FORCES.

In this section we will cover the following topics:

- Forces
- Moments
- Stress/Strain

Forces:

Everyone who has exerted muscular force in lifting an object, pushing a load or overcoming any kind of resistance, has some concept of force. Force is generally recognised and measured by its effects.

Force exerted on a body tends to change its motion, and, unless resisted by another force, will do so.

Having said that force is recognised and measured by its effects, and that the effect will cause the object to change shape, to start moving, to stop moving, to accelerate or decelerate, we now need to represent these quantities and effects on paper.

- First we need to be able to represent the direction:

Forces are represented on a Cartesian plane.

The Cartesian plane is divided into 4 quadrants represented in the illustration below by roman numerals **I, II, III, IV.**

Note: Angles are read from the 90° right (East) in an anti-clockwise direction.

Each force can be divided into its vertical and horizontal components using trigonometric ratios.

Let us consider the force A
The vertical component of the 60° angle is **y**

Trigonometrically we can express the vertical component as follows:

$$\text{Sin } 60° = \frac{Vertical\ comp\ (y)}{A\ (force)}$$

Let us consider the force A
The horizontal component of the 60° angle is **x**

Trigonometrically we can express the horizontal component as follows:

$$\text{Cos } 60° = \frac{Horizontal\ comp\ (x)}{A\ (force)}$$

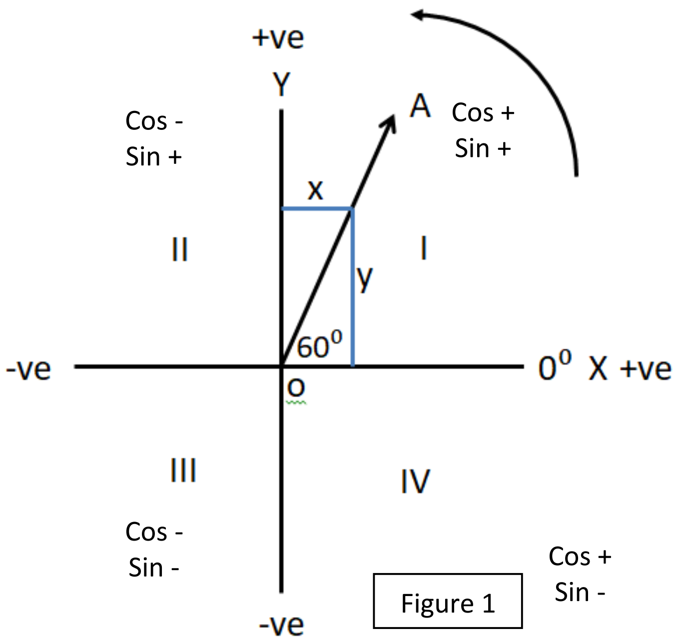


Figure 1

Thus –
Vertical components (VC) may be expressed by the following formula: y = **F**sinθ
Horizontal components (HC) can be expressed by the following: x = **F**cosθ

The vertical axis is labelled the **Y-axis** and the vertical components are thus referred to using the lower case **y** and the horizontal axis is labelled the **X-axis,** the horizontal component uses the lower case **x**.

All values above the X axis are positive and all those below the X axis are negative. Likewise all values to the left of the Y-axis are negative and all values to the right of the Y-axis are positive. This is important when determining the resultant and equilibrant by calculation.

When we consider figure 2 we recognise the following:

The 60° angle is expressed as 60° whilst the 10° angle is expressed as 190° (180° + 10°)

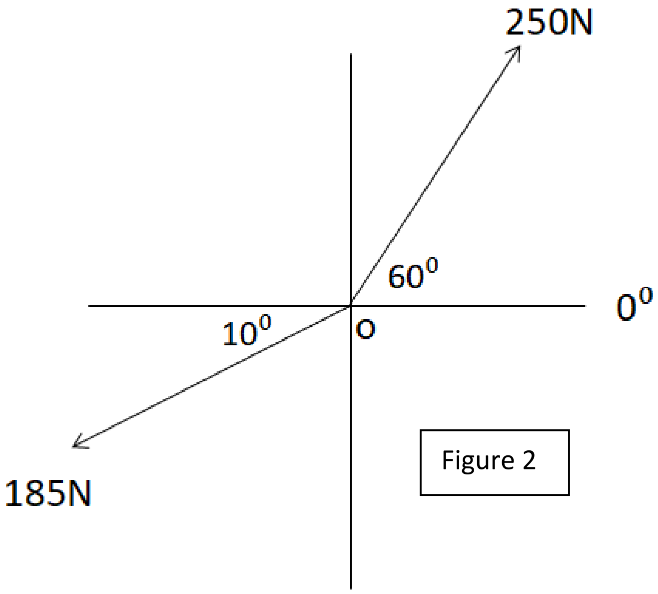


Figure 2

Therefore, by calculation the horizontal components of 250N and 185N will be expressed as:

Cos 60° = $\frac{Horizontal\ comp(HC)x}{250}$

250Cos60° = HC(250N)

HC = 125N

Cos 190° = $\frac{Horizontal\ comp(HC)x}{185}$

185Cos190° = HC(185N)

HC = - 246.2N

Therefore, by calculation the vertical components of 250N and 185N will be expressed as:

Sin 60° = $\frac{Vertical\ comp(VC)y}{250}$

250sin60° = VC(250N)

VC = 216.5N

Sin 190° = $\frac{Vertical\ comp(VC)y}{185}$

185sin190° = VC(185N)

VC = - 32.12N

The following table will simplify the calculation process

Force	Angle	Vertical components		Horizontal components	
250N	60°	250sin 60°	216.5N	250cos60°	125N
185N	190°	185sin 190°	-32.12N	185cos190°	-246.19N
Sum of components:		VC(y)	183.88N	HC(x)	-121.19N

From the calculation we can deduce the following:

y lies either in the 1st or 2nd quadrant as both are positive for the vertical components, and
x lies either in the 2nd or 3rd quadrants. We can now deduce that the resultant lies in the second quadrant as it has y as positive and x as negative and pulls opposite to the original forces.

To determine the resultant we need to use Pythagoras (the square of the Hypotenuse equals the sum of the square of the other two sides). If we go back to the original triangle we write it as follows $H^2 = x^2 + y^2$ (H is the resultant force which is expressed as R)

$R^2 = 183.88^2 + 121.22^2$

$R^2 = 33609.8889 + 14689.44$

$R^2 = 48506.1428$

$R = \sqrt{48506.1428}$

R = 220.24N

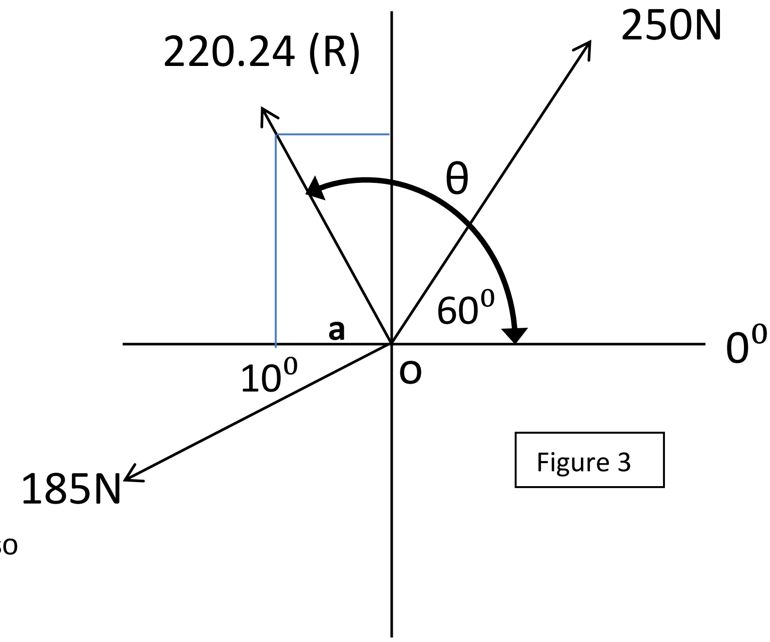


Figure 3

Find angle **a:** Sin**a** = $\frac{183.88}{220.24}$

Sin**a** = 0.8349

a = $\sin^{-1}0.8349$

a = 56.61°

Considering the triangle, we can also use **tan** to determine the angle **a**

tan**a** = $\frac{sum\ of\ VC}{sum\ of\ HC}$

tan**a** = $\frac{183.88}{121.19}$

Tan**a** = 1.517

a = $\tan^{-1}1.517$

a = 56.61°

As we determine direction from 0° the angle that the resultant lies on is expressed as follows: θ = 180° - a

θ = 180° - 56.61°

θ = 123.39°

Let us test the results:

Cos123.39° = $\frac{Horizontal\ comp(HC)x}{220.24}$

220.24Cos123.39° = HC

HC = - 121.2N →

Sin 123.39° = $\frac{Vertical\ comp(VC)y}{220.24}$

220.24sin123.39° = VC

VC = 183.88N →

Activity 1:

Referring to Figure A1, determine, by calculation, the resultant and the equilibrant for the system of forces.

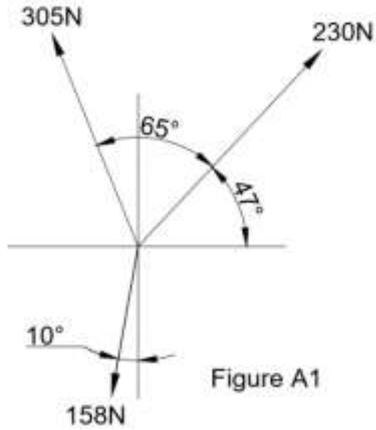


Figure A1

Activity 2:

Referring to Figure A2, determine, by calculation, the resultant and the equilibrant for the system of forces

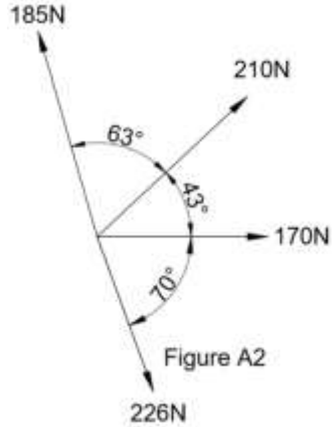


Figure A2

Activity 3:

Referring to Figure A3, determine, by calculation, the resultant and the equilibrant for the system of forces

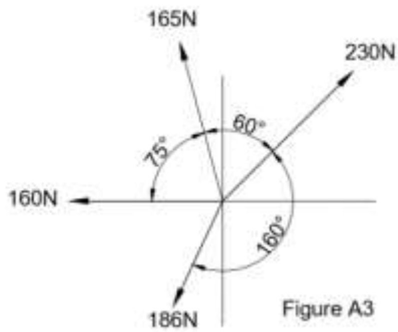


Figure A3

Activity 4

Referring to Figure A4, determine, by calculation, the resultant and the equilibrant for the system of forces

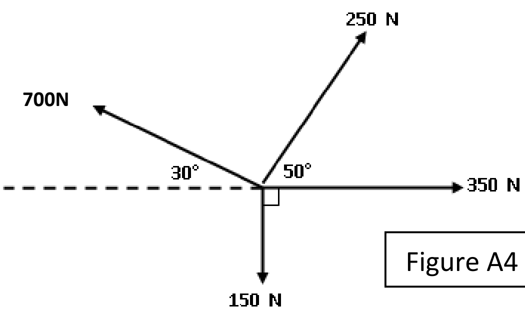


Figure A4

Moments

You are struggling to loosen a tight nut. In order to loosen it you use a spanner, the longer the spanner the easier it is to loosen the nut. This is because of the leverage effect of the long spanner.

From this explanation we can see that the product of the length of the spanner and the distance from the nut is the turning moment of a force. We can therefore define a moment as the turning effect of a force and can be expressed as follows: Moment = Force (f) x distance(d). Forces can make objects turn if there is a pivot (as in the spanner example above.) The unit for moment is **Nm**.

In this note we will concentrate on simply supported beams with point loads and uniformly distributed loads.

Some concepts used in this document:

- A **simply supported beam** is a beam supported at the ***ends***, which are free to rotate, and have no moment of resistance.
- Point load, any load that acts on a specific point along a beam and causes a reaction within that beam
- A **U**niformly **D**istributed **L**oad (also referred to as a **UDL**) that is a load that lies along the length or part of the length of a beam. **UDL**s are always converted to point loads for purposes of calculation.

You may visit this site for examples: https://www.bbc.com/bitesize/guides/zttfyrd/revision/6

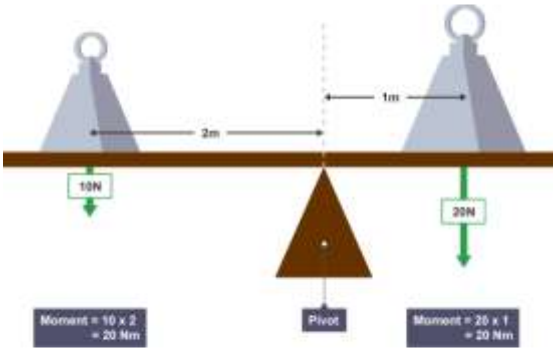


Figure 1

The moments shown in figure 1 above are balanced as the turning effect is equal on both sides.

We speak of clockwise and anti-clockwise moments, when both are equal the system is in equilibrium.

Clockwise moments = anti-clockwise moments (moments about the pivot which would cause the beam to rotate clockwise or anti-clockwise.)
Therefore: Take moments about the pivot.
S↻mom: $f_1 x d_1$ = S↺mom: f_2 x d_2
20N x 1m = 10N x 2m
20Nm = 20Nm the system is in equilibrium

In the problems that we encounter you will be required to balance all forces acting on a simply supported beam.

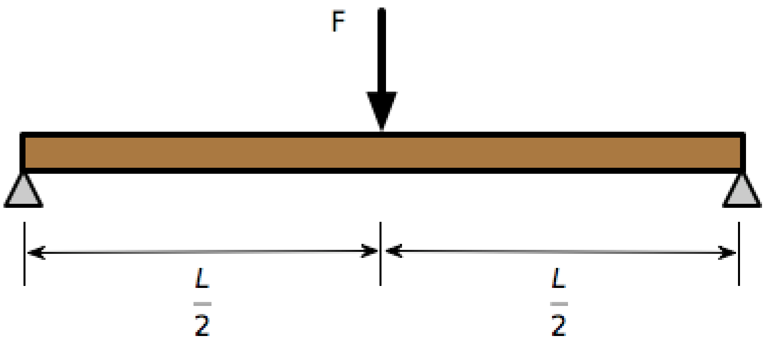


Figure 2

In figure 2 above, we have a simply supported beam with a point load F. As can be seen the turning moment is the same at each support as the load is in the middle. Point loads can be placed anywhere along the length of the beam.

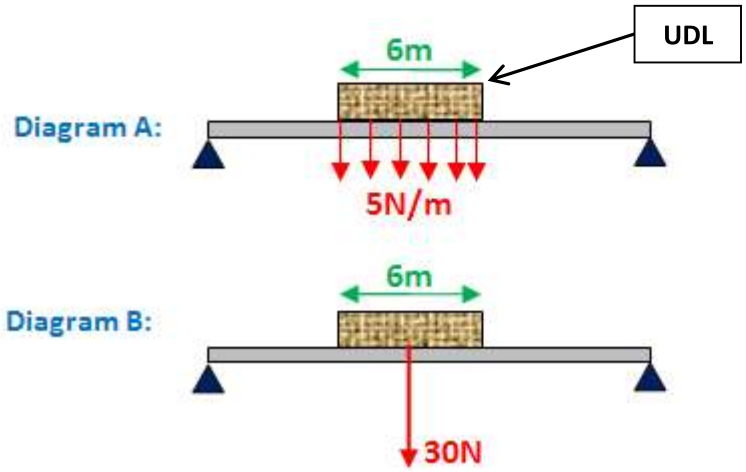


Figure 3

In figure 3 above we have a load of bricks supported by a beam, the total force (load) due to the mass of the bricks is 5N/m. As we cannot work with a distributed load when doing calculations, we change the distributed load to a point load, this point load is then placed in the centre the original UDL and measurements are then taken from the point load.

For calculations, you may be expected to calculate all the reactions in a beam for two point loads and one UDL.

Example:
The diagram in FIGURE 4 below shows a simply supported beam supported by two vertical supports, **A** and **B**. Two vertical point loads, as well as a uniformly distributed load of 60 N/m are exerted on the beam.

Draw a sketch of the set-up to help you calculate the reactions at A and B so that the beam is in equilibrium

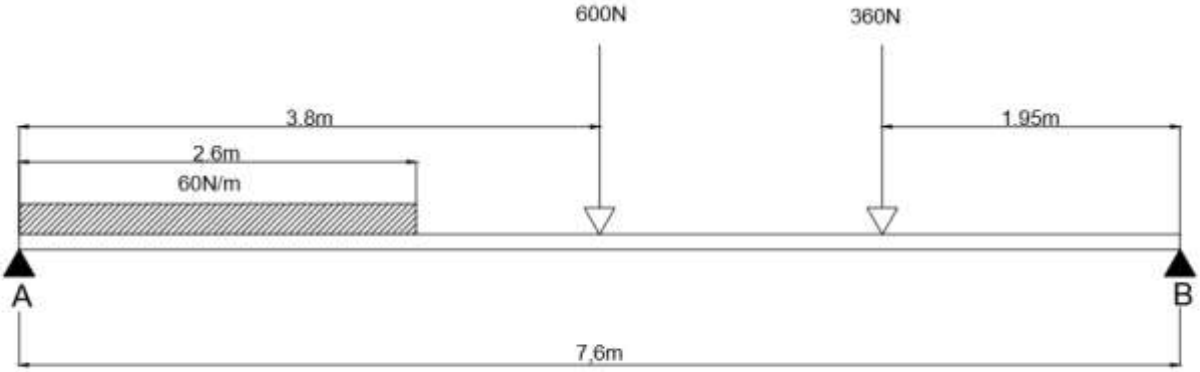
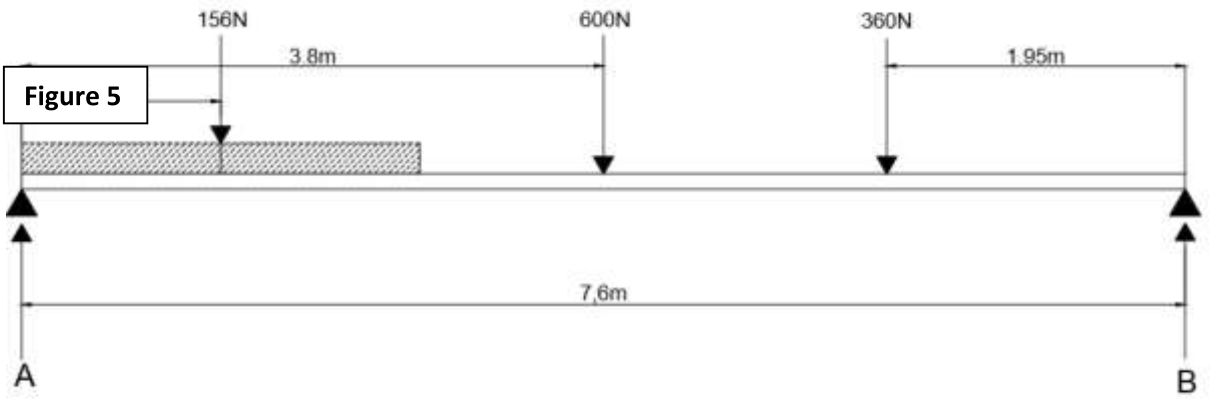


Figure 4



In figure 5 above, the UDL has been converted to a point load of 156N at a distance of 1.3m from support A.

NOTE: It is always a good idea to work from the closest force to the furthest when doing calculations.

We can now calculate the reactions at A & B

Take moments about A:
↻M = ↺M

(156 x 1.3) + (600 X 3.8) + (360 X (7.6-1.95)) = B x 7.6
202.8 + 2280 + 2034 = B X 7.6
$\frac{4516.7}{7.6}$ = B
Reaction at B: B = $\frac{4516.7}{7.6}$
Reaction at B = 594.35N upwards.

Take moments about B:
↻M = ↺M

(360 x 1.95) + [600 X (7.6 - 3.8)] + [156 X (7.6-1.3)] = A x 7.6
(360 x 1.95) + (600 X 3.8) + (156 X (6.3) = A x 7.6
702 + 2280 + 982.8 = A X 7.6
$\frac{3964.8}{7.6}$ = A
Reaction at A: A = $\frac{3964.8}{7.6}$
Reaction at A = 521.68N upwards.

To test the accuracy of the answer we can say that all upward forces = all downward forces as the system should be in equilibrium.

Upward reactions = downward forces
521.68 + 594.32 = 156 + 600 + 360
1116N = 1116N
The beam is in equilibrium.

ACTIVITY:

The diagram in FIGURE A1 below shows a simply supported beam supported by two vertical supports, **A** and **B**. Two vertical point loads, as well as a uniformly distributed load of 85 N/m are exerted on the beam.

Draw a sketch of the set-up to calculate the reactions and calculate the reactions at A and B so that the beam is in equilibrium

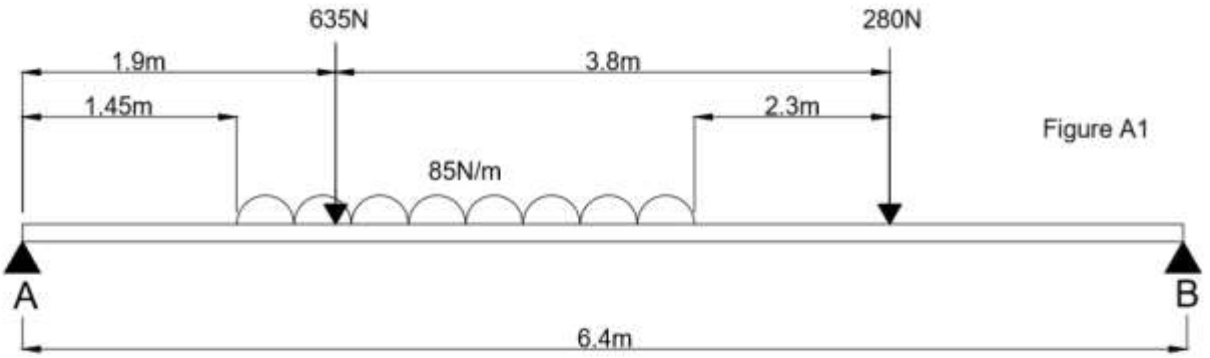


Figure A1

ACTIVITY:

The diagram in FIGURE A2 below shows a simply supported beam supported by two vertical supports, **A** and **B**. Two vertical point loads, as well as a uniformly distributed load of 63 N/m are exerted on the beam.

Draw a sketch of the set-up to calculate the reactions and calculate the reactions at A and B so that the beam is in equilibrium. (First convert millimetres to metres.)

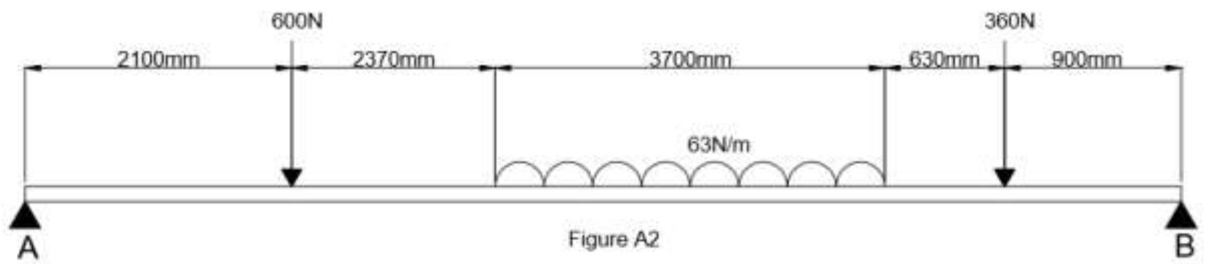


Figure A2

ACTIVITY:

The diagram in FIGURE A3 below shows a beam supported by two vertical supports, **A** and **B**. Two vertical point loads of 800 N and 300 N as well as a uniformly distributed load of 70 N/m over the total length of the beam are exerted onto the beam. Calculate the magnitude of the reactions in support **A** and support **B**.

Draw a sketch of the set-up to calculate the reactions and calculate the reactions at A and B so that the beam is in equilibrium.

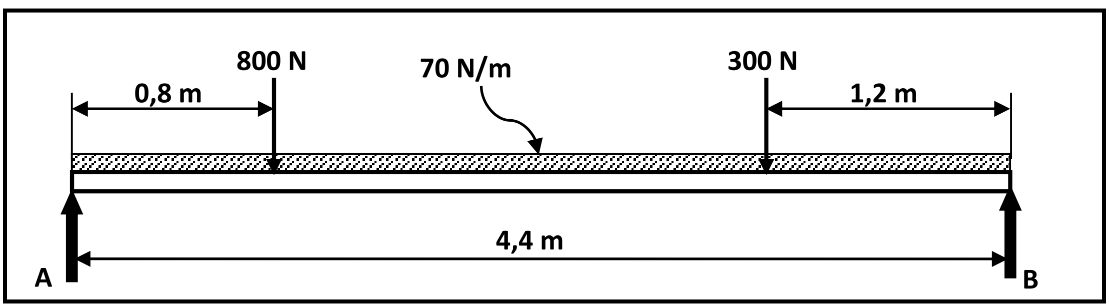


FIGURE A3

STRESS, STRAIN & E

Basic calculations

Stress

Stress(s) = $\frac{Force}{Area}$ Nm^{-2} $OR\ Pa$

Area = $\frac{Force}{Stress}$ m^2

Area = $\frac{\pi d^2}{4}$ m^2

Force = Stress x Area

Example

Calculate the compressive stress in a 200 mm diameter shaft when it is subjected to a force of 10 k N.

Solution

S = $\frac{F}{A}$

A = $\frac{\pi d^2}{4}$

= $\frac{\pi x 0.2^2}{4}$

=0.03141593 m^2

S = $\frac{10\ 000}{0.03141593}$

= 318 309.851 Pa

318.31 x 10^3 $Pa\ \ or\ 318.31kPa$

Strain

Example

Calculate the strain in a steel rod that has elongated by 0.3 mm while its original length was 4m.

1. Write down the formula

   $\mathcal{E}$ = $\frac{change\ in\ length\ \Delta l}{L}$
2. Convert either of the units to be same eg. mm to m $\frac{0.3}{1000} = 0.0003m$
3. Substitute in the formula $\mathcal{E}$ = $\frac{0.0003\ m}{4\ m}$

   = 0.000075 or 7.5 x 10^{-5}

Young's modulus of Elasticity - E

Young's modulus (E) = $\frac{Stress}{strain}$

Example

A 32 mm round bar lengthens by 0.5 mm in a tensile test when subjected to a load of 100 k N. Calculate the Young's modulus for the bar if its original length was 120 mm.

E = $\frac{Stress}{Strain}$

Calculate Stress first :

Stress (σ) = $\frac{force}{area}$

Area = $\frac{\pi d^2}{4}$

A = $\frac{\pi x 0.0032^2}{4}$

A = 0.0008042478 m^2

σ = $\frac{100\ 000}{0.000804278}$

= 124.34 x $10^6\ Pa$

ε = $\frac{0.5}{120}$

= 0.004166666

E = $\frac{124.34\ x\ 10^6}{0.004166666}$

= 29.82 x $10^9\ Pa$

E = 29.82 GPa

Activity

A mild steel tie bar has a diameter of 15 mm and is 4 m long. The breaking stress of steel is 540 MPa and E = 200 GPa. Use a safety factor of 3 and calculate:

1.1 The maximum load the tie bar may carry.
1.2 The deformation of the tie bar when it carries a maximum load.

Activity

A brass bush, 80 mm long with an inside diameter of 30 mm and an outside diameter of 40 mm, is used in a press to push out bearings. A force of 23 kN is exerted onto the bush.

Draw a sketch of the bush and:

2.1 Name the type of stress that the bush material is subjected to.
2.2 Calculate the stress in the material. Indicate the answer in MPa.
2.3 Calculate how much the bush will shorten under the given load, given that Young's modulus of elasticity for brass is 90 GPa.
2.4 Determine the maximum (working) load that can be carried by the bush if the maximum compressive stress for brass is 315 MPa, and a factor of safety of 2.5 is allowed.

Solution

1.1 Stress(safe) = $\frac{load}{area}$

Load = Stress(safe) x area

S_{safe} = $\frac{breaking\ stress}{factor\ of\ safety}$

= $\frac{540\ MPa}{3}$

= 180 MPa

Area = $\frac{\pi d^2}{4}$

= $\frac{\pi x 0.015^2}{4}$

= 1.7671 x $10^{-4} m^2$

Load = 180 x $10^6\ x\ 1.7671\ x\ 10^{-4}$

= 31 807.8 N

= 31.81 k N

1.2 Strain ($\mathcal{E}$) = $\frac{deformation\ \Delta l}{L}$

$\mathcal{E}$ = $\frac{stress}{E}$

= $\frac{180\ X\ 10^6}{200\ X\ 10^9}$

= 0.9 x 10^{-3}

ΔL = Strain x Original length L

= 0.9 x $10^{-3}\ x\ 4$

= 0.0036 m

= 0.0036 x 1000

= 3.6 mm

Solution:

2.1 Compressive stress

2.2

A = $\frac{\pi(D2-d2)}{4}$

= $\frac{\pi(0.042-0.032)}{4}$

= 0.55 x 10^{-3} m^2

σ = FA

= 23 x 10^3 x 0.55 x 10^{-3}

= 41.84 MPa

2.3

$\mathcal{E}$ = σE

= 41.84 × 10^6 x 90 × 10^9

= 0.46 X 10^{-3}

Δl = L_o x E

= 80 x (0.46 X 10^{-3})

= 36.8 x 10^{-3} mm

2.4

Stress = $\frac{load}{area}$

Load = Stress x area

S_{safe} = $\frac{maximum\ (yield)\ stress}{factor\ of\ safety}$

= $\frac{315\ MPa}{2.5}$

= 126 MPa

$Load_{max}$ = 126 x $10^6\ x\ 0.55$ x $10^{-3} m^2$

= 69 300 N

= 69.3 kN