CURRICULUM FET – MATHEMATICS

MATHEMATICS

REVISION MATERIAL – FROM 2019 ONWARDS

A COMPILATION OF 2019 TRIAL EXAMINATIONS FROM PROVINCES GROUPED ACCORDING TO TOPICS!

It does not matter what percentage you got in the past, you can always improve if you practice!

Practice makes perfect! Collect the marks!

YES YOU CAN!

NOTE: PLEASE USE THIS BOOKLET TOGETHER WITH OTHER REVISION MATERIAL LIKE BOOKLET 1 OF 2017, BOOKLET 2 OF 2018, PREVIOUS YEARS QUESTION PAPERS' REVISION MATERIAL (WITH QUESTION PAPERS FROM 2015 – 2019 JUNE)

TO BE USED FROM OCTOBER 2019 ONWARDS!

BOOKLET 1 OF 2019

ALGEBRA

PREP 2019_LIMPOPO

QUESTION 1

1.1 Solve for x:

    1.1.1. $(x+2)(x-1)=0$ (2)

    1.1.2. $x^2 - x = 5$ (correct to two decimal places)   (4)

    1.1.3. $x - \sqrt{5+x} = 7$ (4)

    1.1.4 $4x^2 + 1 \geq 5x$ (4)

1.2   Given: $P = \dfrac{a^{-x+2} - 16a^{-x}}{a^{-x} + 4a^{-x-1}}$

    1.2.1   Simplify P and show $P = a(a-4)$ (4)

    1.2.2   Hence solve for $P = 0$ (2)

1.3   Prove that the roots of $a^2 x^2 + abx + b^2 = 0$

     are non-real for all real values of a and b, $a \neq 0$ and $b \neq 0$ (3)

1.4   Solve for x and y:

     $3 + x = 2y$ and $x^2 + 4y^2 = 2xy + 7$ (5)

                                             [28]

PREP 2019_EASTERN CAPE

QUESTION 1

1.1   Solve for x:

    1.1.1 $x^2 - 3x - 4 = 0$ (3)

    1.1.2 $2x^2 - x - 7 = 0$ (correct to TWO decimal places)   (3)

1.1.3 $5^{x+1} - 5^x = 2500$ (3)

1.1.4 $(x-3)(x+1) < 12$ (4)

1.2 Solve the following equations simultaneously:

$2x = y + 1$
$3x^2 - xy - y^2 = 1$ (6)

1.3 Given that $f(x) = x^2 - 2px + 8 + 2p$ has two equal roots and $p < 0$, determine the coordinates of the turning point of h, if $h(x) = f(x) - 3$. (5)

[24]

PREP 2019_FREE STATE

QUESTION 1

1.1 Solve for x:

1.1.1 $-\dfrac{2}{x^2} + \dfrac{1}{8} = 0$ (3)

1.1.2 $2x^2 + 9x - 24 = 0$ (Correct to TWO decimal places) (4)

1.1.3 $x = 2\sqrt{x} + 3$ (4)

1.2 Given that: $f(x) = 2x^2 + x - 6$

1.2.1 Solve for x if $f(x) \geq 0$ (3)

1.2.2 Determine the sum of all integers satisfying $f(x) < 0$ (2)

1.3 Given that $x^2 - 2xy - 8y^2 = 0$

1.3.1 Determine the value of the ratio $\dfrac{x}{y}$ (3)

1.3.2 Hence, solve for x and y, if $y + 2x = 4$ (6)

[25]

PREP 2019_GAUTENG

QUESTION 1

1.1 Solve for x:

    1.1.1 $2x^2 + 3 = 8x$ (correct to TWO decimal places)   (4)

    1.1.2 $4x - 2x(x-3) \leq 0$ (4)

    1.1.3 $2^x - 5 \cdot 2^{x+1} = -144$ (4)

1.2 If $f(2) = 0$ and $f(-6) = 0$, determine an equation for $f(x)$ in the form $f(x) = x^2 + bx + c$.   (2)

1.3 Solve for x and y simultaneously:

$$2x + y = 17 \quad \text{and} \quad xy = 8$$

    (6)

1.4 Given: $2mx^2 = 3x - 8$ where $m \neq 0$.
Determine the value(s) of m for which the roots of the equation are non-real.   (4)

    [24]

PREP 2019_NORTH WEST

QUESTION 1

1.1 Solve for x, correct to TWO decimal places where necessary:

    1.1.1 $2x^2 - 7x + 3 = 0$ (3)

    1.1.2 $17x - 8 = 3x^2$ (5)

    1.1.3 $(2x - 3)(4 - x) \geq 0$ (3)

1.2 Given: $4^{a+b} = 2^{b+4}$

    1.2.1   Show that: $b = 4 - 2a$ (2)

    1.2.2   Hence, solve for a and b simultaneously if it is further given that

        $2a^2 - 3ab = -4$ (5)

1.3   Determine the value of $(0,04)^{0,5}$ without using a calculator.   (2)

1.4   If $x = \sqrt{3 - 2\sqrt{2}}$ and $y = \sqrt{2} - 1$, prove that $y = x$ without using a calculator.   (3)

[23]

PREP 2019_KZN

QUESTION 1

1.1   Solve for x:

   1.1.1 $x(4 - x) = 0$ (2)

   1.1.2 $2x^2 + 5x = 1$ (rounded off to 2 decimal places)   (4)

1.2   Given: $\sqrt{x - 2} = 2 - x$

   1.2.1   Solve for x.   (4)

   1.2.2   Hence, or otherwise, determine the value(s) of p if $\sqrt{p^2 - p - 2} = 2 + p - p^2$ (4)

1.3   Solve: $-2x^2 + 5x \leq 0$ (4)

1.4   If $2^{x+1} + 2^x = 3^{y+2} - 3^y$, and x and y are integers, calculate the value of $x + y$.   (6)

[24]

PREP 2019_MPUMALANGA

QUESTION 1

1.1   Solve for x:

    1.1.1 $x^2 - 2x = 0$ (3)

    1.1.2 $2x - \dfrac{8}{x+1} = 3$ (correct to 2 decimal places)   (5)

    1.1.3 $2x^2 + x - 3 > 0$ (3)

    1.1.4 $x^{\frac{2}{3}} - x^{\frac{1}{3}} = 6$ (3)

1.2   Given: $4^{x+2} \cdot 8^{y+1} = 2^{1-x}$ and

        $x^2 + y^2 + xy = 7$

    1.2.1   Show that $y = -x - 2$ (3)

    1.2.2   Hence, solve for x and y simultaneously.   (5)

1.3   If the roots of $mx^2 + 5x + 4 = 0$ are non-real, calculate the lowest integral value of m.   (4)

                                                         [26]

PREP 2019_WESTERN CAPE

QUESTION 1

1.1 Solve for x:

    1.1.1 $(x - 3)(3x + 2) = 0$ (2)

    1.1.2 $2x(2x - 1) = 3$ (correct to TWO decimal places)   (4)

    1.1.3 $3^{x+1} - 3^{x-1} - 7 = 2^{-x}.2^x$ (4)

    1.1.4 $\sqrt{2 - x} = \dfrac{x^2 - x - 2}{\sqrt{2 - x}}$ (5)

1.2 Given $f(x) = -x^2 + 7x + 8$ and $g(x) = -3x + 24$

Calculate the coordinates for $f(x) = g(x)$ (6)

1.3 The roots of a quadratic equation are given by:

$$x = \frac{-4 \pm \sqrt{k(3 - k)}}{2}$$

    1.3.1   Determine the nature of the roots if $k = 2$ (2)

    1.3.2   For which values of k will the roots be non-real?   (3)

    [26]

PREP 2019_NORTH WEST 2

QUESTION1

1.1 Solve for x:

    1.1.1 $3x^2 - 18x = 0$ (3)

    1.1.2 $7x^2 - 4x = 5$ (Leave your answer correct to TWO decimal places.)   (4)

    1.1.3 $(x + 5)(x - 2) > 0$ (2)

    1.1.4 $26 - 5^{2x} = (5^x - 6)^2$ (6)

1.2 Solve simultaneously for x and y:

    $x - 4y = 5$ and $3x^2 - 5xy + 2y^2 = 25$ (6)

1.3 Solve for x if: $x = \sqrt{12 + \sqrt{12 + \sqrt{12 + \sqrt{12 + \ldots}}}}$ (4)

                                                                       [25]

NUMBER PATTERN
PREP 2019_LIMPOPO

QUESTION 2

The sequence $3 ; 9 ; 17 ; 27 ; \ldots\ldots$ is a quadratic sequence.

2.1 Write down the next term. (1)

2.2 Determine an expression for the nth term of the sequence. (4)

2.3 What is the value of the first term of the sequence that is greater than 269? (4)

[9]

QUESTION 3

The first two terms of an arithmetic series A, and a geometric series to infinity B, are the same:

A: $-2 + a + \ldots\ldots$ and   B: $-2 + a + \ldots\ldots$

3.1   write down in terms of a:

3.1.1   the third term of the geometric series, B.   (2)

3.1.2   the third term of the arithmetic series, A.   (2)

3.2   The sum of the first three terms of the arithmetic series A is the same as the third

term of the geometric series B.

Write down an equation and determine the value of a.   (5)

3.3   If $a = -6$, determine if the geometric series converge.   (3)

[12]

QUESTION 4

4.1   The sum to $n-$terms of a sequence of numbers is given as : $S_n = \dfrac{n}{2}(5n + 9)$.

4.1.1   Calculate the sum to 23 terms of the sequence.   (3)

4.1.2   Hence calculate the 23nd term of the sequence.   (2)

4.2   If x is a real number, show that the following sequence can NOT be geometric:

1 ; $x+1$; $x-3$; $\ldots\ldots$ (4)

[9]

PREP 2019_EASTERN CAPE

QUESTION 2

2.1 Given the quadratic number pattern: $3 ; 1 ; -3 ; -9 ; \ldots$

    2.1.1 Write down the next 2 terms of the pattern. (1)

    2.1.2 Determine T_n, the nth term of the pattern, in the form $T_n = an^2 + bn + c$. (4)

    2.1.3 Which term of the pattern has a value of -809? (3)

2.2 Given the arithmetic sequence: $-1 ; 1 ; 3 ; 5 ; \ldots$

    2.2.1 Determine T_{53}, the 53rd term of the sequence. (2)

    2.2.2 Determine the sum of the first 29 terms of the sequence. (2)

    2.2.3 Hence, write your answer in sigma notation. (2)

2.3 In an arithmetic sequence, $T_4 = 2x + y$ and $T_{10} = 8x - 2y$. Determine the first
term of the sequence in terms of x and y. (5)

[19]

QUESTION 3

Given that: $p = \sum_{k=1}^{x}(x-1)^k$

(2)

3.1 Determine the values of x for which p converges.

(4)

3.2 Calculate the value of p when $x = \dfrac{2}{3}$.

[6]

PREP 2019_FREE STATE

QUESTION 2

2.1 Given the quadratic sequence $1; 6; 15; 28; \ldots$

    2.1.1 Write down the second difference. (1)

    2.1.2 Determine the nth term. (4)

2.1.3 Calculate which term of the sequence equals 2701. (3)

2.2 Given the arithmetic series: $10 + 15 + 20 + 25 + \ldots + 185$

2.2.1 How many terms are there in the series? (3)

2.2.2 Calculate the sum of all the natural numbers from 10 to 185 that are NOT divisible by 5. (6)

[17]

QUESTION 3

Given that: $\displaystyle\sum_{n=1}^{\infty} 63p^{n-1} = \frac{189}{2}$

3.1 Solve for p. (4)

3.2 If it is further given that $p = \dfrac{1}{3}$, determine the **smallest value of n** such that

$T_n < \dfrac{1}{6561}$. (5)

[9]

PREP 2019_GAUTENG

QUESTION 2

Given the quadratic sequence: $-\dfrac{1}{2}$; 2 ; $\dfrac{11}{2}$; 10 ; $\ldots$

2.1 Show that the nth term of this sequence can be written as $T_n = \dfrac{1}{2}(n^2 + 2n - 4)$. (4)

2.2 Determine the value of $T_{75} - T_{74}$. (2)

2.3 The first differences of the given sequence above forms another number sequence.

2.3.1 Is the sequence of the first differences arithmetic or geometric? Give a reason for your answer. (2)

2.3.2 Which term in the sequence of the first differences will be equal to $\dfrac{151}{2}$? (1)

2.3.3 Calculate the value of the 30th first difference. (2)

2.3.4   Calculate the number of terms in the quadratic sequence if the sum of the
first n first differences is 2 176.   (4)

[15]

QUESTION 3

3.1   The following geometric series is given:

$$2(3x-1)+2(3x-1)^2+2(3x-1)^3 \ ...$$

Determine the value(s) of x for which the series converges.   (3)

3.2   The first two terms of a convergent geometric series are k and 6 respectively where $k \neq 0$.
The sum of the infinite series is 25.
Calculate the value(s) of k.   (5)

3.3   Given the series:

$$(1\times2)+(5\times6)+(9\times10)+(13\times14)+...+(81\times82)$$

Write the series in sigma notation. (It is not necessary to calculate the value of the series).   (4)

[12]

PREP 2019_NORTH WEST

QUESTION 2

2.1   Evaluate:

$$\sum_{n=3}^{20}(15-4n)$$ (4)

2.2   A water tank contains 216 litres of water at the end of day 1. Because of a leak,
the tank loses one-sixth of the previous day's contents each day.
How many litres of water will be in the tank by the end of:

2.2.1   the 2nd day?   (2)

2.2.2   the 7th day?   (3)

2.3   Consider the geometric series: $2(3x-1)+2(3x-1)^2+2(3x-1)^3+...$

2.3.1   For which values of x is the series convergent?   (3)

2.3.2   Calculate the sum to infinity of the series if $x = \dfrac{1}{2}$.   (4)

2.4 $2 ; x ; 12 ; y ; \ldots$ are the first four terms of a quadratic sequence. If the second
difference is 6, calculate the values of x and y.   (5)

                                                                                              [21]

PREP 2019_KZN

QUESTION 2

The first four terms of a quadratic sequence are $8 ; 15 ; 24 ; 35 ; \ldots$

2.1   Write down the next TWO terms of the quadratic sequence.   (1)

2.2   Determine the nth term of the sequence.   (4)

                                                                                              [5]

QUESTION 3

The first three terms of an arithmetic sequence are $2p - 3 ; p + 5 ; 2p + 7$.

3.1   Determine the value(s) of p.   (3)

3.2   Calculate the sum of the first 120 terms.   (3)

3.3   The following pattern is true for the arithmetic sequence above:

$T_1 + T_4 = T_2 + T_3$

$T_5 + T_8 = T_6 + T_7$

$T_9 + T_{12} = T_{10} + T_{11}$

$\therefore T_k + T_{k+3} = T_x + T_y$

3.3.1   Write down the values of x and y in terms of k. (2)

3.3.2   Hence, calculate the value of $T_x + T_y$ in terms of k in simplest form. (4)

[12]

QUESTION 4

4.1   Given: $\displaystyle\sum_{k=1}^{\infty} 5(3^{2-k})$

4.1.1   Write down the value of the first TWO terms of the infinite geometric series. (2)

4.1.2   Calculate the sum to infinity of the series. (2)

4.2   Consider the following geometric sequence:

$$\sin 3\,0°; \cos 3\,0°; \frac{3}{2}; \dots; \frac{81\sqrt{3}}{2}$$

Determine the number of terms in the sequence. (5)

[9]

PREP 2019_MPUMALANGA

QUESTION 2

2.1   A quadratic number pattern and a constant value is combined to form the sequence:

$$8\,;1\,;18\,;1\,;30\,;1\,;44\,;1\,;\,\dots.$$

2.1.1   Write down the next TWO terms of the sequence. (1)

2.1.2   Calculate the nth term of the quadratic sequence. (4)

2.1.3   Which term of the given sequence is 368? (4)

2.2 The sum of the second and third terms of a geometric series is 280 and the sum of the fifth and sixth terms is 4375.

Determine the common ratio. (5)

2.3 The first term of an arithmetic series is increased by 3, but the common difference remains the same. By how much will the sum of the first twelve terms increase? (4)

2.4 A converging geometric series is given by

$1 + 4m + 16m^2 + ...$

2.4.1 Determine the values of m for which the given series will converge. (3)

2.4.2 Calculate the value of m if $1 + 4m + 16m^2 + ... = \dfrac{2}{3}$ (3)

[24]

PREP 2019_WESTERN CAPE

QUESTION 2

2.1 The arithmetic sequence $6 \; ; \; 8 \; ; \; 10 \; ; \; 12 \; ; ...; 164$ is given.

2.1.1 Determine the nth term of the sequence. (2)

2.1.2 How many terms are in the sequence? (2)

2.1.3 It is further given that the above sequence forms the row of first differences for a quadratic sequence $T_n = an^2 + b + c$ where $T_5 = 39$. Determine the values of a, b and c. (4)

2.2 The first three terms of a geometric sequence are: $p - 1 \; ; \; p$ and $p + 2$. Calculate the value of p. (3)

2.3 If $n \in$ N, calculate the smallest value of n for which:

$$\sum_{k=1}^{n} \frac{1}{32} (2)^{k-1} > 900$$

(5)

2.4 Given:

$$\sum_{p=1}^{n} (2p - 5) = m \quad \text{and} \quad \sum_{p=1}^{1} (2p - 5) = 140$$

Determine, in terms of m, the value of

$$\sum_{p=1}^{n} (2p - 5)$$

(2)

[18]

QUESTION 3

The diagram represents a sketch of circles. Half of the area of each circle is shaded.
The radius of the largest circle is $1 u$. The diameter of each smaller circle is equal to the radius of the previous circle.
Determine the sum of the shaded areas of all the circles, if this process is going on forever.
(Give your answer in terms of π.)
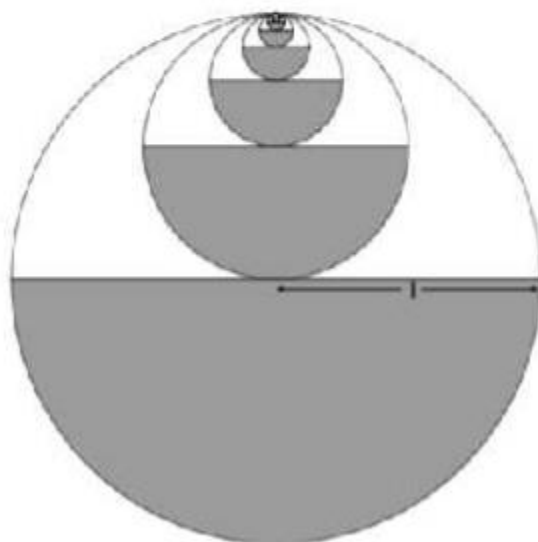
(4)

[4]

PREP 2019_NORTH WEST 2

QUESTION 2

2.1   Given the following arithmetic sequence: $-11 ; -4 ; 3 ; ...$

Determine the:

      2.1.1   General term in the form $T_n = bn + c$. (2)

      2.1.2   Value of the 60th term. (2)

      2.1.3   Sum of the first 60 terms. (2)

2.2   Hence, or otherwise, write $-4 + 3 + 10 + ... + 486$ in sigma notation. (4)

2.3   This arithmetic sequence $-11 ; -4 ; 3 ; ...$ forms the first three first differencesof a quadratic sequence. Which term in this quadratic sequence will be the smallest? Show all your calculations. (5)

                                                                                    [15]

QUESTION 3

Consider the geometric series: $5(3x + 1) + 5(3x + 1)^2 + 5(3x + 1)^3 + ...$

3.1   For which value(s) of x will the series converge? (3)

3.2   Calculate the sum to infinity of the series if $x = -\dfrac{1}{6}$ (4)

                                                                                          [7]

FUNCTION & INVERSES

PREP 2019_LIMPOPO

QUESTION 5

5.1 Consider the function: $f(x) = \dfrac{-6}{x-3} - 1$.

    5.1.1 Calculate the y-intercept of f.   (2)

    5.1.2 Calculate the x-intercept of f.   (3)

    5.1.3 Sketch the graph of f, showing the asymptotes and the intercepts

          with the axes. Use the ANSWER SHEET given.   (4)

    5.1.4 For which values of x is $f(x) > 0$?   (2)

    5.1.5 Calculate the average gradient of f between $x = -2$ and $x = 0$.   (4)

5.2 Draw a sketch graph of $y = ax^2 + bx + c$, $a < 0$, $b < 0$, $c < 0$ and $ax^2 + bx + c = 0$

    has only ONE solution.   (4)

                                    [19]

QUESTION 6

In the diagram below $f(x) = 2^x$ and f^{-1} are given. A and C are points on f.

B is a point on f^{-1}. CB and AB are perpendicular to each other.
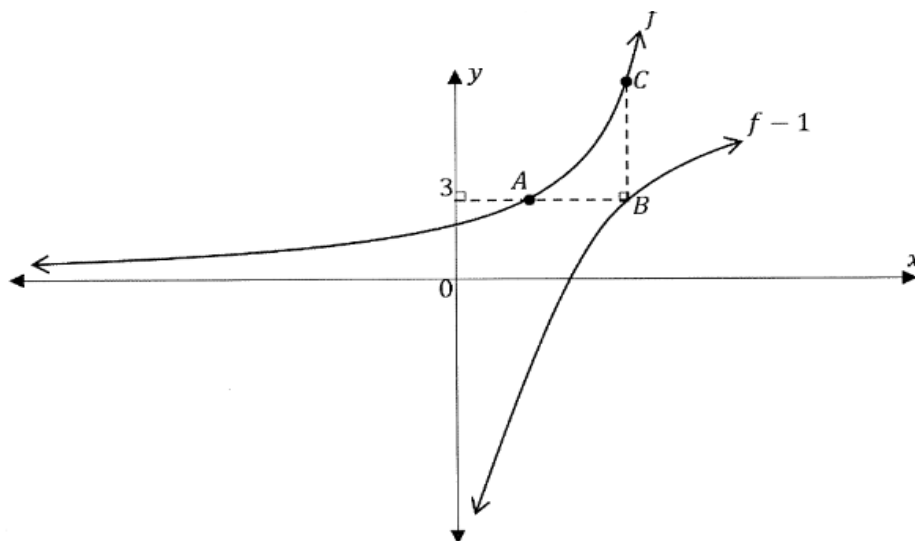
6.1 Write down the equation of $f^{-1}(x)$ in the form $y = \ldots\ldots$ (2)

6.2 Calculate the length of AB. (5)

6.3 Calculate the length of CB, where CB $\perp$ AB. (3)

6.4 Write down the domain of $f^{-1}(x)$. (1)

[11]

PREP 2019_EASTERN CAPE

QUESTION 3

Given that: $p = \sum_{k=1}^{\infty}(x-1)^k$

(2)

3.1 Determine the values of x for which p converges.

(4)

3.2 Calculate the value of p when $x = \dfrac{2}{3}$.

[6]

QUESTION 4

Given: $f(x) = 1 + \dfrac{2}{x+3}$

4.1 Write down the equations of the asymptotes of f. (2)

4.2 Calculate the x and y intercepts of f. (3)

4.3 Draw a neat sketch of f, clearly indicating all intercepts with the axes and any asymptotes. (4)

4.4 Given that h is a reflection of f in the x-axis, determine the equation of the axis of symmetry of h having a positive gradient. (4)

[13]

QUESTION 4

Given: $f(x) = 1 + \dfrac{2}{x+3}$

4.1 Write down the equations of the asymptotes of f.   (2)

4.2 Calculate the x and y intercepts of f.   (3)

4.3 Draw a neat sketch of f, clearly indicating all intercepts with the axes and any
      asymptotes.   (4)

4.4 Given that h is a reflection of f in the x-axis, determine the equation of the axis of
      symmetry of h having a positive gradient.   (4)

                                                                                            [13]

QUESTION 5

The diagram below shows the graphs of $f(x) = x^2 - 7x - 8$ and $g(x) = mx + c$.
P and R are x-intercepts of f, and V is the x-intercept of g. S is the turning point of f.
f and g intersect on the y-axis at Q and also at T(9; 10).
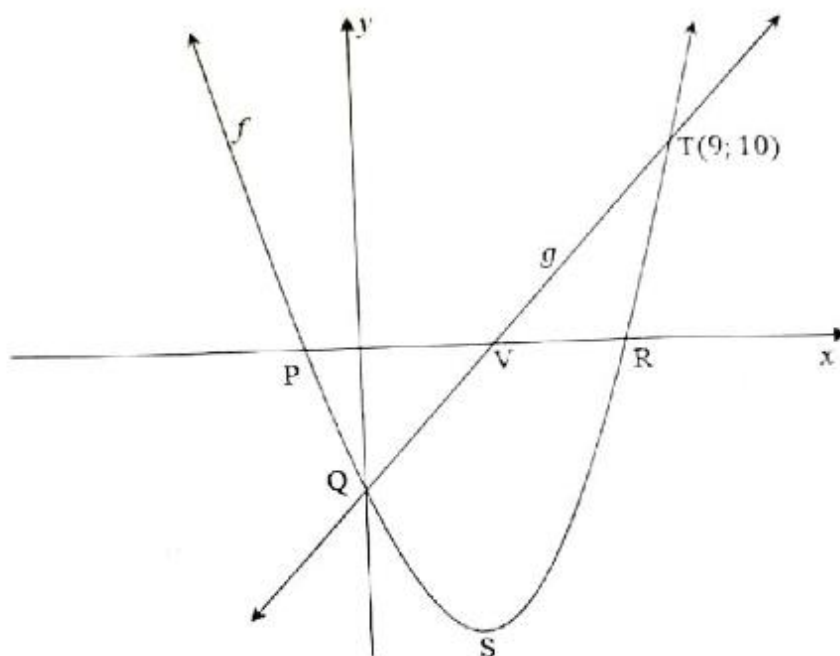
5.1 Write down the coordinates of Q.   (1)

(3)

5.2   Determine the equation of g.

(2)

5.3   Write down the equation of f in the form $y = a(x + p)^2 + q$.

(2)

5.4   Hence, or otherwise, determine the coordinates of S, the turning point of f.

5.5   Determine the coordinates of a point W, on f, such that the average gradient   (5)
        between T and W is 1.

                                                                                        (4)

5.6   Determine the values of x for which $f(x) \cdot g(x) < 0$.   [17]

                                                                                        (2)

6.1   Determine the value of m, if the point $(64 ; 3)$ lies on f.

                                                                                        (2)

6.2   Determine the equation of f^{-1} in the form $y = \ldots$

6.3   Draw a neat sketch of f^{-1}, showing all intercepts with the axes. Indicate at least   (2)
        one other point on your graph.

                                                                                        (1)

6.4   Write down the range of h if: $h(x) = f^{-1}(x) - 2$ [7]

PREP 2019_FREE STATE

QUESTION 4

In the diagram, the graph of $f(x) = \log_a x$ is drawn. $B\left(\dfrac{25}{9};2\right)$ is a point on f.
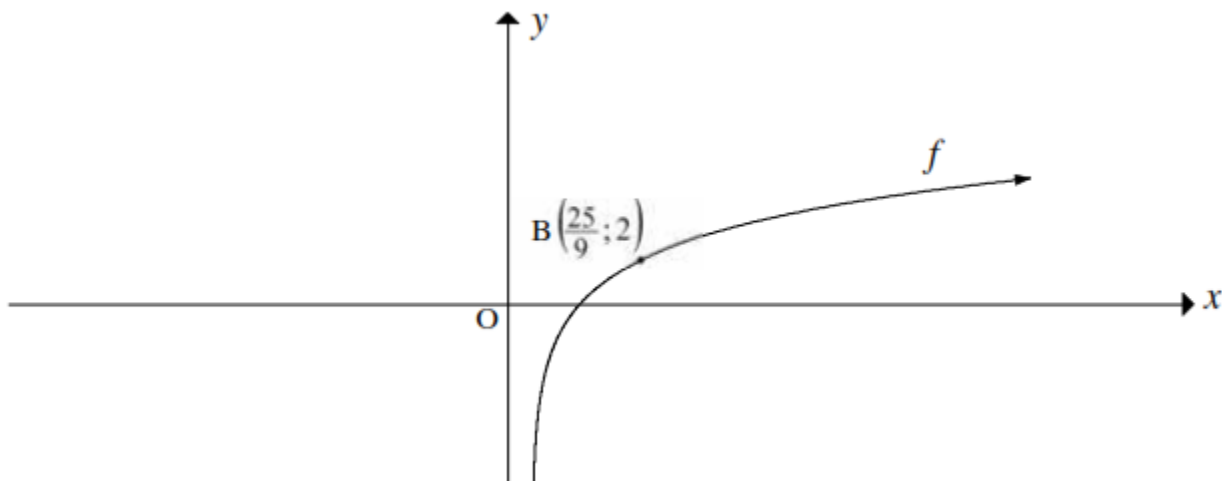
4.1 Determine the value of a. (2)

4.2 Determine the value(s) of x for which $f(x) \leq 0$. (2)

4.3 Write down the equation of f^{-1}, the inverse of f, in the form $y = \ldots$ (2)

4.4 B'' is the reflection of B on the graph $g(x) = \left(\dfrac{3}{5}\right)^x$.

Write down the coordinates of B''. (2)

4.5 Determine for which value(s) of x will $f^{-1}(x) > \dfrac{25}{9}$. (2)

[10]

QUESTION 5

In the diagram below, the graph of $g(x) = \dfrac{-2}{x+4} - 3$ is drawn. The graph f passes through A, the point of intersection of the asymptotes of g, and cuts the x-axis and the y-axis at L and R respectively. K is the y-intercept of g.
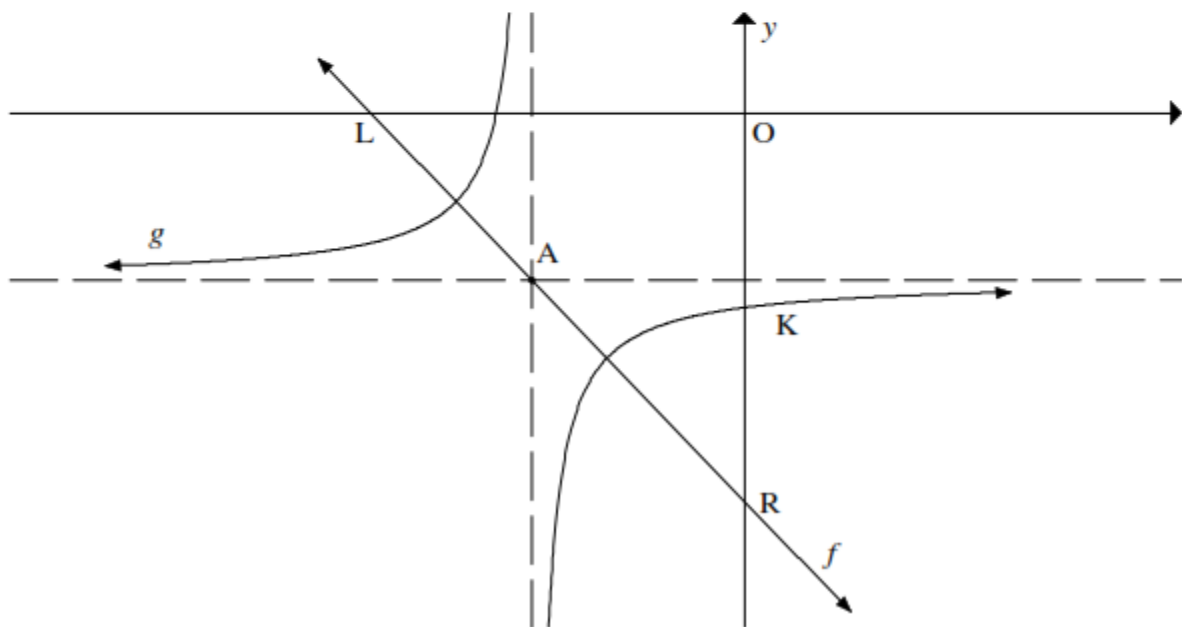
5.1 Determine the equation of f in the form $y = mx + c$. (3)

5.2 Write down the equation of the asymptotes of $g(x - 2) + 1$. (2)

5.3   Calculate the length of KR.   (3)

5.4   The graph of h, where h is the reflection of f in the line $y = -7$, passes through the
       point S(-4 ; p). Calculate the area of $\triangle$ARS.   (4)
                                                                                  [12]

QUESTION 6

In the diagram below, the graphs of $f(x) = ax^2 + bx + 16$ and $g(x) = -12x + 24$ are drawn. The
graph of g is a tangent to the graph of f at B. A and B are the x-intercepts of f and C, the turning
point.
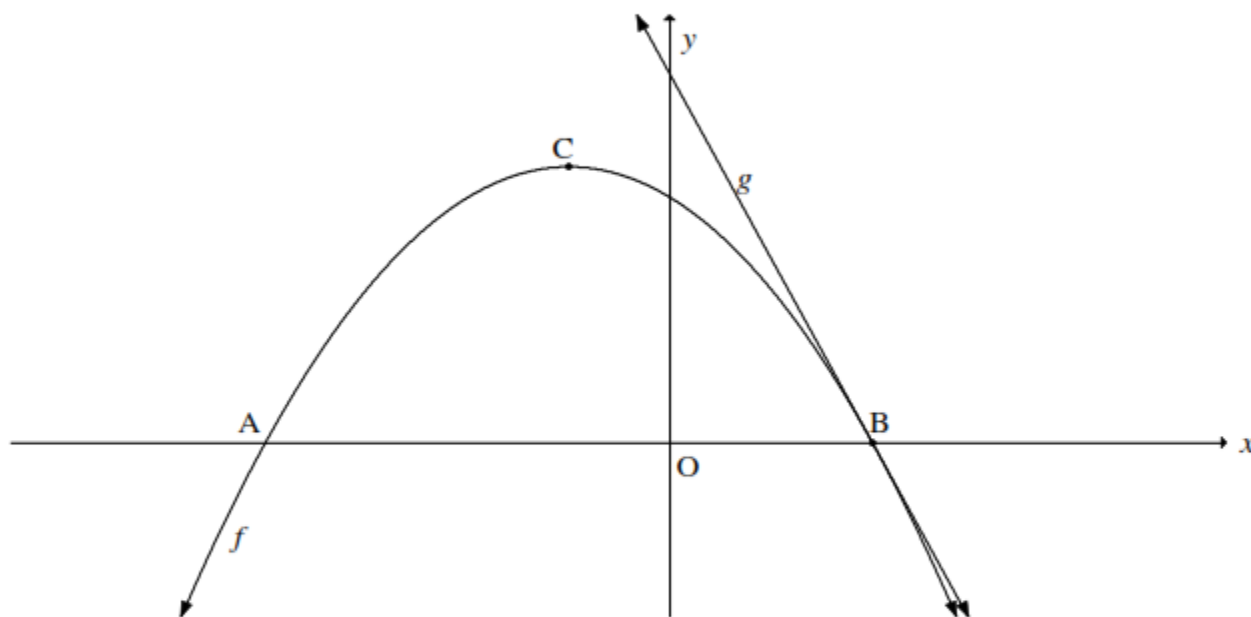
6.1   Calculate the coordinates of B.   (2)

6.2   Determine the values of a and b.   (6)

6.3   If it is given that $f(x) = -2x^2 - 4x + 16$, determine:

       6.3.1   The range of f (5)

       6.3.2   The value(s) of x for which $f'(x) . g(x) > 0$ (2)
                                                                                  [15]

PREP 2019_GAUTENG

QUESTION 5

Given: $f(x) = \dfrac{3}{x-1} - 2$

5.1 Calculate the coordinates of the x-intercept of f. (2)

5.2 Calculate the coordinates of the y-intercept of f. (1)

5.3 Sketch the graph of f in your ANSWER BOOK. Clearly show the asymptotes and the intercepts with the axes. (3)

5.4 ONE of the axes of symmetry of f is a decreasing function. Write down the equation of this axis of symmetry. (3)

[9]

QUESTION 6

The graphs of $f(x) = -2x^2 - 5x + 3$ and $g(x) = ax + q$ are sketched below. The angle of inclination of g is $135°$. Graph g is a tangent to f at point P.
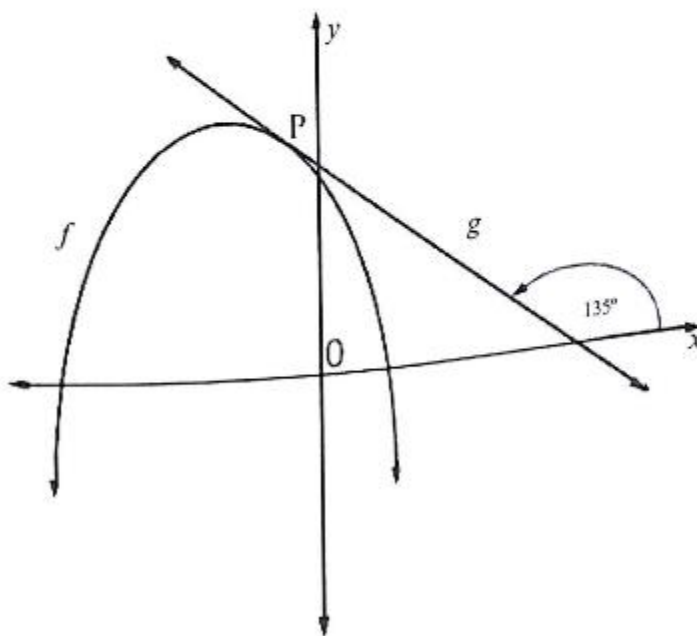
6.1 Calculate the coordinates of the turning point of f. (3)

6.2 Write down the range of f. (1)

6.3 Calculate the coordinates of point P, the point of contact of f and g. (4)

6.4 Determine the value(s) of k for which the straight line $y = k$ is NOT a tangent to $y = 2x^2 + 5x - 3$. (2)

[10]

QUESTION 7

Given $f(x) = a^x$, where $a > 0$, passing through the point $(2 ; \frac{1}{4})$ and $g(x) = 4x^2$.

7.1 Prove that $a = \frac{1}{2}$. (2)

7.2 Determine the equation of $y = f^{-1}(x)$ in the form $y = \ldots$ (2)

7.3 Determine the equation of $y = h(x)$ where h is the reflection of f in the x-axis. (1)

7.4 How must the domain of $g(x)$ be restricted so that $g^{-1}(x)$ will be a function? (2)

[7]

QUESTION 8

The graphs of $f(x) = -x + 3$ and $g(x) = \log_2 x$ are drawn below.
Graphs f and g intersect at point $K(2 ; 1)$.
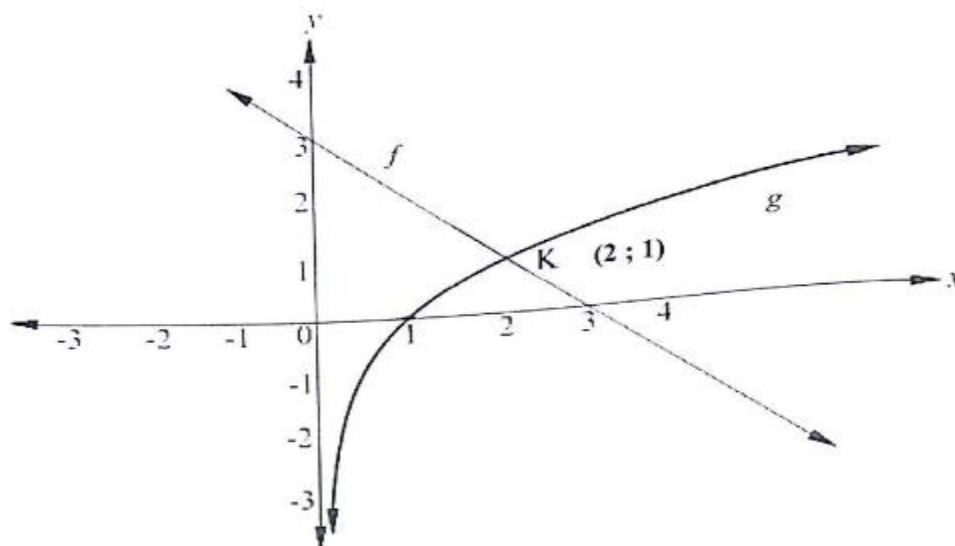
8.1 Write down value(s) of x for which:

    8.1.1 $f(x) - g(x) > 0$ (2)

    8.1.2 $g(x) \cdot g^{-1}(x) \leq 0$ (2)

8.2   8.2.1   Write down the equation of g^{-1} in the form $y = \ldots$ (2)

    8.2.2   Explain how you could use the given sketch to solve the equation $\log_2(3-x) = x$.   (2)

    8.2.3   Write down the solution to $\log_2(3-x) = x$.   (1)

    [9]

PREP 2019_NORTH WEST

QUESTION 4

4.1   Sketched below are the functions: $f(x) = 2x^2 - 6x - 20$ and $g(x) = -2x + k$.
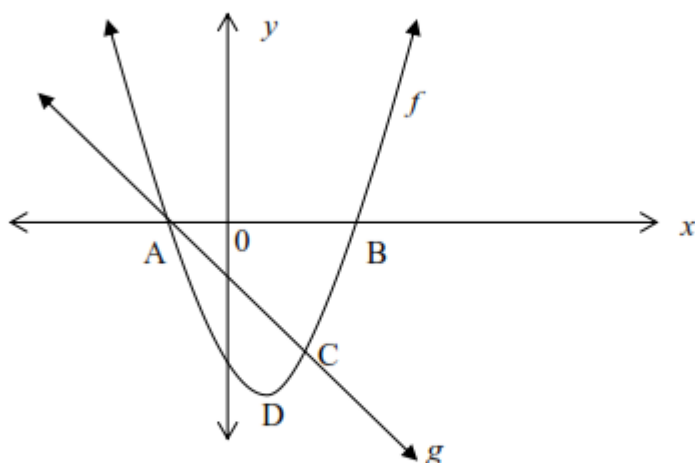
Determine:

    4.1.1   the coordinates of turning point D.   (2)

    4.1.2   the coordinates of A and B.   (3)

    4.1.3   the value of k.   (2)

4.1.4   the values of p if $2x^2 - 6x + p = 0$ has no real roots.   (2)

4.1.5   for which values of x is $f(x).g(x) \leq 0$.   (2)

4.1.6   the value of t if $y = -2x + t$ is a tangent to f.   (4)

4.2   Consider the following two functions: $p(x) = x^2 + 1$ and $r(x) = x^2 + 2x$.

         4.2.1   How will you shift p to become the function r?   (3)

         4.2.2   Write down the range of p.   (1)

                                                                                                                [19]

QUESTION 5

The sketch below represents the inverses of $g(x) = 4^x$ and $f(x) = ax^2$; $x \geq 0$.
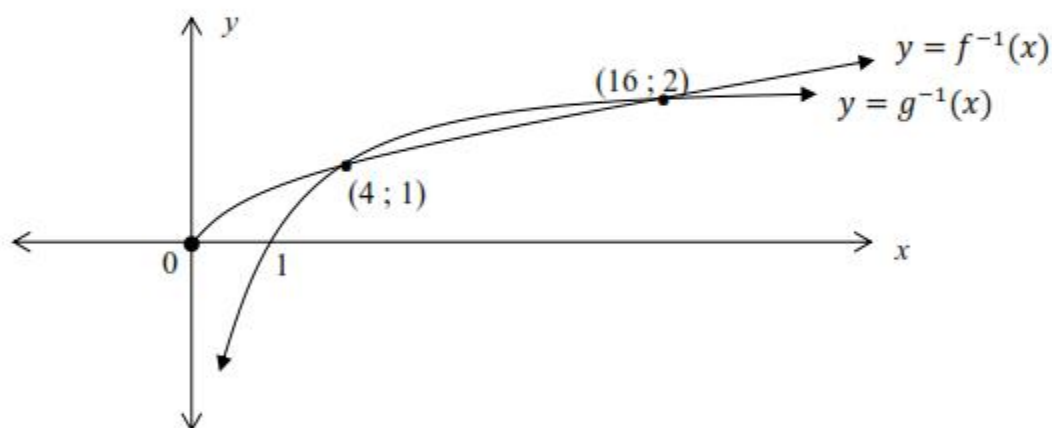
5.1   Write down the coordinates of ONE point through which both f and g will pass. (1)

5.2   Determine the equation of f.   (3)

5.3   Calculate x if $g(x + 2) = 16$.   (3)

5.4   If $h(x) = g^{-1}(x - 2)$, for which values of x will $h(x) \leq 0$?   (2)

                                                                                                                [9]

PREP 2019_KZN

QUESTION 5

Given $f(x) = \dfrac{-4}{2-x} - 1$

5.1   Write down the equations of the vertical and horizontal asymptotes of f.   (2)

5.2   Determine the intercepts of the graph of f with the axes.   (3)

5.3   Draw the graph of f. Show all intercepts with the axes as well as the asymptotes of the graph.   (4)

                                                                                 [9]

QUESTION 6

In the diagram, the graphs of $f(x) = -x^2 + 5x + 6$ and $g(x) = x + 1$ are drawn below.

The graph of f intersects the x – axis at B and C and the y – axis at A.

The graph of g intersects the graph of f at B and S. PQR is perpendicular to the x – axis with points P and Q on f and g respectively. M is the turning point of f.
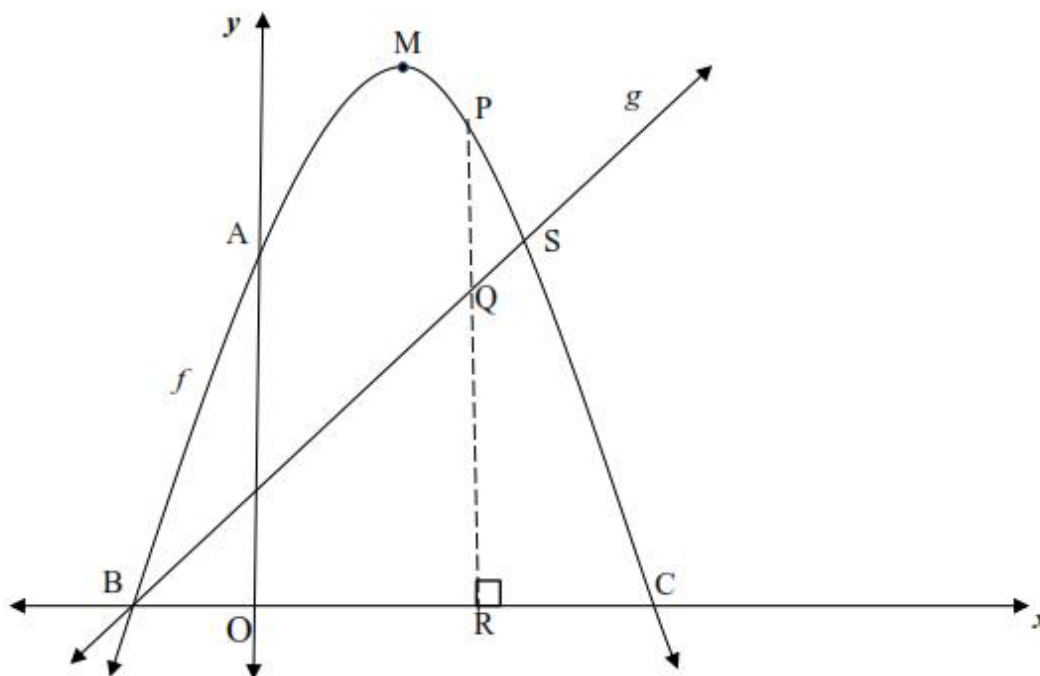
6.1   Write down the co-ordinates of A.   (1)

6.2   S is the reflection of A about the axis of symmetry of f. Calculate the coordinates of S.   (2)

6.3   Calculate the coordinates of B and C.   (3)

6.4   If PQ = 5 units, calculate the length of OR.   (4)

6.5   Calculate the:
        6.5.1   Coordinates of M.   (4)
        6.5.2   Maximum length of PQ between B and S.   (4)

                                                                                [18]

QUESTION 7

In the diagram, the graph of $g(x) = \log_5 x$ is drawn.
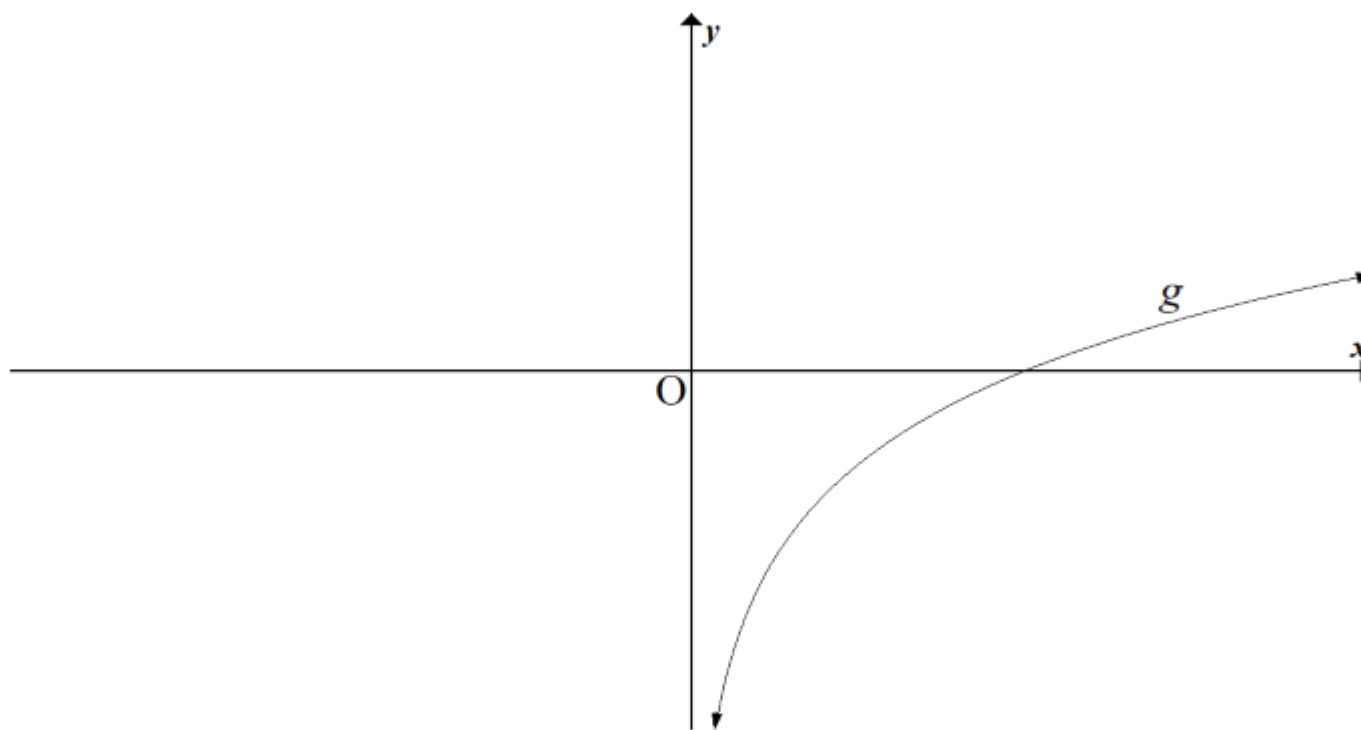
7.1   Write down the equation of g^{-1}, the inverse of g, in the form $y = \ldots$ (2)

7.2   Write down the range of g^{-1}.   (1)

7.3   Calculate the value(s) of x for which $g(x) \leq -4$.   (4)

         [7]

PREP 2019_MPUMALANGA

QUESTION 4

In the diagram, the graphs of $f(x) = m^x + k$ and $g(x) = \dfrac{-4}{x+2} - 1$ are drawn.

The two graphs intersect at A $(0; -3)$. The point C$(2; 5)$ lies on f and B$(-6;0)$ is the x- intercept of g.
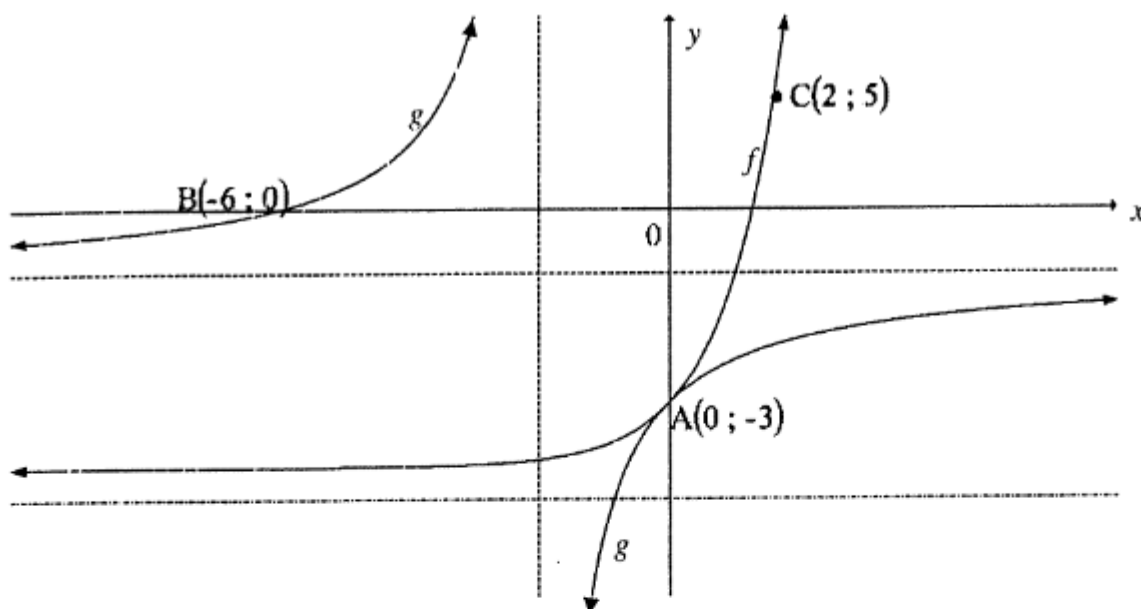
4.1   Determine:

   4.1.1   the equation of f (4)

   4.1.2   the equation of h, the axis of symmetry of g with a negative gradient   (3)

4.2   Describe the transformation that g has to undergo to form the graph of

$$p(x) = \frac{-4}{x+4} + 4$$ (2)

4.3   For which value of x is:

$$h(x) \le g(x)\,?$$ (5)

                                                                                              [14]

QUESTION 3

In the diagram below, the graph of $f(x) = ax^2 + bx + c$ and $g(x) = \log_m x$ intersect at M(4 ; 2)

The axis of symmetry of f is $x = 2$. N is the turning point of the parabola. R and Q (5 ; 0) are the x-intercepts of f and P is the intercept of f. K is the x-intercept of g.
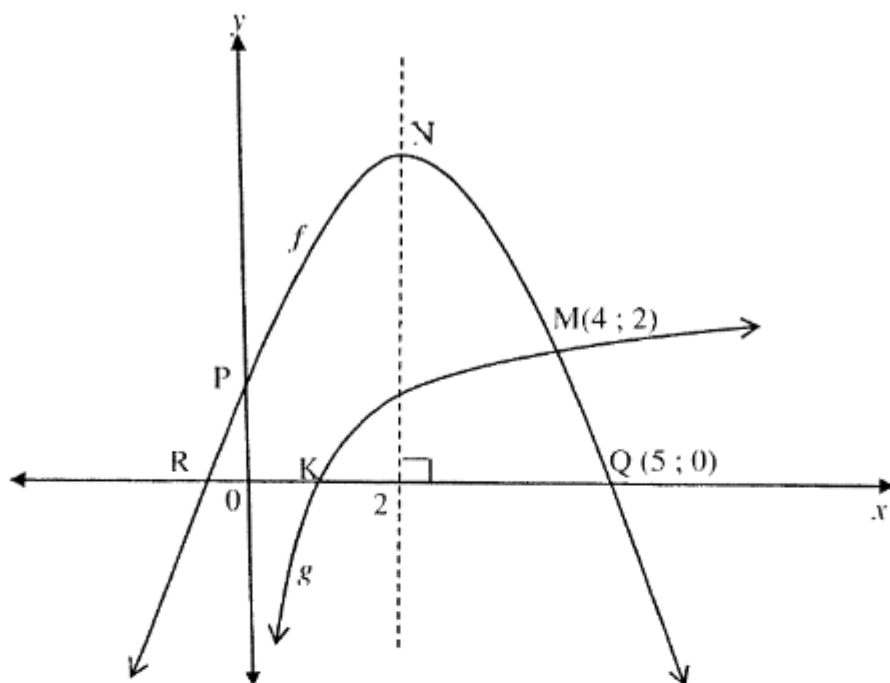
3.1   Determine the value of m.   (2)

3.2   Write down the equation of $g^{-1}(x)$, the inverse of g, in the form $y = \ldots$ (2)

3.3   Determine the equation of the parabola in the form $f(x)=ax^2+bx+c$.   (5)

3.4   Determine the value of $f(0) - g^{-1}(0)$.   (1)

3.5   Write down the co-ordinates of the turning point of $f(x+3)$ (4)

3.6   Determine the values of x for which:

     3.6.1 $f'(x) \cdot g(x) \geq 0$ (2)

     3.6.2 $f(x) - \dfrac{13}{5} = g(x)$ (2)

[18]

PREP 2019_WESTERN CAPE

QUESTION 4

Given: $f(x) = \dfrac{3}{x+1} + 1$

4.1 Determine the equations of the asymptotes of f. (2)

4.2 Calculate the y-intercept of f. (2)

4.3 Sketch the graph of f for $x > -1$.
Clearly show the intercept(s) with the axis and any asymptotes. (4)

4.4 Determine the value(s) of x for which $f(x) \geq 4$. (2)

[10]

QUESTION 5

Sketched below are the graphs of $g(x) = -3x + 20$ and $f(x) = ax^2 + b + c$. Graph f has a turning point at $A(3; k)$. Graph f and g intersects at A and $B(6; 2)$.
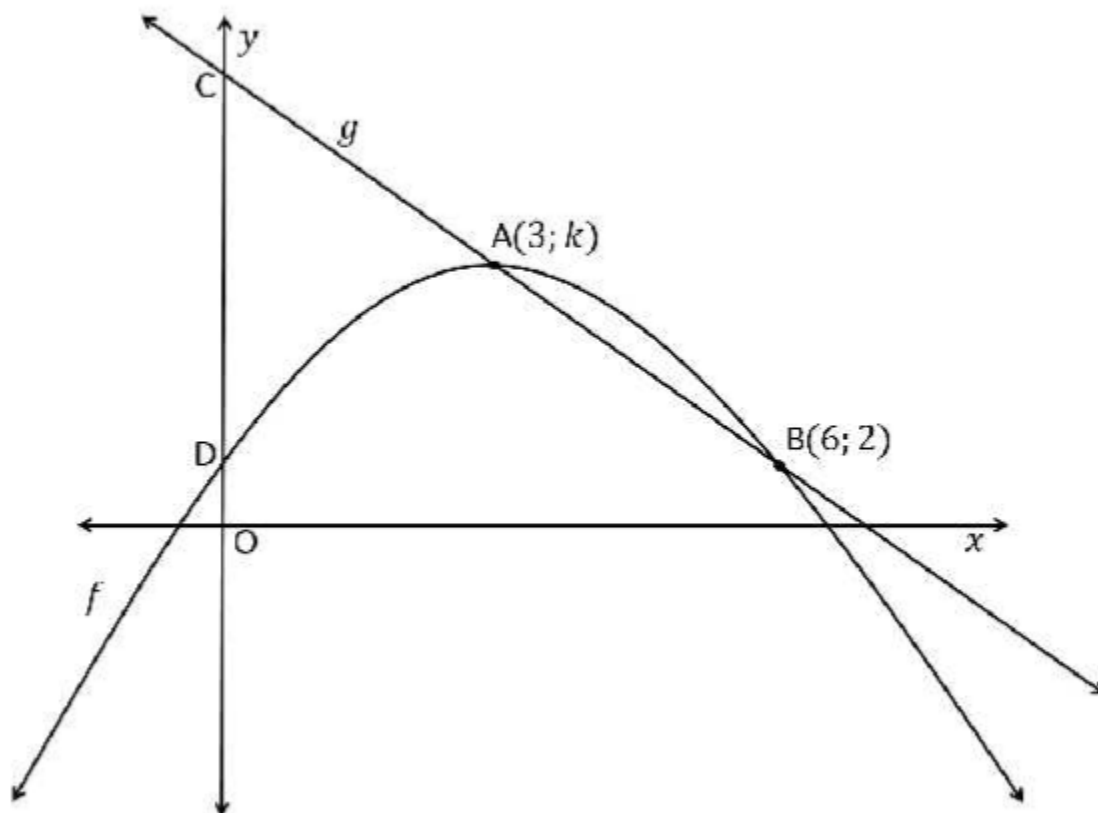
5.1 Calculate the numerical value of k, the y-coordinate of A. (2)

5.2 Determine the range of $y = -f(x)$. (2)

5.3 Calculate the numerical values of a, b and c. (6)

5.4 Determine the value(s) of x for which $f(x) > g(x)$. (2)

5.5 Describe the nature of the roots for $f(x) - 11$. (2)

5.6 Determine the value(s) of x for which $f'(x) \cdot g'(x) > 0$. (2)

[16]

QUESTION 6

Sketched below are the graphs of $h(x) = \left(\frac{1}{2}\right)^x + q$ and $f(x) = \log_2 x$.
Graph f and the asymptote of h intersect at $B(4; p)$.
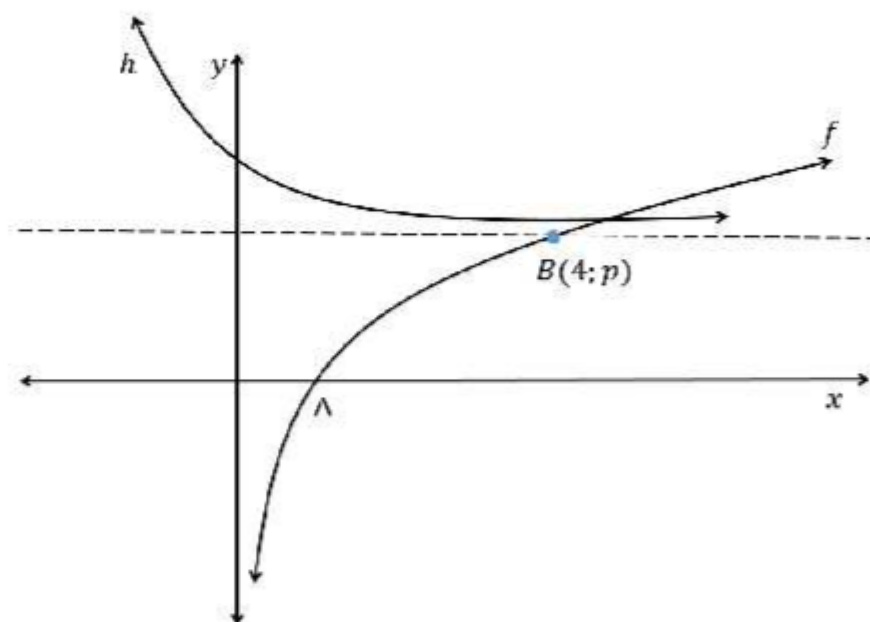
6.1 Write down the coordinates of A, the x −intercept of f. (1)

6.2 Determine the domain of f. (1)

6.3 Determine the equation of f^{-1} in the form $y = \ldots$ (2)

6.4 Sketch the graph of f^{-1}. Clearly labelling the intercept(s) with the axes as well as the coordinates of any one other point on the graph. (3)

6.5 Determine the equation of the asymptote of h. (2)

6.6 Describe, in words, the transformation of h to f^{-1}. (2)

[11]

PREP 2019_NORTH WEST 2

QUESTION 4

The graphs of $g(x) = \frac{1}{2}(x - 2)^2 - 9$ and $h(x) = \frac{a}{x + p} + q$ are sketched below.

The axis of symmetry of graph g is the vertical asymptote of graph h. The line f is an axis of symmetry of graph h. B is the y-intercept of h, g and f.
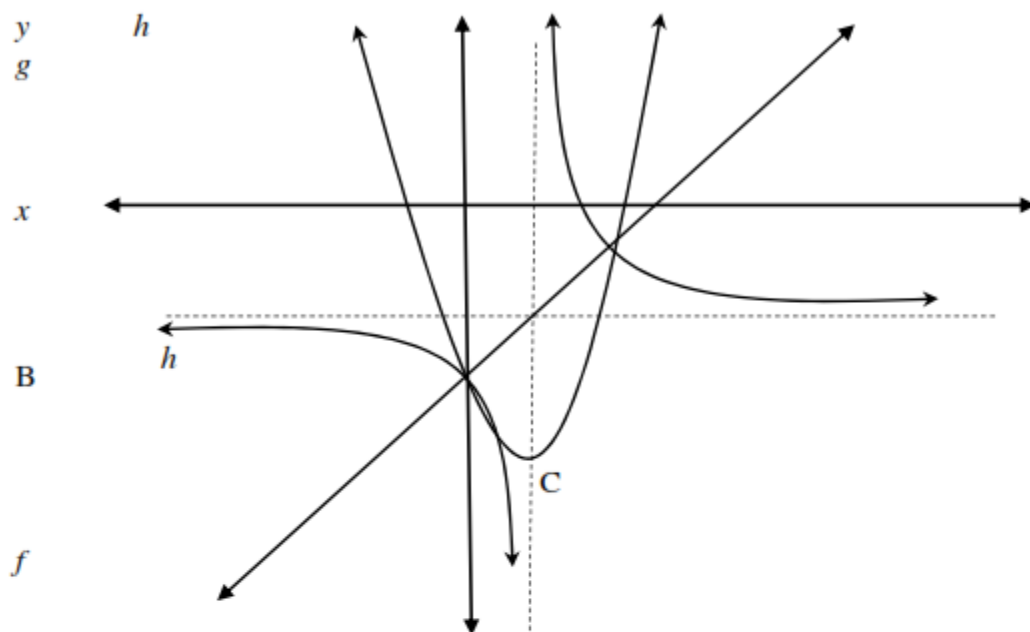
4.1   Write down the coordinates of C, the turning point of g.   (2)

4.2   Determine the coordinates of B.   (2)

4.3   Write down the equation of f.   (2)

4.4   Determine the equation of h.   (5)

4.5   Write down the equations of the vertical and horizontal asymptotes of
$k(x) = 3h(x) - 2$.   (2)

4.6   Determine the x-intercept of h.   (3)

4.7   For which values of x will:

4.7.1 $\dfrac{g'(x)}{h(x)} \geq 0$ (3)

4.7.2 $f^{-1}(x - 1) < 2$ (4)

4.8   Calculate the value(s) of k for which $g(x) = f(x) + k$ has two unequal positive roots.   (6)

[29]

QUESTION 5

5.1   Consider the function $f(x) = \left(\dfrac{5}{6}\right)^{x}$

     5.1.1   Write down the equation of h, the reflection of f in the y-axis.   (1)

     5.1.2   Write down the equation of $f^{-1}(x)$ in the form $y = ...$ (2)

     5.1.3   For which value(s) of x will $f^{-1}(x) \geq 0$?   (2)

5.2   The function defined as $f(x) = ax^{2} + bx + c$ has the following properties:

- $f'(-2,5) = 0$
- $f(1) = 0$
- $b^{2} - 4ac > 0$
- $f(-2,5) = 6$

Draw a neat sketch graph of f. Clearly show all x-intercepts and turning point.   (4)

                                                           [9]

FINANCIAL MATHS

PREP 2019_LIMPOPO

QUESTION 7

7.1 Peter invests R12 500 for k years at a compound interest rate of 9% p.a. compounded quarterly. At the end of the k years his investment is worth R30 440. Calculate the value of k.

(4)

7.2 Sam bought a car for R500 000 on an agreement in which he will repay it in monthly instalments at the end of each month for 5 years, starting one month after the loan was granted. Interest is charged at 18% p.a. compounded monthly.

7.2.1 Calculate the annual effective interest rate of the loan.

(3)

7.2.2 Calculate Sam's monthly instalments.

(4)

7.2.3 Sam decided to pay R12 700 each month as his repayment. Calculate the outstanding balance of the loan after 2 years.

(4)

[15]

PREP 2019_EASTERN CAPE

QUESTION 7

7.1   Kamva bought a motorbike valued at R40 000. After 5 years the value of the motorbike had depreciated to R26 700, at a rate of r % p.a. on the reducing balance method, compounded annually. Calculate r, the rate of depreciation.   (3)

7.2   A bank granted Nathan a loan for R1 200 000 to buy a house. He agreed to repay the loan over a period of 15 years at an interest rate of 11,5% p.a. compounded monthly. He made his first payment at the end of the first month after the loan was granted.

       7.2.1   Calculate Nathan's monthly instalment.   (3)

       7.2.2   Due to unforeseen circumstances, Nathan could not pay his 76th, 77th, 78th, 79th and 80th instalments. He resumed his payments at the end of the 81st month.

            (a)   Calculate the outstanding balance at the end of the 80th month.   (5)

            (b)   If Nathan continues paying the same monthly instalment, how many months will it take him to pay the balance outstanding at the end of the 80th month?   (4)

            [15]

PREP 2019_FREE STATE

QUESTION 7

The Northern Cape Department of Education bought 50-tablets for a total amount of R800 000 in order for teachers to do a coding course in the province.

7.1   Calculate:

    7.1.1   The price of one tablet (1)

    7.1.2   The book value of a tablet after 3 years, if the rate of depreciation is 18% p.a. on reducing-balance method. (2)

    7.1.3   The number of years it would take for the price of a tablet to be R21 200, if the rate of inflation is 5,8% p.a. compounded annually. (Give an answer to the nearest year.) (3)

7.2   John bought a house and took out a loan for R900 000. The loan is repaid over 20 years and the interest on the loan is 8% p.a. compounded monthly.

    Calculate the:

    7.2.1   Monthly payments (4)

    7.2.2   Interest paid on the last two years (5)

        [15]

PREP 2019_GAUTENG

QUESTION 4

4.1   How many years will it take for an investment of R3 000 to accumulate to R4 500, if it is invested at 8% p.a. compounded monthly? (4)

4.2 Bongani paid off a 20-year loan of R40 000. During the period of the loan the interest rate changed from 24% p.a. compounded monthly for the first five years to 18% p.a. compounded monthly for the remaining years.

    4.2.1   Calculate the initial monthly payment before the interest rate changed. (4)

    4.2.2   What is the outstanding balance of the loan after the FIRST five years? (4)

    4.2.3   Determine the monthly payment after the interest rate changed. (4)

[16]

PREP 2019_NORTH WEST

QUESTION 6

6.1 Convert an interest rate of 14% p.a. compounded monthly to an interest rate per annum compounded quarterly. (3)

6.2 Nelson deposits R3 500 into a savings account. Three years later he adds R5 700 to the account. The interest for the first two years is 7% p.a. compounded quarterly. The interest for the last three years is 8% p.a. compounded monthly. Calculate the balance in the savings account at the end of five years. (4)

[7]

QUESTION 7

Mr Daniels wants to take out a loan for a house over twenty years. He has approached two financial institutions and was offered two different options. The two options are shown in the table below:

VARIABLES	OPTION 1	OPTION 2
Loan amount	R950 000	R950 000
Interest rate (compounded monthly)	12%	11,8%
Repayments	Rx per month	R10 328,16 per month
Bank charges	R0	R200 per month
Commissions	R6 000	R0

7.1 Determine the total cost for Option 1. (7)

7.2 Which option is the best? Provide relevant calculations to justify your answer. (3)

[10]

PREP 2019_KZN

QUESTION 8

8.1 A car depreciated at the rate of 13,5 % p.a. to R250 000 over 5 years according to the reducing balance method. Determine the original price of the car, to the nearest rand. (3)

8.2 Melissa takes a loan of R950 000 to buy a house. The interest is 14,25 % p.a. compounded monthly. His first instalment will commence one month after taking out the loan.

    8.2.1 Calculate the monthly repayments over a period of 20 years. (4)

    8.2.2 Determine the balance on the loan after the 100th instalment. (4)

    8.2.3 If Melissa failed to pay the 101st, 102nd, 103rd and 104th instalments, calculate the value of the new instalment that will settle the loan in the same time period. (4)

[15]

PREP 2019_MPUMALANGA

QUESTION 5

5.1 John wishes to save money for his child's education. He will deposit R500 into a savings account on his child day of birth. Thereafter he deposits R500 at the end of each month for the next 18 years.
Determine the final value of this investment, if the rate of interest is 9% p.a. compounded monthly? (4)

5.2 John invests R600 into an account earning 8% per annum, compounded quarterly. His friend, Peter, also invests R600 and earns interest compounded semi-annually (every 6 months). John and Peter's investments are the same at the end of 12 months. Calculate the interest rate that Peter receive. (4)

5.3 The Mahlangu family is eager to possess their own home. They can afford a monthly payment of R4000. They want to buy a house for R650 000 and the bank offers them a home loan at 14,3% p.a. compounded monthly, over a period of 20 years. Will they be able to afford the buy the house? (4)

5.4 Mr Brown receives an amount of R1 000 000 at retirement which he invests at a rate of 9,5% per annum, compounded monthly. His monthly expenditure is R25 000 per month to keep up his normal way of living.
For how many months will he be able to live from his investment? (4)

[16]

PREP 2019_WESTERN CAPE

QUESTION 7

7.1   A car costs x rand. The value of the car decreased by r% annually on the reducing-balance method. After 42 months the book value of the car is half of what it was worth when it was bought. Calculate r.   (4)

7.2   A man plans to buy a car for his family. The car will cost R90 000.
He has two options:

Option 1:
He applies for a car loan.
The bank charges interest at 14% per annum, compounded monthly. He can afford a monthly instalment of R3 000. He will make the first monthly repayment one month after the loan is granted.

Option 2:
He deposits a fixed amount in the bank at the end of each month for 3 years. The bank pays interest at 10% per annum, compounded monthly. He will make the first deposit at the end of the first month.

     7.2.1   Calculate, to the nearest year, how long it will take the man to pay back the loan, if he chooses Option 1.   (5)

     7.2.2   Suppose he chooses Option 1. Calculate the outstanding balance immediately after the 24th payment.   (3)

     7.2.3   Calculate the monthly deposit if he chooses Option 2.   (4)

                                                                         [16]

PREP 2019_NORTH WEST 2

QUESTION 6

On 1 July 2010, David bought a tractor for R2 000 000. On that day, he paid a deposit of 25% of the purchase price and the bank granted him a loan at an interest rate of 9,5% per annum, compounded quarterly, to pay off the balance of the purchase price. David agreed to pay quarterly instalments of R58 000, starting on 1 January 2011.

6.1	How much money did David borrow from the bank?	(2)
6.2	How many quarterly instalmentsare required to pay off the loan?	(6)
6.3	Calculate the amount owing on the loan immediately after David paid his quarterly instalment on 1 July 2016, i.e. six years after he bought the tractor.	(4)
6.4	Hence, calculate the amount of interest that David had paid on this loan until immediately after paying his quarterly instalment on 1 July 2016.	(4)

[16]

CALCULUS

PREP 2019_LIMPOPO

QUESTION 8

8.1   Given : $f(x) = 3x^2 - 1$.

      Determine $f'(x)$ from first principle.   (5)

8.2   Determine:

    8.2.1 $\dfrac{dy}{dx}$ if $y = 5x^2 + \sqrt{x}$ (3)

    8.2.2 $D_x \left[\dfrac{6x - 4}{3x} \right]$ (3)

8.3   Given $s(t) = t^3$.

      Show that the gradient of any tangent to s will never be negative.   (2)

      [13]

QUESTION 9

The sketch below shows the graph of the function $f(x) = x^3 - x^2 - 8x + 12$.

A is an x-intercept and B a turning point of the graph. The points $C(0;12)$ and $D(2;0)$,

the other turning point, are given on the sketch.
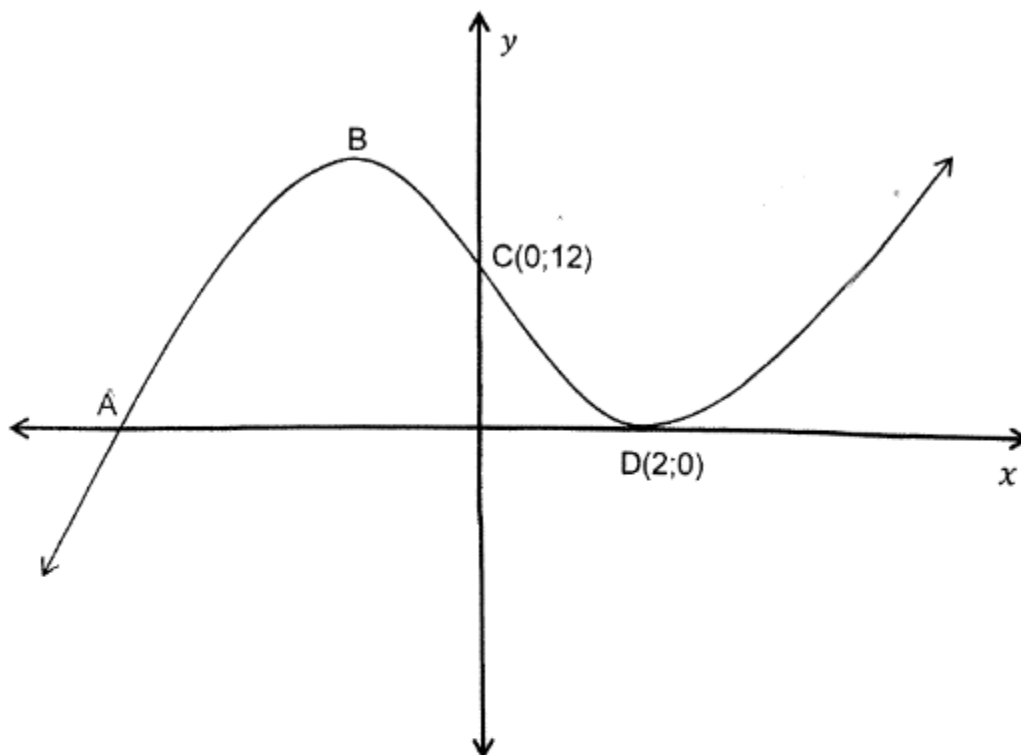
9.1   Determine the coordinate of A. (4)

9.2   Determine the coordinate of B. (5)

9.3   Calculate the $x-$coordinate of the point of inflection. (2)

9.4   Write down the value(s) of x for which $f'(x) > 0$ (2)

9.5   How many real roots will the equation $x^3 - x^2 - 8x + 12 = k$

have if $k < 0$? (2)

[15]

QUESTION 10

PQRS is a rectangle with P on the curve $h(x) = x^2$ and with the x-axis and the line $x = 6$ as boundaries.
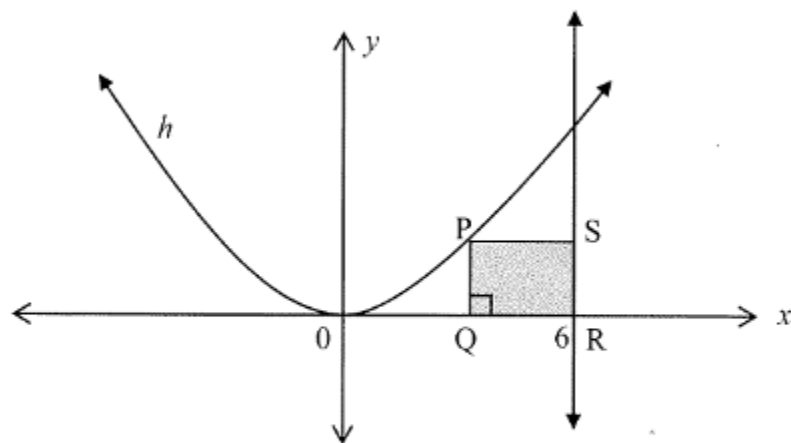
10.1   Show that the area of rectangle PQRS can be expressed as:

$$A = 6x^2 - x^3 .$$ (3)

10.2   Determine the largest possible area for rectangle PQRS. Show all your calculations.   (4)

[7]

PREP 2019_EASTERN CAPE

(5)

8.1   Determine $f'(x)$ from first principles if $f(x) = 3 - 2x^2$

8.2   Determine:

8.2.1 $D_x\left[x(x-2)^2\right]$ (4)

8.2.2 $\dfrac{dy}{dx}$ if $y = ax^{\frac{3}{7}} - \dfrac{2x}{\sqrt{x}} + 3$ (3)

[12]

QUESTION 9

The diagram below shows the graph of $y = g'(x)$ where $g(x) = ax^3 + bx^2 + cx + d$.
The graph $g'(x)$ cuts the y-axis at $(0; 1)$ and the x-axis at $(-\frac{1}{3}; 0)$ and $(1; 0)$.
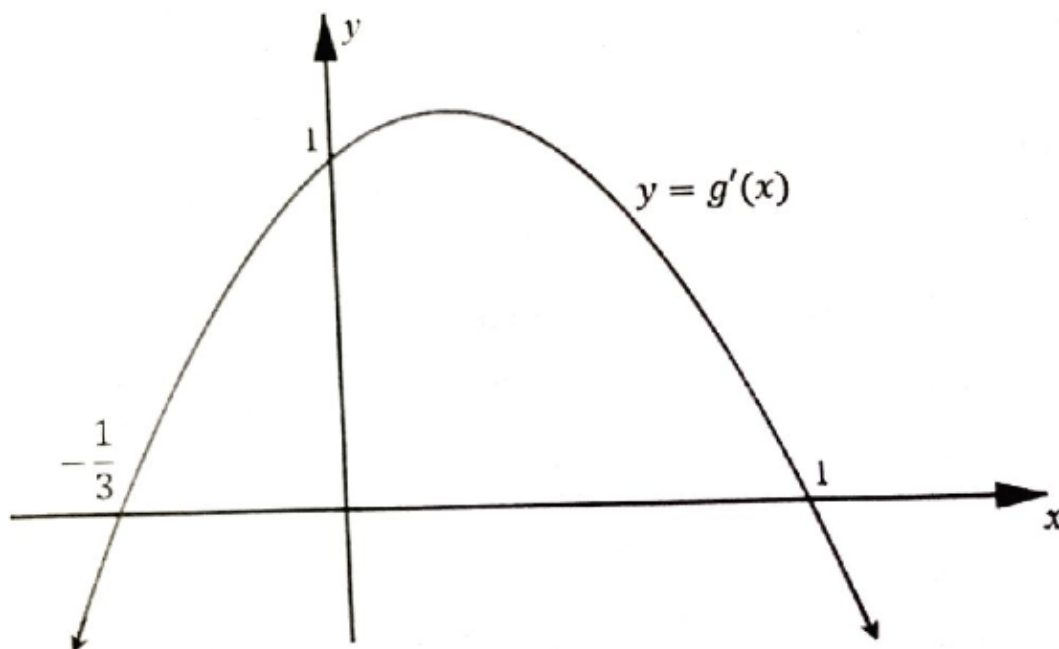
9.1 Write down the x-coordinate(s) of the stationary point(s) of g. (2)

9.2 Determine the x-coordinate of the point of inflection of g. (2)

9.3 Determine the values of x for which g is an increasing function. (2)

9.4 Determine the equation of $g'(x)$ in the form: $g'(x) = px^2 + qx + r$. (4)

9.5 Given that:
- $g(x) + 1$ passes through $(0; 0)$ and
- $g'(x) = -3x^2 + 2x + 1$

Show that for $g(x)$, $a = -1, b = 1, c = 1$ and $d = -1$. (5)

[15]

PREP 2019_FREE STATE

QUESTION 8

8.1   Determine:

    8.1.1 $f'(x)$ from first principles if $f(x) = 3x^2$ (5)

    8.1.2 $\dfrac{d}{dx}\left(\sqrt{x^3} - x + \dfrac{3}{x^2} \right)$ (3)

8.2   Given that $g(x) = -4x + 12$ and $g(x) = f'(x)$.

    8.2.1   Calculate the x coordinate of the turning point of f.   (2)

    8.2.2   Determine the values of x for which the graph of f will be decreasing.   (2)

        [12]

QUESTION 9

In the diagram, the graph of $f(x) = -x^3 + 10x^2 - 17x - 28$ intersects the y-axis at A.
B and C are the turning points of f.
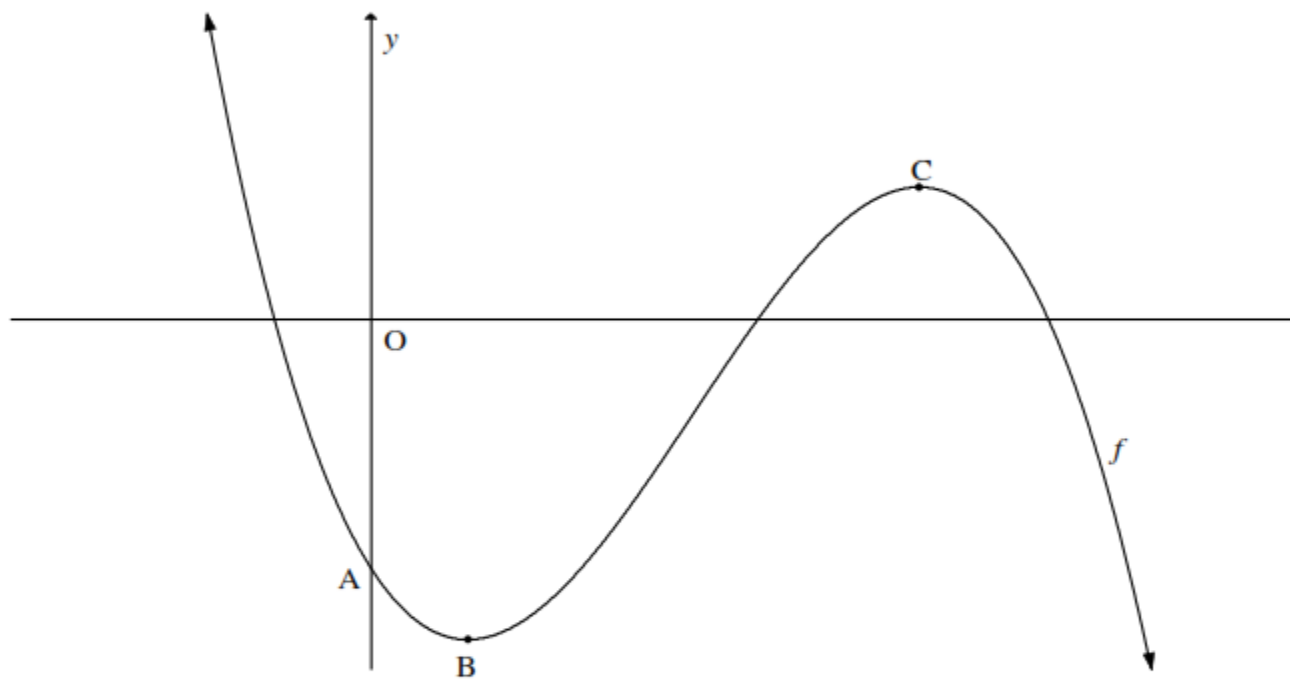
9.1   Write down the coordinates of A.   (1)

9.2   Calculate the coordinates of B and C.   (6)

9.3   For which value(s) of x is f concave up?   (4)

9.4   Determine the value(s) of p for which $f(x) = p$ has only one positive root.   (2)

                                                                                         [13]

QUESTION 10

In the diagram below, TUVW is a rectangular picture. The picture is framed such that there is a 3 cm space around the picture. The perimeter of the rectangle PQRS is 70 cm.
PQ = x units and QR = y units.
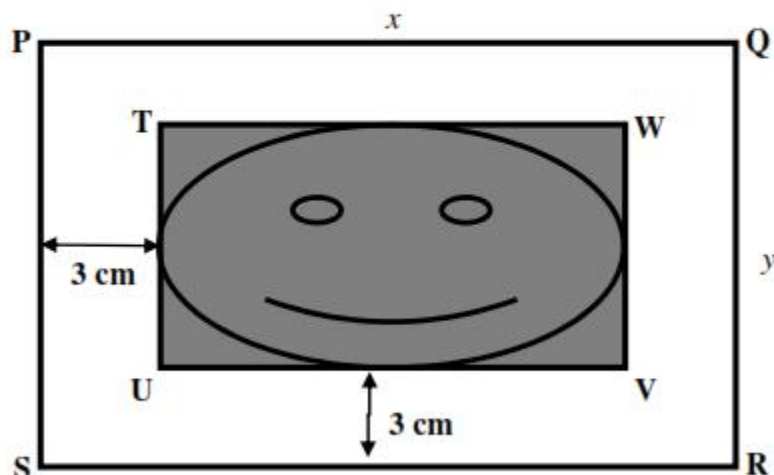
Calculate the maximum area of the picture TUVW. [8]

PREP 2019_GAUTENG

QUESTION 9

9.1 Given: $f(x) = 3x - x^2$

   9.1.1 Determine $f'(x)$ from FIRST principles. (5)

   9.1.2 Determine the average gradient of f between $x = 1$ and $x = 3$. (3)

9.2 Determine:

   9.2.1 $\frac{dy}{dx}$ if $y = \frac{8 - 3x^6}{8x^5}$ (3)

   9.2.2 $D_x\left[\sqrt[3]{x^2} + \frac{1}{x} + 2x\right]$ (4)

[15]

QUESTION 10

A cubic function has the following essential properties:

- $f(0)=8$
- $f(4)=f(1)=0$
- $f'(3)=f'(1)=0$
- $f(3)=8$

10.1   Sketch the graph of f in your ANSWER BOOK clearly indicating the turning point(s) and the points of intersection of the graph with the axes.   (3)

10.2   Show that the defining equation of f is $f(x)=-2x^3+12x^2-18x+8$.   (4)

10.3   Determine the value(s) of x for which graph of f is concave down.   (3)

    [10]

QUESTION 11

The diagram below shows a garden in the form of a rectangle and a semi-circle.
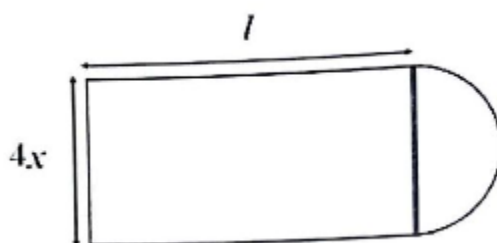
The rectangular section of the garden has the dimensions length (l) and width ($4x$).

The perimeter of the garden is 32m.

11.1   Express the length (l) in terms of x.   (3)

11.2   Show that the area of the garden can be written as $A(x)=-8x^2-2\pi x^2+64x$.   (2)

11.3   Determine the value of x for which the area of the garden is minimum.   (3)

    [8]

PREP 2019_NORTH WEST

QUESTION 8

8.1   Given: $f(x) = 3 - x^2$. Determine $f'(x)$ from first principles. (5)

8.2   Determine $\dfrac{dy}{dx}$ if:

   8.2.1 $y = \dfrac{2}{\sqrt[3]{x}} - \pi x$ (3)

   8.2.2 $xy - y = x^2 - 1$ (4)

8.3   Determine the value of x where the gradient of the tangent to $f(x) = (x - 5)^2$
      is equal to -8. (4)

[16]

QUESTION 9

9.1   The function defined by $f(x) = x^3 + px^2 + qx + 30$ is represented by the sketch
      below. A $(-1\,;36)$ and B are the turning points of f, while g is a tangent to f at A
      which cuts f at point C.
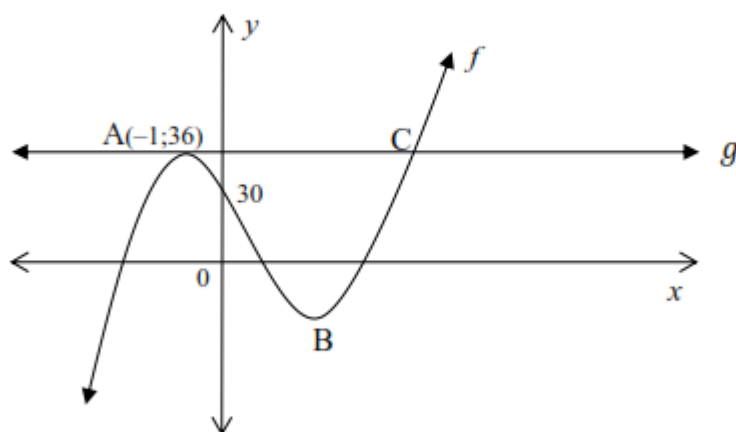
9.1.1   Show that $p = -4$ and $q = -11$. (7)

9.1.2   Determine the coordinates of C. (3)

9.1.3 Write down the coordinates of a turning point of k, if
$$k(x) = f(x) - 10.$$
(2)

9.2 The turning points of the graph of a cubic polynomial $h(x)$ are $(2\,;-3)$ and $(5\,;4)$. Draw a sketch graph of the derivative function $h'(x)$, clearly showing the x-intercepts.
(3)
[15]

QUESTION 10

PQRS is a rectangle with P on the curve $h(x) = x^2$ and with the x-axis and the line $x = 6$ as boundaries.
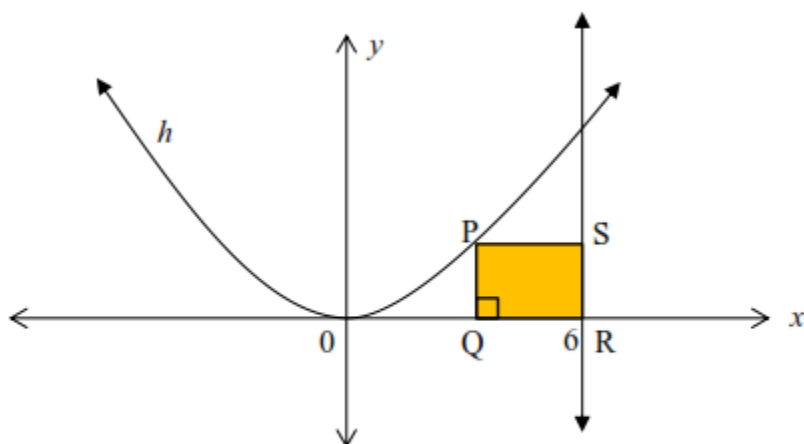
10.1 Show that the area of rectangle PQRS can be expressed as:
$$A = 6x^2 - x^3.$$
(3)

10.2 Determine the largest possible area for rectangle PQRS. Show all your calculations.
(4)
[7]

PREP 2019_KZN

QUESTION 9

9.1 Determine $f'(x)$ from first principles given $f(x) = x^2 - \dfrac{1}{2}x$.
(5)

9.2 Determine:

9.2.1 $\dfrac{d}{dx}[3x^4 + \sqrt[5]{x} + a^2]$
(3)

9.2.2 $\dfrac{dy}{dx}$, if $xy = x + x^2 - 1$. (4)

[12]

QUESTION 10

In the diagram, the graph of $f(x) = x^3 + 5x^2 - 8x - 12$ is drawn. A and B are the turning points and C the y - intercept of f. $g(x) = mx + c$ is a tangent to the graph of f at C.
D is the intersection of f and g.
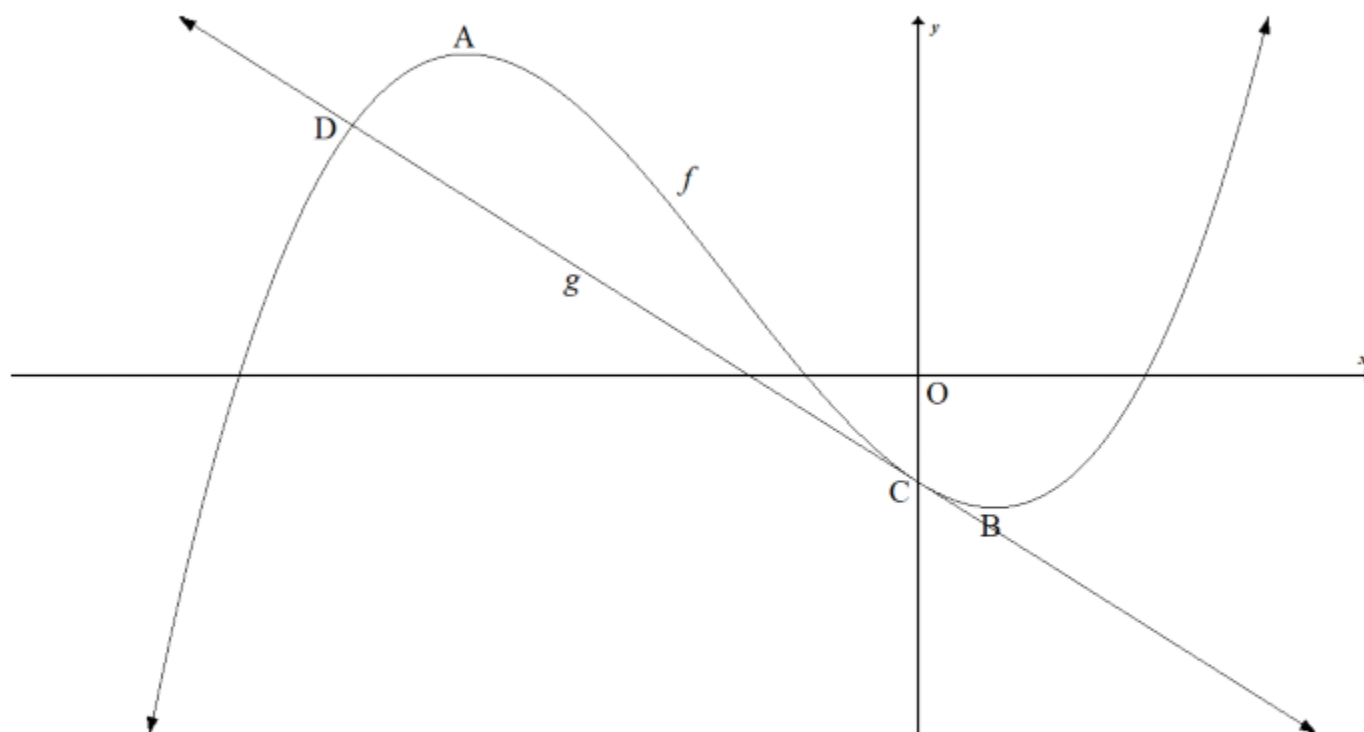
10.1   Calculate the:

    10.1.1   co-ordinates of the x-intercepts of f. (6)
    10.1.2   co-ordinates of B. (4)
    10.1.3 x – coordinate of the point of inflection of f. (2)

10.2   Determine the:

    10.2.1   equation of the g. (2)
    10.2.2   values of x for which $f'(x) \cdot g'(x) > 0$. (3)

[17]

QUESTION 11

In the diagram below, ΔABC is an equilateral triangle with sides d units long. P and S are points on sides AB and AC respectively. Q and R are points on BC such that PQRS is a rectangle. BQ = RC = $2y$ units.
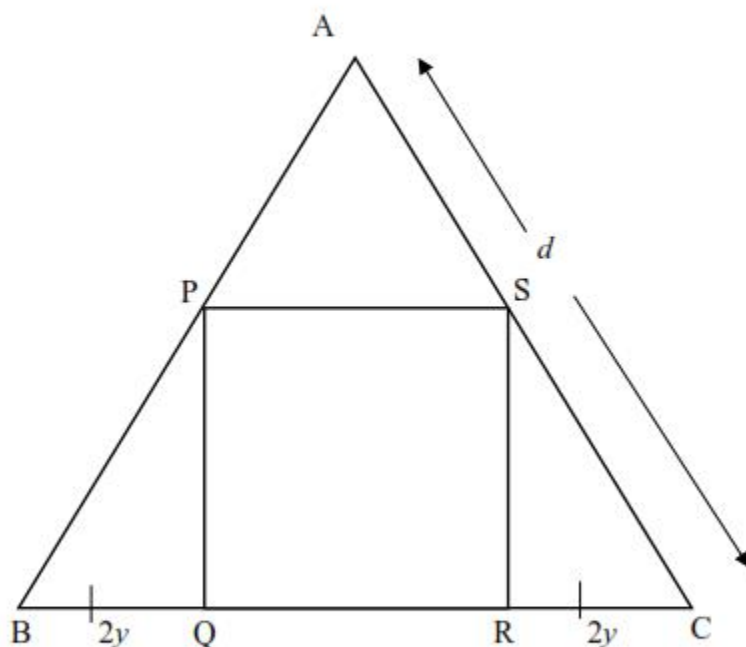
11.1   Show that the area of the rectangle PQRS is given by $A = 2\sqrt{3}y(d - 4y)$.   (4)

11.2   Determine the maximum area of the rectangle in terms of d.   (6)

[10]

PREP 2019_MPUMALANGA

QUESTION 6

6.1   Given: $f(x) = -2x^2 + 2$

Determine $f'(x)$ from FIRST PRINCIPLES.   (5)

6.2   Determine:

6.2.1 $\frac{dy}{dx}$, if $y = (x - 2)^2$ (3)

6.2.2 $f'(x)$ if $f(x) = \dfrac{2x^2 + \sqrt{x^5}}{6x}$ (4)

[12]

QUESTION 7

7.1 The cubic graph , f, has the following properties.

$f'(x) > 0$ when $x < 1$ and $x > 5$

$f'(x) < 0$ when $1 < x < 5$

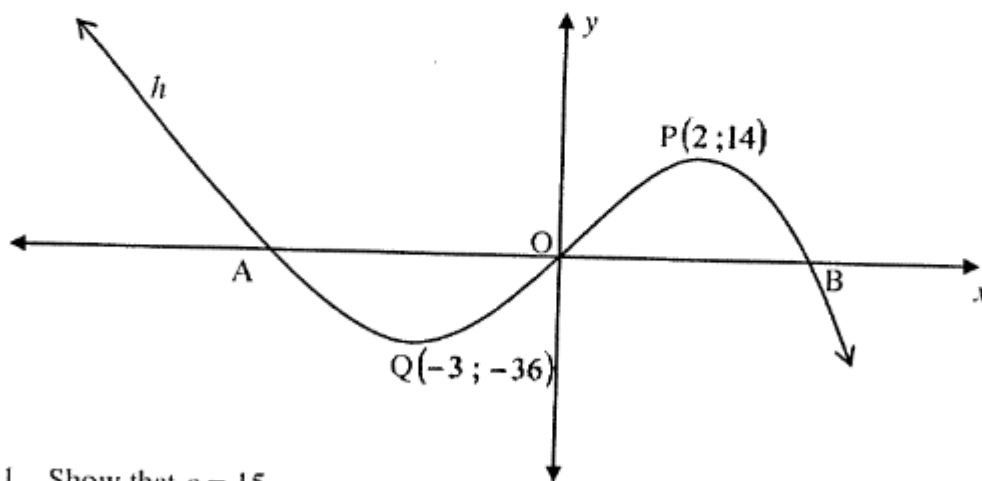$f'(1) = 0$ and $f'(5) = 0$

$f(3) = 0$ and $f(0) = -12$

Sketch the graph of f, showing all the critical points. (4)

7.2 In the diagram, the graph of $h(x) = -x^3 - 2x^2 + cx$ is drawn.

Graph h cuts the x- axis at A and B and the origin.

$P(2;14)$ and $Q(-3;-36)$ are the turning points of h.

7.2.1 Show that $c = 15$. (2)

7.2.2 Determine the length of AB (3)

7.2.3 For which value(s) of n will the equation $h(x) = -x^3 - 2x^2 + 15x$ have only one negative root? (2)

7.2.4 Determine the interval on which h will be concave up. (3)

7.2.5 Determine the x coordinate where the tangent, parallel to the AP will touch the graph of h. (5)

QUESTION 8

The depth, D, of water in a kettle t minutes after it starts to boil, is given by $D(t) = 86 - \frac{1}{8}t - \frac{1}{4}t^3$,

where D is measured in millimetres.

8.1   What is the depth of the water in the kettle just before it starts to boil?   (1)

8.2   As the water boils, the level in the kettle drops. Find the rate at which the water level

is decreasing when $t = 2$ minutes.   (3)

8.3   How many minutes after the kettle starts boiling will the water level be dropping at

a rate of 12⅛ ml/minute?   (2)

[6]

PREP 2019_WESTERN CAPE

QUESTION 8

8.1   Given: $f(x) = 1 - 3x^2$.

Determine $f'(x)$ from first principles.   (5)

8.2   Given:

$$g(x) = \frac{x^4 - x}{x}$$

Determine the gradient of the tangent to g drawn at $x = -2$.   (4)
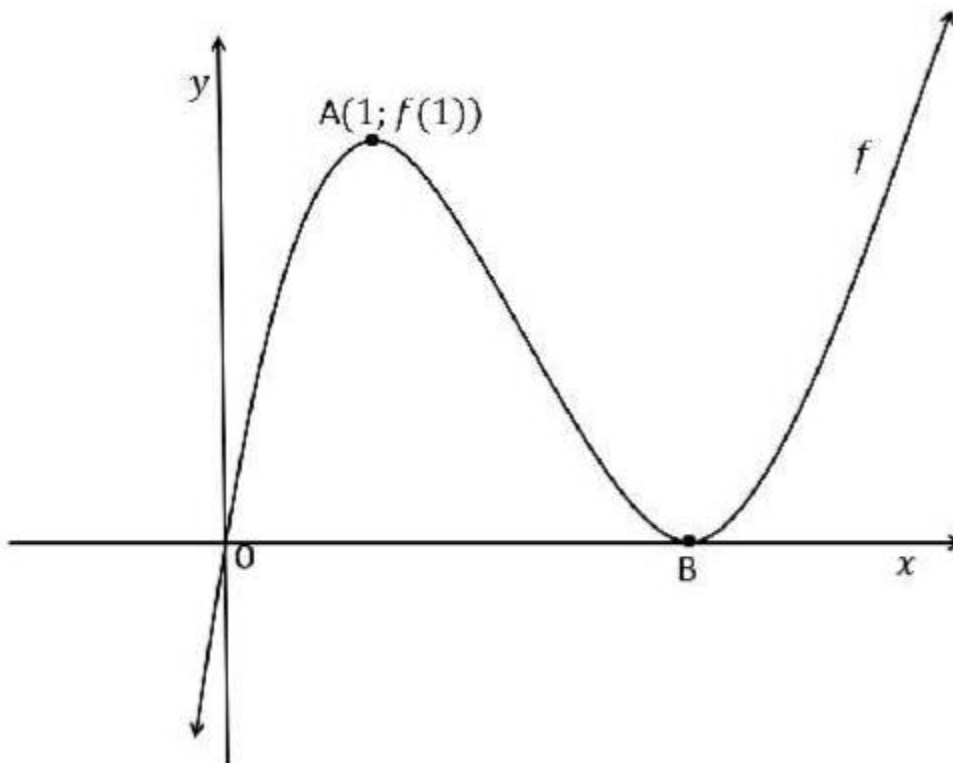
8.3   Determine $\frac{d}{d}$ if:

8.3.1 $y = \sqrt[3]{x} + \dfrac{5}{x^2}$ (4)

8.3.2 $x = \sqrt{y} + \dfrac{1}{x}$ (4)

[17]

QUESTION 9

Sketched below is the graph of $f(x) = 3x^3 + bx^2 + 27x + d$. $A(1; f(1))$ and B are the turning points of f. The graph of f cuts the x−axis at $(0; 0)$ and at B.

9.1   Give the value of d.   (1)

9.2   Show that the numerical value of $b = -18$.   (4)

9.3   Calculate the coordinates of B.   (4)

9.4   Determine the value(s) of x for which $f'(x) > 0$.   (2)

9.5   Calculate the x−coordinate of the point of inflection of f.   (3)

9.6   For which values of x will the graph of f be concave down?   (2)
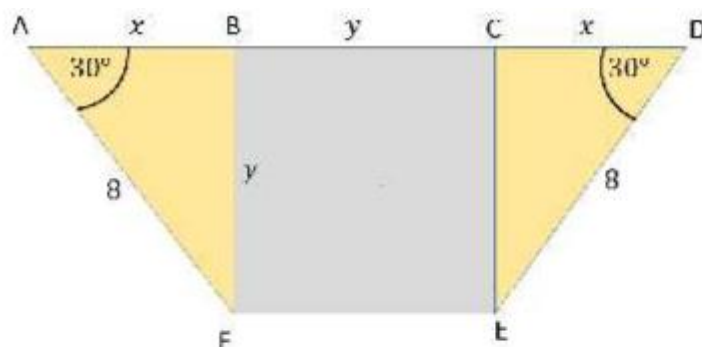
    [16]

QUESTION 10

A certain high school wants to build a stage as shown in the diagram below:

- The stage floor consists of a square (BCEF) and two congruent triangles (ΔABF and ΔDCE)
- Each side of the square is equal to $y\,u$.
- The length of the hypotenuse of the congruent triangles is equal to 8 units each.
- AB = CD = $x\,u$.
- $\hat{A} = \hat{D} = 30°$


10.1 Show that the area of the stage floor can be written as:
$$A = 8x \sin 30° + 64 - x^2$$
(3)

10.2 Calculate the value of x, for which the area of the floor will be a maximum. (3)

[6]

PREP 2019_NORTH WEST 2

QUESTION 7

7.1 Given: $f(x) = -x^2 + 3x - 7$

Determine $f'(x)$ from first principles. (6)

7.2 Determine: $D_x \left[15 \sqrt[5]{x^4} - \dfrac{3x^7 + x}{4x^3} \right]$ (6)

[12]

QUESTION 8

The graph of $f'(x) = x^2 + bx + c$, where f is a cubic function, is sketched below.
The derivative function f' cuts the x-axis at $x = -3$ and $x = 2$.
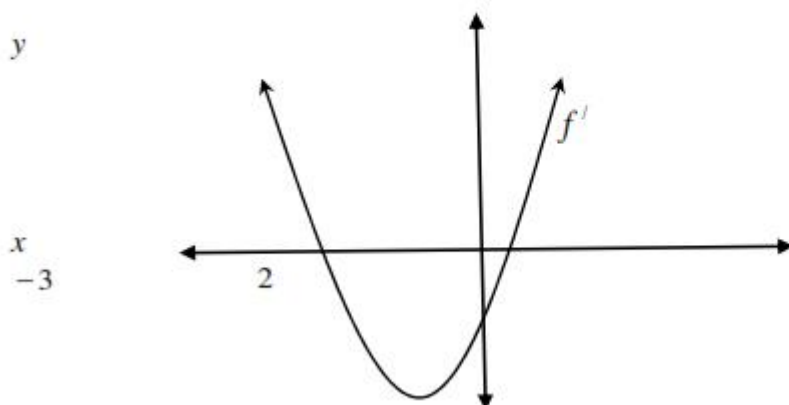
8.1 For which values of x is graph f increasing? (2)

8.2 At which value of x does graph f have a local maximum value? (1)

8.3 Determine the equation of $f'(x)$. (2)

8.4 If $f(x) = px^3 + qx^2 + rx + 10$, show that $p = \dfrac{1}{3}$, $q = \dfrac{1}{2}$ and $r = -6$. (4)

8.5 For which value(s) of x is graph f concave down? (3)

[12]

QUESTION 9

A hunter was standing at point A, along the fence of a rectangular game enclosure, when he spotted a deer standing at point B, the corner of the rectangular enclosure. The distance from A to B is 1200m. At exactly the same time as the hunter started to move in an easterly direction towards B, the deer started to move in a southerly direction towards D. The hunter moves at 4metres per second and the deer moves at 5metres per second. After t seconds, the hunter is at a point H and the deer is at point D.
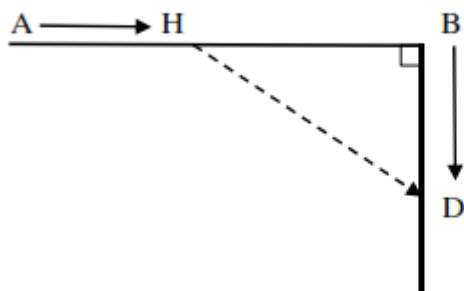
The hunter tries to shoot the deer but with his caliber rifle he must be at most 800m from the deer.

9.1   Show that the distance between the hunter and the deer (HD) at t seconds after they both started moving can be written as:

$$HD(t) = \sqrt{41t^2 - 9\,600t + 1\,440\,000}$$ (4)

9.2   How long after they started walking, were they the nearest to one another? Show all calculations.   (3)

9.3   The calibre of the hunter's rifle allows him to be at most 800m from his target. Was the hunter within shooting range of the deer at the time when they were nearest to each other? Show all calculations.   (3)

[10]

PROBABILITY

PREP 2019_LIMPOPO

QUESTION 11

11.1 A and B are mutually exclusive events. If it is given that the P(A) = 0,35

and P(B) = 0,52 determine:

11.1.1 P(A$'$) (2)

11.1.2 P(A and B) (1)

11.1.3 P(A or B) (2)

11.2 Three married couples – Mr and Mrs Brown, Mr and Mrs Green and

Mr and Mrs White are to be seated on a bench.

11.2.1 How many different arrangements are possible? (1)

11.2.2 If Mr and Mrs Green must be seated in the middle, how many different

arrangements are possible for the remaining persons? (2)

11.2.3 Determine the probability that Mr and Mrs Green will sit next to each other. (4)

[12]

PREP 2019_EASTERN CAPE

QUESTION 10

Two numbers are such that their sum is 18. One of the numbers is multiplied by the square of the other. Calculate the numbers that make this product a maximum. [7]

QUESTION 11

11.1 A school has 530 learners. Each learner is expected to choose his/her summer extra-curricular activity from the following:
- Athletics
- Cricket
- Tennis

The choices for 2019 were recorded in the following partially completed table:

	Athletics	Cricket	Tennis	Total
Girls	120	a	57	288
Boys	b	108	28	242
Total	226	219	85	530

11.1.1 Determine the values of a and b. (2)

11.1.2 A learner is chosen at random. Determine the probability that:

(a) It is a boy who plays cricket (2)

(b) It is a girl or **not** a tennis player (3)

11.2 Consider the letters of the word: NUMERATOR.

11.2.1 How many 9 letter word-arrangements can be formed, if repetition of letters is allowed? (1)

11.2.2 How many 9 letter word-arrangements can be formed, if all 4 vowels are never together and repetition of letters is not allowed? (3)

11.2.3 An 8 letter word-arrangement is made from the word NUMERATOR. All the vowels must be included in this word-arrangement and repetition of letters is not allowed. What is the probability that all odd-number spaces are occupied by vowels? (4)

[15]

PREP 2019_FREE STATE

QUESTION 11

11.1 Mandisa is visiting a restaurant. The probability that she will order tea is 0,4 and the probability that she will order cake is 0,5. The probability that she will order tea or cake is 0,8.

     11.1.1 Calculate the probability that:

         (a)   Mandisa will order neither tea nor cake   (1)

         (b)   She will order tea and cake   (2)

     11.1.2 Let T and C represents the events of Tea and Cake respectively. Are the events T and C mutually exclusive? Motivate your answer.   (2)

11.2 Seven learners, Bonolo, Jeffrey, Themba, Richard, Thandeka, Godfrey and Palesa are standing in a line.

     11.2.1 In how many ways can the learners be arranged in a line?   (2)

     11.2.2 In how many ways can the learners be arranged if Bonolo, Jeffrey and Themba must be next to one another in any order?   (3)

     11.2.3 What is the probability that Thandeka and Palesa will not be standing next to each other?   (4)

                                        [14]

PREP 2019_GAUTENG

QUESTION 12

12.1 For two events, A and B, it is given that:

- $P(A) = 0,3$
- $P(B) = 0,4$
- $P(A \text{ or } B) = 0,6$

Calculate $P(A \text{ and } B)$.　　　　　　　　　　　　　　　　　　(2)

12.2 A survey was completed among Grade 11 and Grade 12 learners at a certain school to establish the type of cell phone that each learner uses. Some of the results are shown in the table below.

	Gr. 11	Gr. 12	Total
Android	A	33	65
iPhone	53	B	101
Total	85	81	166

12.2.1　Calculate values for A and B in the table.　　　　　　　　(2)

12.2.2　If a learner from this group is selected at random, what is the probability that he/she will use an iPhone?　　　　　(2)

12.2.3　All these Grade 11 and Grade 12 learners attend Mathematics lessons in a particular class at the school. At the end of the day, the Mathematics educator found an iPhone in this class.
What is the probability that the phone belongs to a Grade 12 learner?　　　(2)

　　　　　　　　　　　　　　　　　　　　　　　　　　　　　　[8]

PREP 2019_NORTH WEST

QUESTION 11

11.1 The probability of getting the first answer in a quiz correct, is 60%. If the first answer is correct, the probability of getting the second answer correct, rises to 70%. However, if the first answer is wrong, the probability of getting the second answer correct, is only 40%. Determine the probability of getting the second answer correct. (4)

11.2 A and B are independent events.
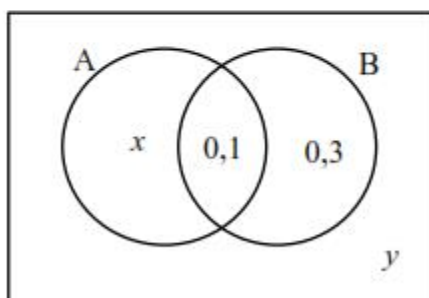
Determine the values of x and y. All calculations must be shown. (5)

11.3 Ping-Pong balls are numbered 1 to 12 and placed in a hat. Every time a ball is drawn, it is placed on a rack, one next to the other.

11.3.1 How many different arrangements of the 12 balls are possible? (2)

11.3.2 Assume that the numbers 8 and 11 must be placed next to each other, in any order. In how many different ways can the numbers then be arranged? (3)

11.3.3 The numbers 2, 5, 6 and 9 are taken from the group of balls. Two-digit numbers must be formed out of these four numbers. How many two-digit numbers can be formed? (2)

[16]

PREP 2019_KZN

QUESTION 12

A bag contains 12 blue balls, 10 red balls and 18 green balls. 2 balls are chosen at random without replacement.

Determine the probability:

12.1	if the two balls chosen at random are green.	(3)
12.2	if the two balls chosen at random are blue and red.	(3)
		[6]

QUESTION 13

The digits 1, 2, 3, 4, 5, 6, 7, 8, 9 are used to form 3 - digit codes, eg. 567, 218, etc.

Determine the number of different codes that can be formed:

13.1	if repetition is allowed.	(2)
13.2	such that the code is greater than 500 and repetition is NOT allowed.	(2)
13.3	such that the middle digit is 5 and repetition is allowed.	(2)
		[6]

PREP 2019_MPUMALANGA

QUESTION 9

9.1   In St Peter's School there are 150 learners in grade 12. The Venn-diagram shows the learners that take Geography (G), Mathematics (M) and History (H). The total number of learners taking History is 49 and those taking Mathematics is 74.
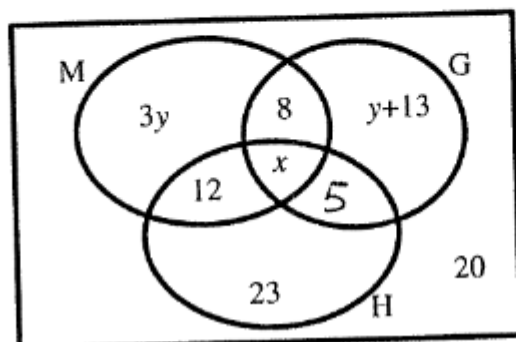
9.1.1 Determine the value of x and y. (4)

9.1.2 Calculate the probability that if a grade 12 learner is randomly selected, the learner takes Mathematics and History, but not Geography. (2)

9.1.3 Are H and G independent events? Give a reason for your answer. (4)

9.2 Lucky, Simon and 5 friends sits in a row of seven seats at the theatre.

9.2.1 In how many different ways can the seven people sit if any person occupies any of the seven seats? (2)

9.2.2 In how many different ways can the seven people sit if Lucky and Simon do not sit next to each other? (3)

[15]

PREP 2019_WESTERN CAPE

QUESTION 11

11.1 A gift basket is made up with one book, one slab of chocolate, one packet of nuts and one bottle of fruit juice.

The person who makes up the gift basket can choose from:
- five different books,
- three different slabs of chocolates,
- four kinds of nuts and
- six flavours of fruit juice.

Calculate the number of different gift baskets that can possibly be produced. (2)

11.2 In a class, the probability of the students studying Mathematics is 60% while the probability of the students studying Economics is 30%. The probability of the students studying Mathematics or Economics is 70%.
Calculate the probability that a student, will study both Mathematics and Economics. (2)

11.3 A photograph needs to be taken of the Representative Council of Learners (RCL) at a school. There are three girls and two boys in the RCL and all of them need to sit in one row for the photograph.
The President of the RCL is a boy and the vice-president is a girl.

11.3.1 In how many ways can the RCL be arranged in a row, if there were no restriction on the order in which the RCL members could sit? (2)

11.3.2 If the members of the RCL are seated randomly, calculate the probability that the President and the Vice-President of the RCL are seated next to each other. (4)

[10]

PREP 2019_NORTH WEST 2

QUESTION 10

The rules for the final game of the North West Hockey tournament specify that there must be a winner. In the event of a draw, the winner will be determined by a penalty-flick shootout.

On the day that the final game of the North West Hockey tournament takes place, there is a 45% chance that it could rain, a 32% chance that it could be cloudy or it could be sunny. The team from Taung, a low rainfall area, has an 18% chance of winning the tournament on a rainy day, a 39% chance of winning on a cloudy day and a 63% chance of winning on a sunny day.

10.1 Draw a tree diagram to represent all outcomes of the above information. (2)

10.2 What is the probability of the Taung hockey team winning the final game of the tournament? (4)

[6]

A horse breeder has 9 single horse stables in a row next to each other. He has 4 stallions (male horses) and 5 mares (female horses), where one of the stallions is his breeding stallion and one of the mares his breeding mare. The horses are placed randomly in the stables.

11.1 In how many different ways can the 9 horses be placed in the 9 stables? (1)

11.2 In how many different ways can the 9 horses be placed if the breeder wants to place the breeding stallion and breeding mare next to each another? (2)

11.3 What is the probability that there will be a mare placed on both ends of the row stables? (3)

11.4 If 5 stables became unavailable due to renovations, in how many different ways can the breeder place his horses in the remaining single stables such that there will be at least one mare in these stables? (3)

[9]