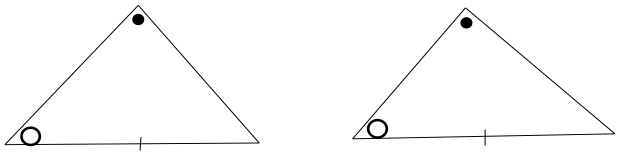
Two triangles are congruent if two sides and the included angle are equal to two sides and the included angle of the other triangle (SAS).



**Condition 3**

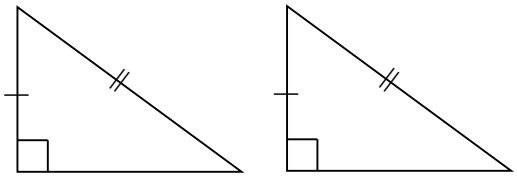
Two triangles are congruent if two angles and one side are equal to two angles and one side of the other triangle (ASA or AAS or SAA).





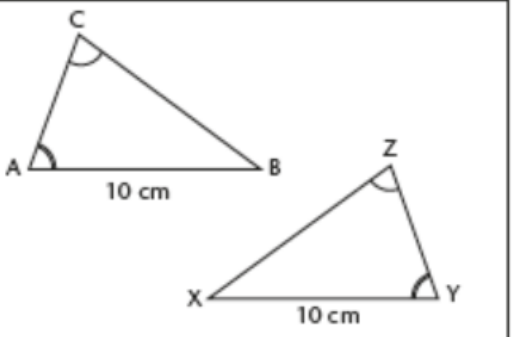
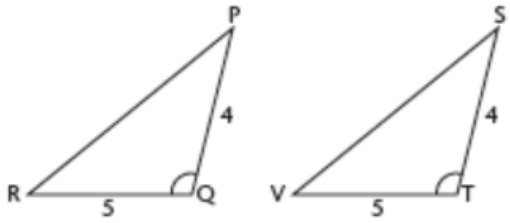
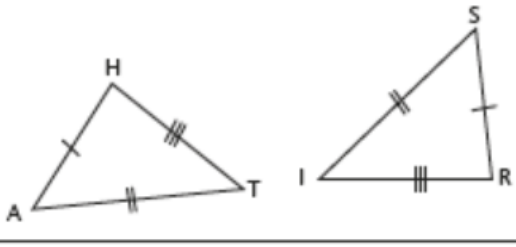
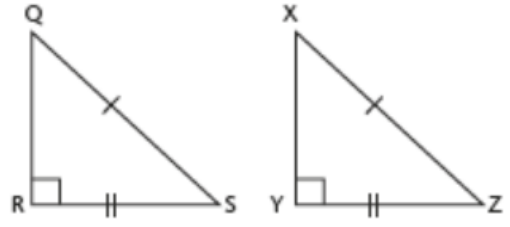
**Condition 4**

Two right-angled triangles are congruent if the hypotenuse and a side of the one triangle is equal to the hypotenuse and a side of the other triangle (RHS).



**(Adopted From EC training manual 2019)**

## How to decide if two triangles are congruent

| Term | Definition | Diagram |
|---|---|---|
| **angle-angle-side [AAS]** | Show that two corresponding angles are the same, and one corresponding side is the same |   |
| **side-included angle-side [SAS]** | Show that two corresponding sides are equal, and the corresponding angle between these sides are equal |   |
| **side-side-side [SSS]** | Show that all three corresponding sides are equal |   |
| **90°-hypotenuse-side [RHS]** | For two right-angled triangles, show that the hypotenuses are equal and that a pair of corresponding sides are equal |   |

**Note:** that congruent triangles are automatically similar, in the ratio 1:1.

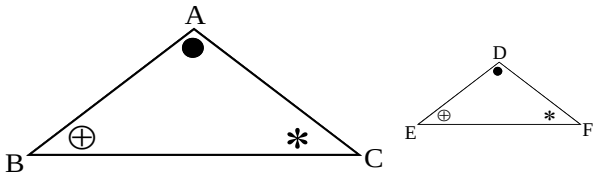
**(Adopted from NZALO PAGE 279)**

**SIMILAR TRIANGLES**

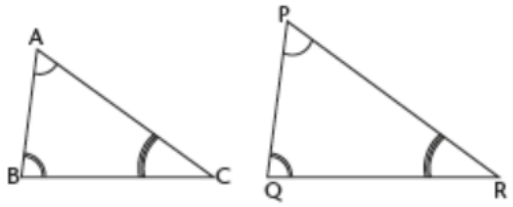
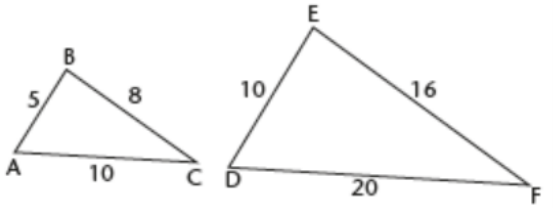
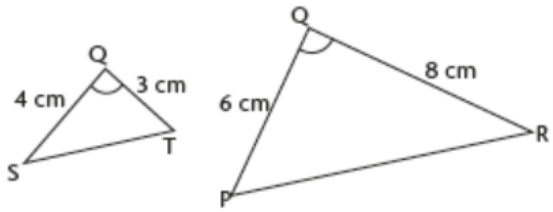
- If two triangles are similar (equiangular), then their corresponding sides are in the same proportion.
- Two triangles are similar even if their sides are not equal, but as long as the corresponding angles are equal.
- The symbol used to denote two triangles that are similar is ///.

If $\Delta ABC|||\Delta DEF\ (AAA)$,

then $\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}$



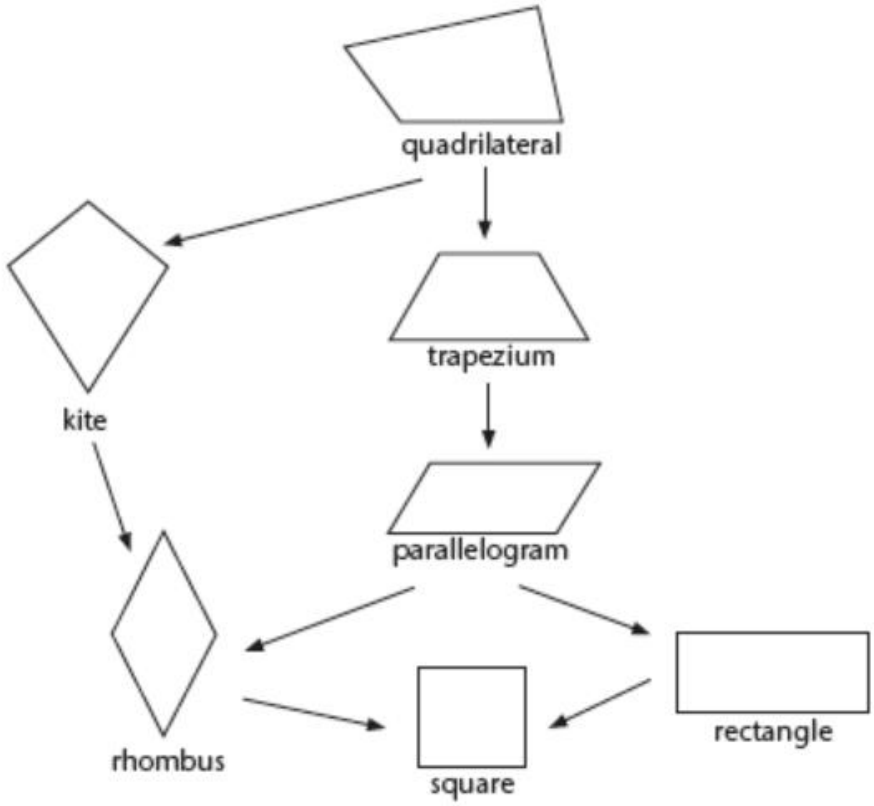

## How to decide if two triangles are similar

| Term | Definition | Example |
|---|---|---|
| **angle-angle-angle [AAA]** | Show that they have two angles in common; when this is true the third is automatically common |   |
| **ratios of the three pairs of corresponding sides are in the same proportion to each other** | Show that all three pairs of corresponding sides are in the same proportion |   |
| **ratios of the corresponding arms are equal to each other** | Show that one angle is the same and the two pairs of arms of the equal angle are in the same proportion |   |

Triangles with the same angle sizes are called **similar triangles**.

(**Adopted from Gr 10 Text book : INZALO Page 286**)

| **THE HIERARCHY OF QUADRILATERALS** |
|---|
| Any quad that is lower down in the hierarchy has all the properties of any quadrilateral that is higher up: |
| ALL QUADRILATERALS INDICATED ARE REGULAR EXCEPT THE TOP MOST WHICH IS IRREGULAR |



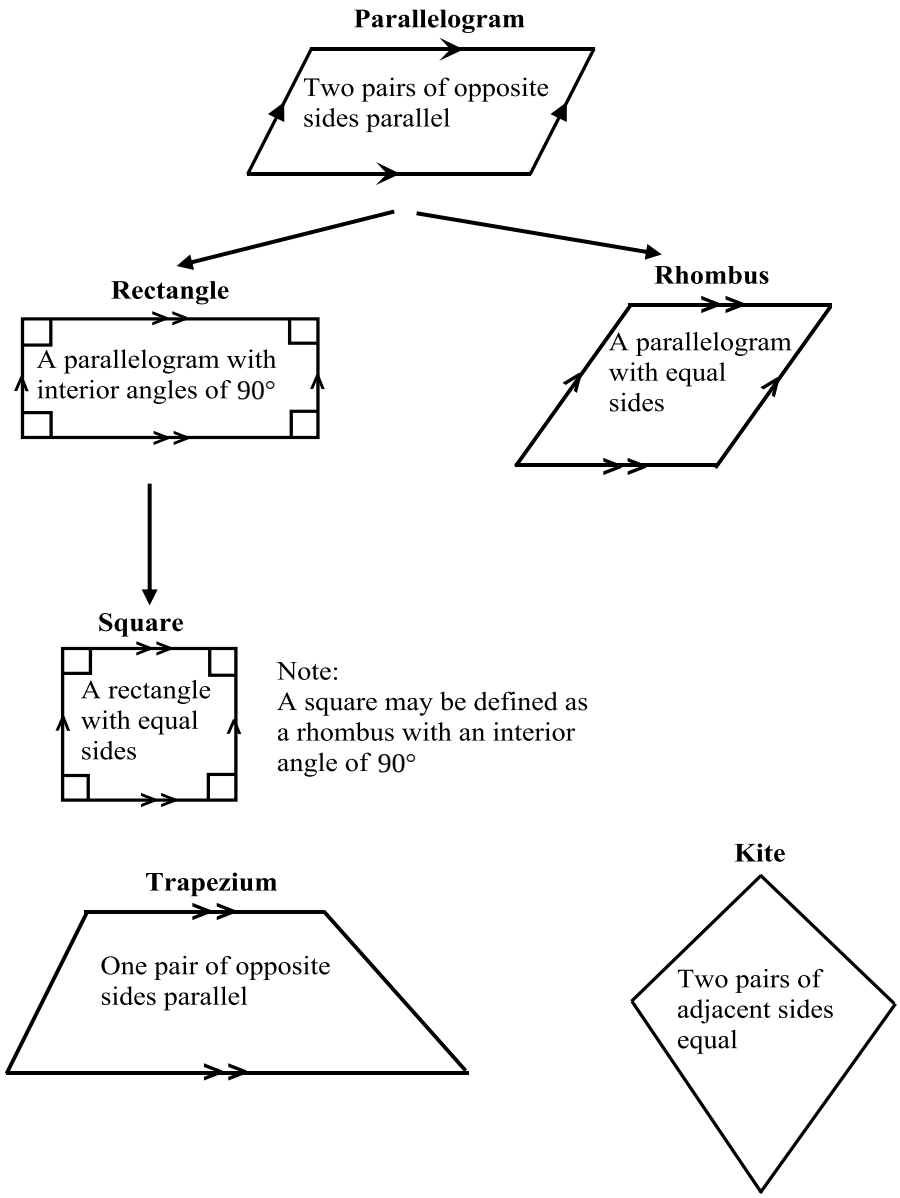

(**Adopted from Gr 10 Text book : INZALO Page 299)**

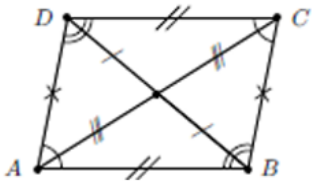
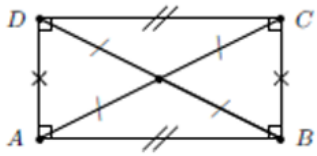
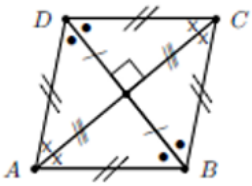
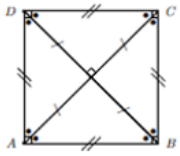
## 1. QUADRILATERALS

A **polygon** is a two-dimensional figure with three or more straight sides.

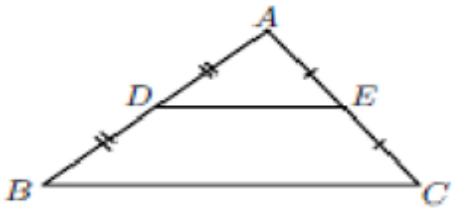
A **quadrilateral** is a polygon with four straight sides.

### Types of quadrilaterals



| PROPERTIES OF SPECIAL QUADRILATERALS | |
|---|---|
| **PARALLELOGRAM**<br>• Both pairs of opposite sides are parallel<br>• Both pairs of opposite side are equal<br>• Both pairs of opposite angles are equal<br>• Diagonals bisect each other |  |
| **RECTANGLE**<br>All properties of **parallelogram** PLUS:<br>• Both diagonals are equal in length<br>• All interior angles are equal to $90°$ |  |
| **RHOMBUS**<br>All properties of **parallelogram** PLUS:<br>• All **sides** are **equal**<br>• Diagonals bisect each other **perpendicularly**<br>• Diagonals **bisect** interior **angles** |  |
| **SQUARE**<br>All properties of a **rhombus** PLUS:<br>• All interior angles are $90°$<br>• Diagonals are equal in length |  |

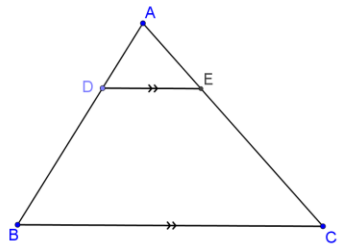
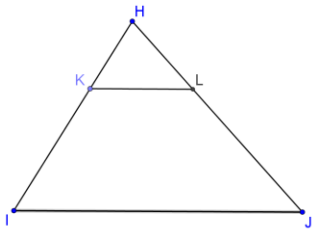
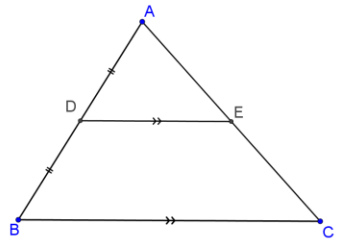
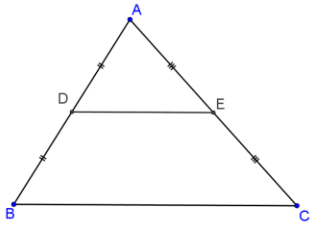
( *Midpt Theorem* )




If AD = DB and AE = EC, then DE ∥ BC and DE = $\frac{1}{2}$BC

**Summary: ON PROPTIONALITY ON TRIANGLES**

| **Diagram** | **Theorem** | **Mathematical statement** |
|---|---|---|
| | | |

| | | |
|---|---|---|
|  | A line drawn parallel to one side of a triangle divides the other two side proportionally. | $\frac{AD}{DB} = \frac{AE}{EC}$ $(\text{line } \perp \text{ one side of } \Delta)$<br>OR<br>$(\text{prop th; DE } \| \text{BC})$ |
|  | If a line divides two sides of a triangle in the same proportion, then the line is parallel to the third side. | $KL \parallel IJ$ $(\text{line divides two sides of } \Delta \text{ in prop})$ |
|  | The line drawn from the midpoint of one side of a triangle, parallel to another side, bisects the third side. | $AE = EC$ $\left(\text{line through midpt } \| \text{ to } 2^{nd} \text{ side}\right)$ |
|  | The line segment joining the midpoints of two sides of a triangle is<br><br>parallel to the third side and equal to half the length of the third side | $DE = \frac{1}{2}BC$ $(\text{Midpt th})$<br>$DE \parallel BC$ $(\text{Midpt th})$ |