



# TECHNICAL MATHEMATICS

## GRADE 12 REVISION BOOKLET 1 OF 2020 (PAPER 1 QUESTIONS MEMORANDA)

### A COLLECTION OF 2017 – 2019 NSC EXAM QUESTIONS AND MEMORANDA GROUPED ACCORDING TO TOPICS



- # MASTER THE BASICS FIRST!
- HARDWORK NEVER KILLS!
- PRACTICE MAKES PERFECT!

YES  
YOU  
CAN!



**PLEASE READ:**

<p><b>Dear Grade 12 Technical Mathematics learner</b></p> <p><b>INTRODUCTION</b> Your final school exam result in Technical Mathematics is extremely important. Good results in Technical Mathematics will surely open doors for you that will influence the quality of your future life. So, practice Technical Mathematics regularly, not only before tests and exams. Always tell yourself that “I Can Do TMAT!”.</p> <p><b>REQUIRED RESOURCES</b></p> <ul style="list-style-type: none"> <li>● A Technical Mathematics textbook</li> <li>● Workbooks/ Revision Material</li> <li>● Past Examination Question Papers</li> <li>● A scientific calculator, etc.</li> </ul> <p><b>CONTENT CHECKLIST</b></p> <p>Below is a checklist you should use to ensure that you have covered the content for Grade 12 Technical Mathematics in full:</p> <p><b>Paper 1</b></p> <p><b>Equations and inequalities</b></p> <ul style="list-style-type: none"> <li>● Quadratic equations and inequalities</li> <li>● Simultaneous equations</li> <li>● Exponents and Surds</li> <li>● Nature of Roots</li> <li>● Logarithms</li> <li>● Binary numbers</li> <li>● Complex Numbers</li> </ul> <p><b>Functions and graphs</b></p> <ul style="list-style-type: none"> <li>● Linear, parabola; hyperbola;</li> <li>● exponential and circle/ semi-circle</li> <li>● finding equation when critical points are given</li> <li>● horizontal and vertical translation and reflection about the <math>x</math> and <math>y</math> axis.</li> <li>● interpret functions and graphs</li> </ul> <p><b>Finance, Growth and Decay</b></p> <ul style="list-style-type: none"> <li>● Simple and compound interest</li> <li>● Different periods of compounding</li> <li>● Analysis of different Loan options</li> <li>● Nominal and effective interest rates</li> <li>● Depreciation (reducing balance and straight line)</li> </ul> <p><b>Differential Calculus and Integration</b></p> <ul style="list-style-type: none"> <li>● Limits and average gradient</li> <li>● First principles and differentiation rules</li> <li>● Gradient at a point and tangents to curves</li> <li>● Polynomials (Remainder and Factor theorems)</li> <li>● Cubic functions</li> <li>● Applications (maxima and minima; rate of change including Calculus of motion)</li> <li>● Definite and indefinite integrals</li> </ul>	<p><b>Paper 2</b></p> <p><b>Analytical Geometry</b></p> <ul style="list-style-type: none"> <li>● Distance, midpoint, gradient, parallel and perpendicular lines</li> <li>● Equation of a line, Co-linear points</li> <li>● Angle of inclination</li> <li>● Equation of a circle (centre at the origin <b>only!</b>)</li> <li>● Equation of a tangent to a circle</li> <li>● properties of geometric figures (triangles, quadrilaterals, etc.)</li> <li>● Ellipse</li> </ul> <p><b>Trigonometry</b></p> <ul style="list-style-type: none"> <li>● Trigonometric definitions, Special angles</li> <li>● Reduction formulae</li> <li>● Identities and equations (simple Trig. equations)</li> <li>● Solution of triangles and problems in 2D and 3D (sine-, cosine-, area rules)</li> <li>● Trigonometric Functions and transformations</li> </ul> <p><b>Euclidean geometry</b> Solving riders using properties of parallel lines, triangles and quadrilaterals Circle geometry – apply theorems with converses in solving problems. Ratio and proportion - apply the concept of:</p> <ul style="list-style-type: none"> <li>● Midpoint theorem</li> <li>● Proportionality</li> <li>● Similarity</li> </ul> <p><b>Mensuration</b></p> <ul style="list-style-type: none"> <li>● Convert units, square and cubic units</li> <li>● Volumes and Surface Areas of solids</li> <li>● Area of irregular figures using the</li> <li>● Mid-ordinate rule</li> </ul> <p><b>Circles, Angles and Angular Movement</b></p> <ul style="list-style-type: none"> <li>● Define Radian</li> <li>● Convert between degrees and radians</li> <li>● Central angles and arcs</li> <li>● Area of Sector</li> <li>● Height of segment</li> <li>● Angular velocity</li> <li>● Circumferential velocity</li> </ul> <p><b>TIPS FOR SUCCESS:</b></p> <p>Ensure that you are fully acquainted with your calculator. It will save you time in the examination. For instance, in converting radians to degrees, finance, trigonometry, etc.</p> <p><i>Attend school every day and any extra tuition offered to you</i> <i>Do Class and Homework every day.</i> <i>Practice, Practice, Practice!</i> <i>‘YES YOU CAN!’</i></p>
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**ENJOY TECHNICAL MATHEMATICS... BECAUSE YOU CAN!**

## ALGEBRA

### 2018 EXEMPLAR

#### QUESTION 1

1.1 Given:  $f(x) = x(x + 2)$

Solve for  $x$  if:

1.1.1  $f(x) = 0$  (2)

1.1.2  $f(x) \geq 0$  and then represent the solution on a number line. (4)

1.2 Solve for  $x$  if  $5x^2 = 2 + x$  (rounded off to TWO decimal places) (4)

1.3 Solve algebraically for  $m$  and  $t$  simultaneously if:  
 $m - t - 1 = 0$  and  $m^2 + t^2 = 5$  (6)

1.4 The two diagrams below represent a metal rod with an initial length  $L_1$  which is then stretched (elongated) to a length  $L_2$ .



The strain measure ( $\varepsilon$ ) is defined as the ratio of elongation with respect to the original length and is given by the formula:

$$\varepsilon = \frac{L_2 - L_1}{L_1}$$

1.4.1 Express  $L_1$  as the subject of the formula. (3)

1.4.2 Hence, or otherwise, calculate the value of  $L_1$  if  $\varepsilon = 0,8$  and  $L_2 = 18 \text{ cm}$  (2)

1.4.3 Convert the value obtained in QUESTION 1.4.2 to a binary number. (2)

1.5 Write the simplified value of  $12 \times 0,00361$  in scientific notation without any rounding. (2)  
**[25]**

**QUESTION 2**

2.1 Given:  $A = \frac{\sqrt{9-3p}}{p+1}$

Determine the value(s) of  $p$  for which  $A$  will be:

2.1.1 Undefined (1)

2.1.2 Non-real (2)

2.1.3 Rational (give only ONE integer) (1)

2.2 Determine the value of  $k$  for which the equation  $x^2 - 4x + (k-1) = 0$  will have equal roots. (4)  
[8]

**QUESTION 3**

3.1 Simplify the following without the use of a calculator (show ALL steps):

3.1.1  $\frac{5 \times 2^{n-1} - 2^n}{2^n}$  (3)

3.1.2  $\sqrt{64+16} - \sqrt{20}$  (4)

3.1.3  $\log_6 216 \times \log 0,001$  (4)

3.2 Solve for  $x$ :  $\log(x-18) - \log x = 1$  (4)

3.3 Express the complex number  $z = 3 + \sqrt{3}i$  in trigonometric (polar) form. (5)

3.4 Solve for  $x$  and  $y$  if  $x + yi = (3 + 5i)(2 - 7i)$  (5)  
[24]

NOV 2018

QUESTION 1

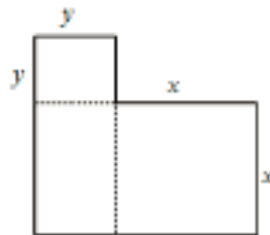
1.1 Solve for  $x$ :

1.1.1  $-2x(x+a)(3-x) = 0$  (3)

1.1.2  $2x = 6 - x^2$  (correct to TWO decimal places) (4)

1.1.3  $5x(x-3) \leq 0$  and then represent the solution on a number line (3)

1.2 The total area represented by the L-shaped diagram below is 21 units<sup>2</sup>. The equation  $y - 2x = -7$  represents the relationship of the sides of the two squares.



Solve for  $x$  and  $y$  (dimensions of the two squares) if:

$y - 2x = -7$  and  $x^2 + xy + y^2 = 21$  (7)

1.3 The formula below represents the moment of inertia ( $E$ ), with mass ( $M$ ) and length ( $L$ ):

$$E = \frac{1}{12}ML^2$$

1.3.1 Make  $L$  the subject of the formula. (2)

1.3.2 Calculate the value of  $L$ , if  $E = 8,3 \times 10^{-2} \text{ kg.m}^2$  and  $M = 1,6 \times 10^3 \text{ kg}$ . (2)

1.4 Express 36 as a binary number. (2)

[23]

**QUESTION 2**

2.1 Given the roots:  $x = \frac{-8 \pm \sqrt{q-3}}{2}$

Describe the nature of the roots if:

2.1.1  $q = 5$  (1)

2.1.2  $q = 3$  (1)

2.1.3  $q < 0$  (1)

2.2 Determine for which value(s) of  $p$  will the equation  $3x^2 + 7x = 2x + p$  have non-real roots. (4)  
[7]

**QUESTION 3**

3.1 Simplify (showing ALL calculations) the following without the use of a calculator:

3.1.1  $\left(2a^{\frac{2}{3}}\right)^3$  (2)

3.1.2  $\log_p p + \log_m 1$  (2)

3.1.3  $\frac{\sqrt{48} - \sqrt{12}}{2\sqrt{75}}$  (3)

3.2 Solve for  $x$ :  $\log_2(x+62) - \log_2 x = 5$  (4)

3.3 Express the complex number  $z = -\sqrt{2} + \sqrt{2}i$  in the polar form  $z = a \operatorname{cis} \theta$  (6)

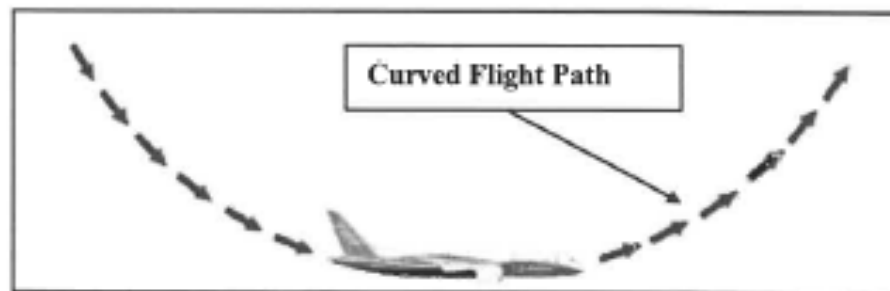
3.4 Solve for  $p$  and  $q$  if  $p + qi = (2 - 3i)^2$ . (4)  
[21]

NOV 2019

QUESTION 1

- 1.1 The picture below shows the curved flight path of an aircraft. The flight path, as indicated by the arrows, is parabolic in shape and is defined by the equation:

$$p(x) = 2x^2 - \frac{8}{81}$$



- 1.1.1 Factorise  $p(x)$  completely. (2)
- 1.1.2 Hence, solve for  $x$  if  $p(x) = 0$  (1)
- 1.2 Solve for  $x$  in EACH of the following:
- 1.2.1  $(3x-5)(x+2) = -13$  where  $x \in \{\text{Complex numbers}\}$  (5)
- 1.2.2  $(4-x)(x+3) < 0$  (3)
- 1.3 Solve for  $x$  and  $y$  if:
- $$y = 3x - 8 \text{ and } x^2 - xy + y^2 = 39$$
- (6)
- 1.4 The following formula represents the relationship between the voltage, the current and the impedance in an alternating current circuit:  $V = I \times Z$
- Where:
- $V$  = Voltage (in volts)  
 $I$  = Current (in amperes)  
 $Z$  = Impedance (in ohms)
- 1.4.1 Express  $I$  as the subject of the formula. (1)
- 1.4.2 Hence, determine in simplified form the value of  $I$  (in amperes) if:
- $$V = 7i \text{ and } Z = 3 - i$$
- (5)
- 1.5 Simplify:  $101_2 \times 11_2$  (2)

[25]

**QUESTION 2**

2.1 Given:  $G = \sqrt{\frac{p+1}{2p-1}}$

Determine the value(s) of  $p$  such that  $G$  will be as follows:

2.1.1 Undefined (1)

2.1.2 Equal to zero (1)

2.2 Determine for which value(s) of  $k$  the equation  $x^2 - k + 4 = 5x$  will have real roots. (5)  
[7]

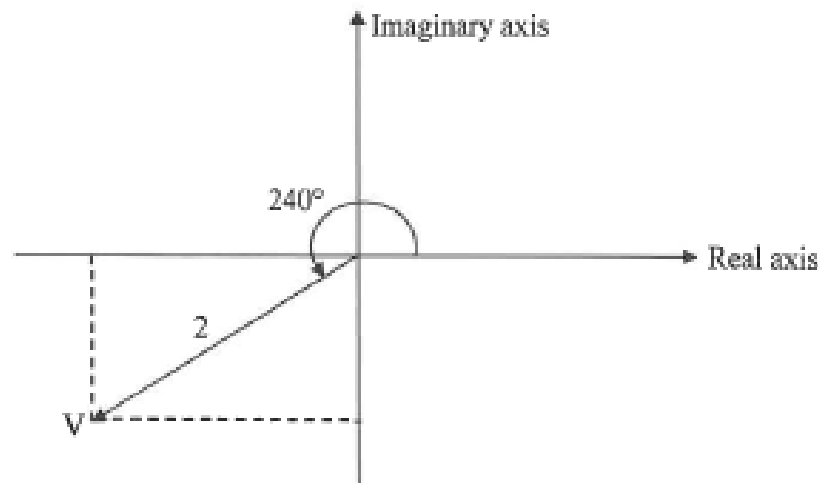
**QUESTION 3**

3.1 Simplify:  $\left(-2\sqrt[4]{a^3}\right)^8$  (2)

3.2 Solve for  $x$ :  $\log_2(3x-2) + \log_2 0,5 = 3$  (4)

3.3 If  $\log 2 = a$  and  $\log 3 = b$ , determine the value of  $\log \sqrt{0,6}$  in terms of  $a$  and  $b$ . (5)

3.4 The voltage ( $V$ ) in an alternating current circuit is represented by the Argand diagram below.



3.4.1 Use the Argand diagram above to write down the voltage in the form  $V = r(\cos \theta + i \sin \theta)$  (1)

3.4.2 Hence, or otherwise, express  $V$  in rectangular form. Leave your answer in simplified surd form. (3)

3.5 Determine the numerical values of  $m$  and  $n$  if  $m + ni = 2(6 - 4i) - (-7i)$  (2)  
[17]

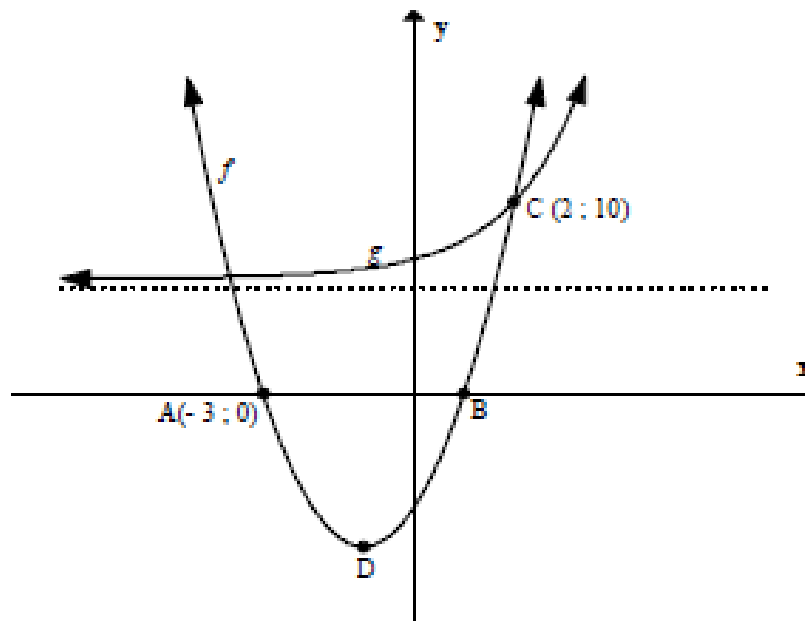


## FUNCTIONS AND GRAPHS

### EXEMPLAR 2018

#### QUESTION 4

- 4.1 The graphs of the functions defined by  $f(x) = 2x^2 + 4x - 6$  and  $g(x) = k^x + 6$  are shown in the figure below.  $C(2;10)$  is a point of intersection of  $f$  and  $g$ . Points  $A(-3;0)$  and  $B$  are the  $x$ -intercepts and  $D$  is the turning point of  $f$ .



Determine:

- |       |  |     |
|-------|--|-----|
| 4.1.1 | The coordinates of $B$                             | (2) |
| 4.1.2 | The coordinates of turning point $D$               | (3) |
| 4.1.3 | The numerical value of $k$                         | (3) |
| 4.1.4 | The equation of the asymptote of $g$               | (1) |
| 4.1.5 | The values of $x$ for which $f(x) \times g(x) < 0$ | (2) |
- 4.2 Given:  $g(x) = \sqrt{4-x^2}$  and  $h(x) = \frac{3}{x} + 1$
- |       |   |     |
|-------|---|-----|
| 4.2.1 | Write down the equations of the asymptotes of $h$ .   | (2) |
| 4.2.2 | Determine the $x$ -intercept of $h$ .   | (2) |
| 4.2.3 | Write down the length of the radius of $g$ .  | (1) |
| 4.2.4 | On the ANSWER SHEET provided, draw neat sketch graphs of $g$ and $h$ on the same set of axes. Clearly show ALL the asymptotes and the intercepts with the axes. | (7) |
| 4.2.5 | Determine the range of $g$ .  | (2) |

[25]

NOV 2018

QUESTION 4

4.1 Given:  $g(x) = 2^{-x} - 1$  and  $h(x) = -\frac{6}{x} - 1$

4.1.1 Write down the equations of the asymptotes of  $h$ . (2)

4.1.2 Determine the coordinates of the  $x$ -intercept of  $h$ . (2)

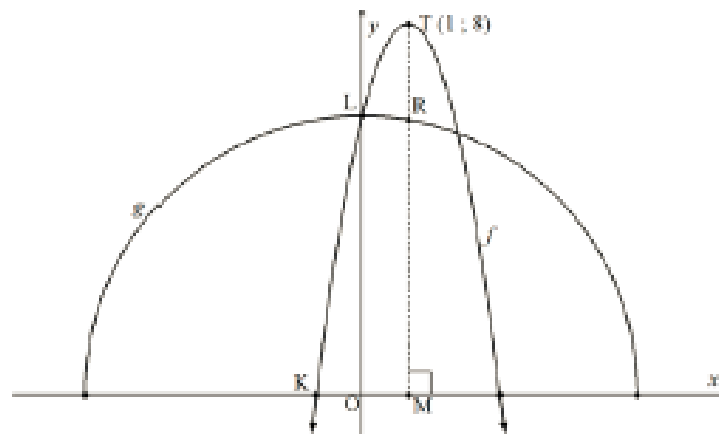
4.1.3 Sketch the graphs of  $g$  and  $h$  on the same set of axes on the ANSWER SHEET provided. Clearly show the asymptotes and the intercepts with the axes. (5)

4.1.4 Show that  $(-2; 3)$  is a point on the graph of  $g$ . (1)

4.1.5 Write down the range of  $g$ . (1)

4.1.6 Write down the domain of  $h$ . (1)

4.2 Sketched below are the graphs defined by  $f(x) = a(x+p)^2 + q$  and  $g(x) = \sqrt{36 - x^2}$  with  $T(1; 8)$  the turning point of  $f$ . Line  $TM$  is drawn such that  $TM$  is perpendicular to the  $x$ -axis. Points  $L$  and  $K$  are the intercepts of  $f$ . Point  $L$  is a point of intersection of  $f$  and  $g$ . Point  $R$  lies on both line  $TM$  and the graph of  $g$ .



4.2.1 Write down the coordinates of  $M$ . (1)

4.2.2 Determine the length of  $TR$ . (leave your answer in surd form). (3)

4.2.3 Show that  $(0; 6)$  are the coordinates of  $L$ . (1)

4.2.4 Hence, show that the graph of  $f$  is defined by  $f(x) = -2(x+1)(x-3)$ . (4)

4.2.5 Hence, give the coordinates of  $K$ . (1)

4.2.6 Determine the values of  $x$  for which  $f(x) \times g(x) > 0$  and  $x < 0$  (2)

[24]

**NOV 2019**

**QUESTION 4**

4.1 Given functions  $k$  and  $q$  defined by  $k(x) = (x - 5)(x + 3)$  and  $q(x) = \frac{12}{x} - 2$  respectively.

4.1.1 Write down the  $x$ -intercepts of  $k$ . (1)

4.1.2 Determine the  $x$ -intercept of  $q$ . (2)

4.1.3 Determine the coordinates of the turning point of  $k$ . (3)

4.1.4 Write down the equations of the asymptotes of  $q$ . (2)

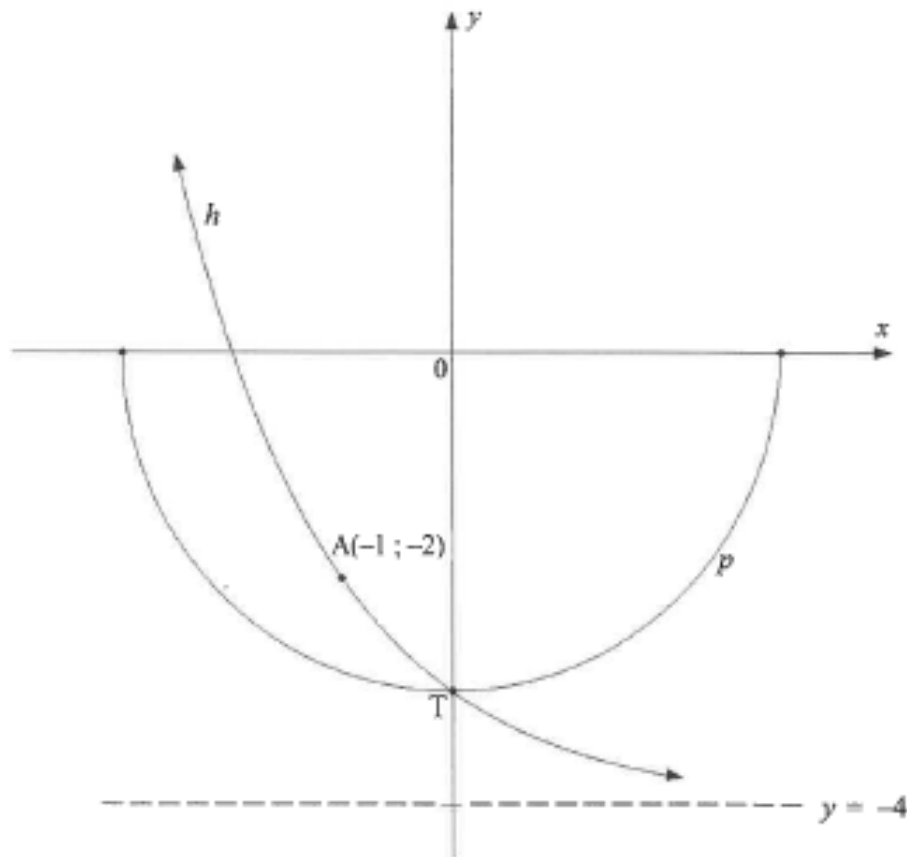
4.1.5 Sketch the graphs of  $k$  and  $q$  on the same set of axes provided on the ANSWER SHEET. Clearly show the asymptotes, the intercepts with the axes, as well as the coordinates of any turning points. (7)

- 4.2 Sketched below are the graphs of  $p$  and  $h$  defined by  $p(x) = -\sqrt{r^2 - x^2}$  and  $h(x) = a^x + d$  respectively.

T is the point of intersection of  $p$  and  $h$ .

A(-1 ; -2) is a point on  $h$ .

The asymptote of  $h$  is indicated by the dotted line.



- 4.2.1 Write down the numerical value of  $d$ . (1)
- 4.2.2 Show that  $h(x) = \left(\frac{1}{2}\right)^x - 4$  (2)
- 4.2.3 Hence, determine the coordinates of T. (2)
- 4.2.4 Write down the range of  $p$ . (1)
- 4.2.5 Hence, determine the defining equation  $w(x)$  of function  $w$ , such that  $w$  is the reflection of  $p$  in the  $x$ -axis. (2)
- 4.2.6 Determine for which values of  $x$  will  $h(x) < p(x)$  (2)

[25]

## FINANCE, GROWTH AND DECAY

### EXEMPLAR 2018

#### QUESTION 5

- 5.1 The nominal interest rate charged on an investment is 7,2 % compounded half yearly. Calculate the annual effective interest rate for the investment. (3)
- 5.2 The air pressure of a punctured tyre deflated from 220 kPa to 70 kPa at a decreasing rate of 8% per minute. Determine (to the nearest minute) how long it took the tyre to deflate from 220 kPa to 70 kPa. (5)
- 5.3 Mrs Rethabile invested an amount of R150 000 to buy a drilling machine for her engineering company. Interest, compounded quarterly, is calculated at a rate of 10,5% p.a. for 5 years. At the end of the third year, Mrs Rethabile withdrew an amount of R30 000 from the investment account and then continued investing the balance for the remaining period.
- Determine the value of the investment at the end of the investment period. (6)  
[14]

### NOV 2018

#### QUESTION 5

- 5.1 The annual effective interest rate charged by a financial institution is 6,7%. Calculate the nominal interest rate charged per annum if compounded monthly. (4)
- 5.2 A company bought a new 3D wheel-alignment machine for R240 000. The machine depreciated at a rate of 16% per annum to half its original value over a certain period.

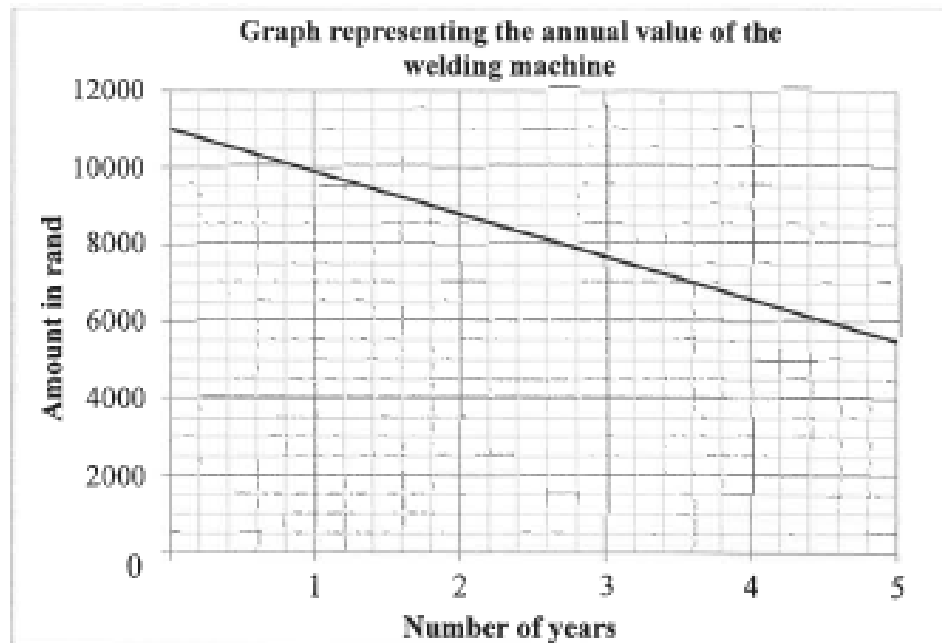


- 5.2.1 Give the depreciated value of the machine at the end of the period. (1)
- 5.2.2 Determine how long it will take for the machine to depreciate to half its original value. Give your answer to the nearest year. (5)
- 5.3 Mr Bohlale invested R40 000 at a bank for 7 years. The interest rate for the first 4 years was 11,2% per annum, compounded quarterly. The interest rate then changed to 13% per annum compounded annually for the remaining years. Calculate the total amount of money that Mr Bohlale will receive at the end of the investment period. (5)  
[15]

NOV 2019

## QUESTION 5

- 5.1 A small engineering business purchased a new welding machine. The value of the welding machine depreciated annually over a period of 5 years, as shown in the graph below.



Use the graph above to answer the following questions:

- 5.1.1 Write down the value of the welding machine when it was new. (1)
- 5.1.2 Calculate the annual constant percentage rate of depreciation. (3)
- 5.2 A mechanic of Model X cars found a data sheet showing that 200 Model X cars had been serviced by the workshop during 2009. The annual compound growth rate of the number of Model X cars serviced by this workshop is 3,5% per annum.
- Determine, showing ALL calculations, the year during which 273 Model X cars were serviced by this workshop. (5)
- 5.3 Anita planned to purchase a truck for her company in 8 years' time and decided to open an investment account to provide for the purchase of the truck. She deposited an initial amount of R293 000 into the account.
- At the end of 2 years, Anita made a further deposit of R95 000 into the account. The interest rate for the first 4 years was 6,7% per annum, compounded quarterly, and for the remaining period the interest rate was 7,5% per annum, compounded monthly. The projected value of the truck at the end of 8 years will be R660 580.
- Determine, showing ALL calculations, whether her investment would accumulate enough funds for her to purchase the truck at the end of the 8-year investment period. (8)

[17]

## DIFFERENTIAL CALCULUS

### EXEMPLAR 2018

#### QUESTION 6

- 6.1 Determine the average gradient of  $f(x) = 2x^2 - 3$  between the points where  $x = -2$  and  $x = 1$ . (4)
- 6.2 Determine  $f'(x)$  from FIRST PRINCIPLES if  $f(x) = 4 - 3x$ . (5)
- 6.3 Determine  $\frac{dy}{dx}$  if  $y = \frac{2}{x^3} + \sqrt{x}$  (4)
- 6.4 Determine the equation of the tangent to the curve defined by  $g(x) = -x^2 - x$  at the point where  $x = 2$ . (5)  
[18]

#### QUESTION 7

Given:  $f(x) = x^3 + 2x^2 - 7x + 4$

- 7.1 Show that  $(x-1)$  is a factor of  $f(x)$ . (2)
- 7.2 Hence, or otherwise, find the  $x$ -intercepts of  $f$ . (3)
- 7.3 Determine the coordinates of the turning points of  $f$ . (5)
- 7.4 Sketch the graph of  $f$  on the ANSWER SHEET provided. Clearly show ALL the intercepts with the axes and the turning points. (4)  
[14]

### QUESTION 8

- 8.1 An industrial open water tank, as shown in the picture below, has an inlet pipe and an outlet pipe. The depth of the water in the tank changes continually.



The equation  $D(t) = 4 + 0,5t^2 - 0,25t^3$  gives the depth (in metres) of the water, where  $t$  represents the time (in hours) that has lapsed since the depth reading was taken at 09:00.

Determine:

- 8.1.1 The depth of the water in the tank at 11:00 (2)
- 8.1.2 The rate of change of the depth of the water in the tank at 12:00 (3)
- 8.2 The profit (in R1000s) yielded by a company, using a machine that produces bottle caps, is dependent on the average speed at which the machine runs.
- The profit ( $P$ ) is calculated using the formula:
- $$P = -3v^2 + 30v,$$
- where  $v$  is the average speed (in kilometres per hour) and  $v > 0$ .
- 8.2.1 Calculate the average speed at which neither a profit, nor a loss is yielded. (3)
- 8.2.2 Determine at what average speed the machine should run so that the maximum profit is obtained. (3)
- 8.2.3 Hence, or otherwise, calculate the resulting maximum profit. (2)

[13]



NOV 2018

QUESTION 6

- 6.1 Determine  $f'(x)$  using FIRST PRINCIPLES if  $f(x) = 7x - 2$  (5)
- 6.2 Determine:
- 6.2.1  $\frac{d}{dx}(\pi^2)$  (1)
- 6.2.2  $D_x(x^4 - \sqrt[3]{x})$  (3)
- 6.2.3  $\frac{dy}{dx}$  if  $y = \frac{x^3 + 2}{x^2}$  (4)
- 6.3 The tangent to the curve of the function defined by  $p(x) = x^3 + 1$  passes through point  $A(2; k)$ .
- 6.3.1 Calculate the numerical value of  $k$ . (2)
- 6.3.2 Determine  $p'(x)$  (1)
- 6.3.3 Hence, determine the equation of the tangent to the curve of the function at point  $A$ . (3)
- [19]

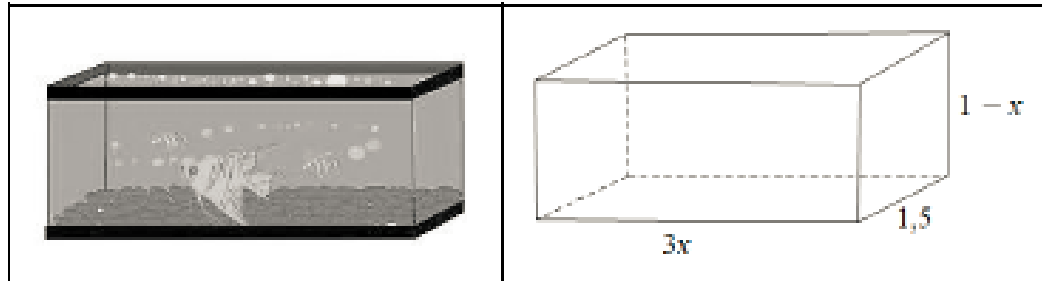
QUESTION 7

Given:  $f(x) = -x(x-3)(x-3)$

- 7.1 Write down the coordinates of the  $x$ -intercepts of  $f$ . (2)
- 7.2 Write down the  $y$ -intercept of  $f$ . (1)
- 7.3 Show that  $f(x) = -x^3 + 6x^2 - 9x$  (2)
- 7.4 Determine the coordinates of the turning points of  $f$ . (5)
- 7.5 Sketch the graph of  $f$  on the ANSWER SHEET provided. Clearly show ALL the intercepts with the axes and the turning points. (4)
- 7.6 Determine the values of  $x$  for which the graph of  $f$  is increasing. (2)
- [16]

**QUESTION 8**

- 8.1 Mr Alexander built a rectangular fish tank. The length, breadth and height of the tank are  $3x$  metres,  $1,5$  metres and  $(1-x)$  metres respectively, as shown in the diagram below.



- 8.1.1 Determine a formula for the volume of the tank in terms of  $x$ . (3)
- 8.1.2 Hence, determine the value of  $x$  that will maximise the volume of the tank. (3)
- 8.2 During an experiment, learners must record the velocity ( $v$ ) of an electronic toy car over a distance ( $m$ ),  $t$  seconds after the experiment has begun. The velocity of the electronic toy car is given by  $v(t) = 8 + 4t - t^2$
- Determine:
- 8.2.1 The initial velocity of the toy car (1)
- 8.2.2 The velocity of the toy car when  $t = 0,2$  seconds (2)
- 8.2.3 The rate at which the velocity changes with respect to time when  $t = 1,2$  seconds (4)
- [13]

**NOV 2019**

**QUESTION 6**

6.1 Determine  $f'(x)$  using FIRST PRINCIPLES if  $f(x) = 5 - \frac{1}{2}x$  (5)

6.2 Determine the following:

6.2.1  $f'(x)$  if  $f(x) = a^3 - 0,5x^3 - x^{-1}$  (3)

6.2.2  $D_x \left[ x \left( \sqrt{x} + 2 \right) \right]$  (4)

6.3 Given:  $xy + 2x^3 = 7x^6$

6.3.1 Make  $y$  the subject of the equation. (2)

6.3.2 Hence, determine  $\frac{dy}{dx}$ . (2)

6.4 A factory producing light bulbs makes a daily profit  $P(x)$  in rands for  $x$  number of light bulbs produced. The formula to calculate the factory's daily profit is given by  $P(x) = 0,8x^3 - 200x$ , where  $x > 0$ .

Calculate:

6.4.1 The daily profit if 300 light bulbs are produced in one day (1)

6.4.2 The number of light bulbs produced that will yield a zero daily profit (2)

6.4.3 The rate of change of the daily profit with respect to the number of light bulbs produced, if 200 light bulbs are produced (3)  
[22]

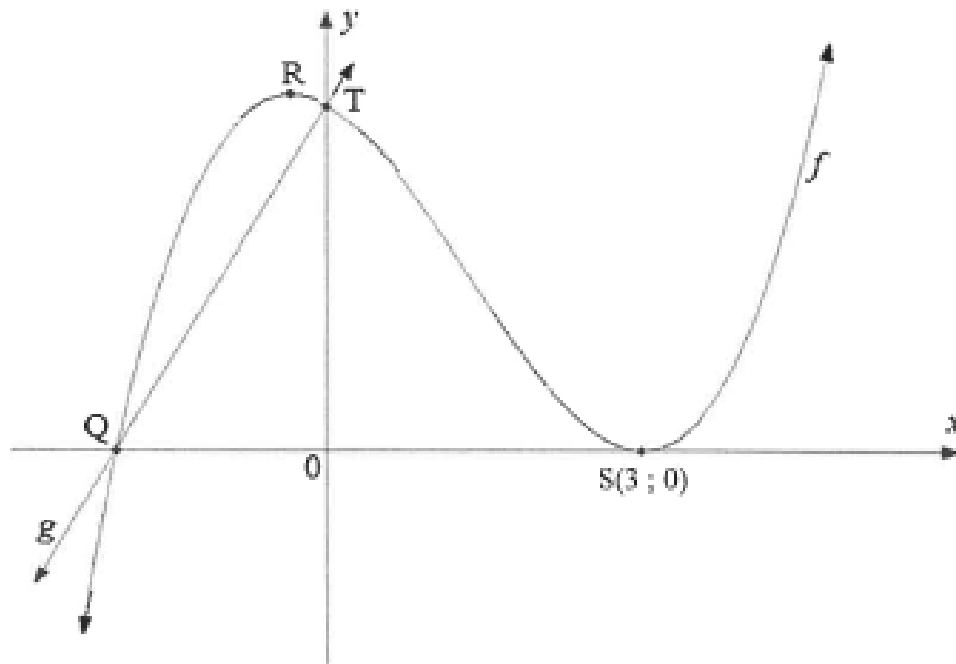
### QUESTION 7

The sketch below represents the graphs of functions  $g$  and  $f$  defined by  $g(x) = 9x + 18$  and  $f(x) = x^3 + bx^2 + cx + d$  respectively.

$S(3; 0)$  and  $R$  are the turning points of  $f$ .

$T$  is the  $y$ -intercept of both  $f$  and  $g$ .

$Q$  is the  $x$ -intercept of both  $f$  and  $g$ .

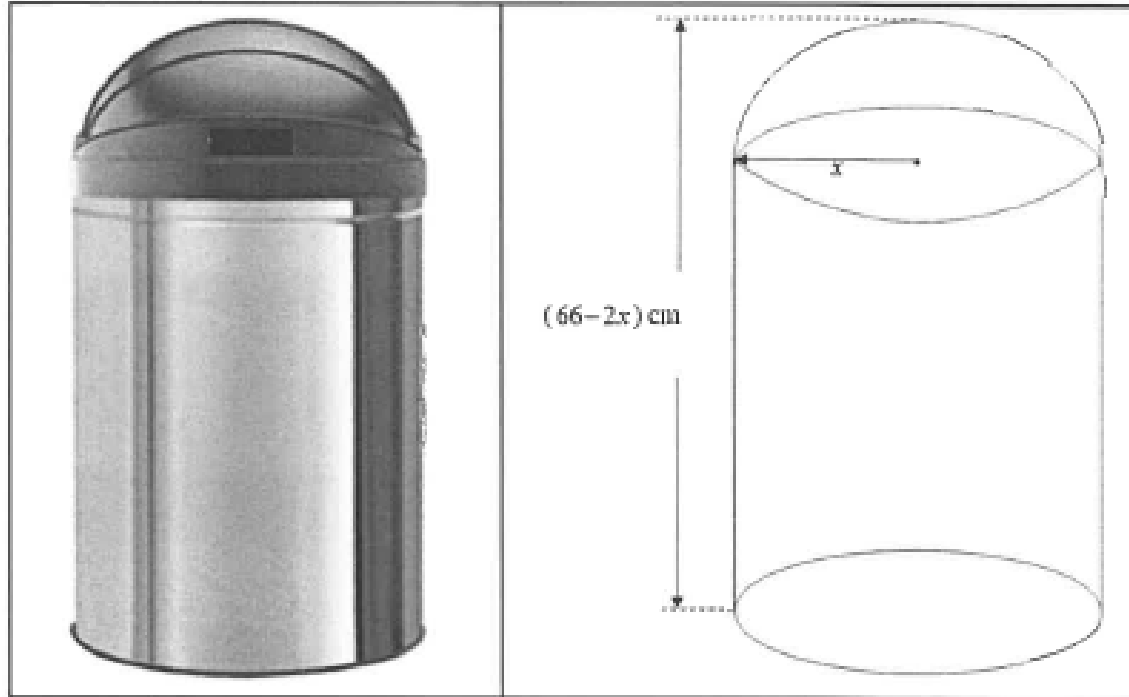


- 7.1 Determine the coordinates of  $Q$  and  $T$ . (3)
- 7.2 Show that  $b = -4$ ,  $c = -3$  and  $d = 18$ . (3)
- 7.3 Hence, determine the coordinates of  $R$ . (5)
- 7.4 Determine:
  - 7.4.1 The equation of the tangent to the curve of function  $f$  at point  $R$ . (1)
  - 7.4.2 The values of  $x$  for which  $g(x) > 0$ . (1)
  - 7.4.3 The values of  $x$  for which  $f'(x) < 0$ . (2)

[15]

### QUESTION 8

A container consists of a right cylindrical part and a hemispherical part at the top, as shown in the picture and diagram below. The radius of both shapes is  $x$  cm and the total height of the container is  $(66 - 2x)$  cm.



The following formulae may be used:

$$\text{Volume of a right cylinder} = \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

8.1 Write down, in terms of  $x$ , the height of the cylindrical part of the container. (1)

8.2 Show that the formula for the total volume (in  $\text{cm}^3$ ) of the container is given by:

$$V = 66\pi x^2 - \frac{7}{3}\pi x^3 \quad (3)$$

8.3 Hence, calculate the value of  $x$  that will maximise the total volume of the container. (4)

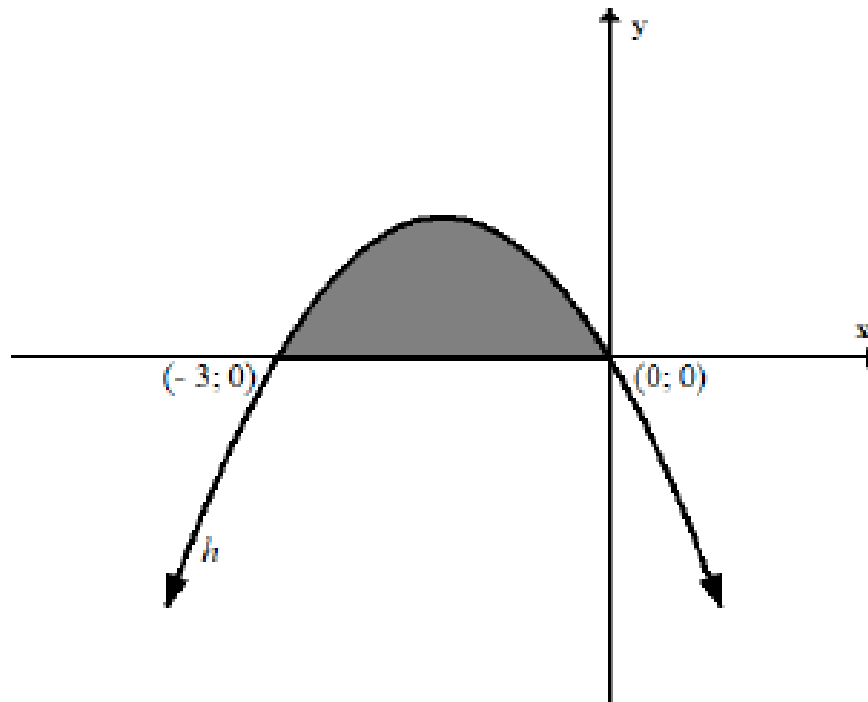
8.4 Hence, determine the maximum total volume of the container. (2)

[10]

**INTEGRATION**  
**EXEMPLAR 2018**  
**QUESTION 9**

9.1 Determine the following integral:  $\int \left( x^{-4} + \frac{7}{x} - 1 \right) dx$  (4)

9.2 The sketch below represents the graph of the function defined by  $h(x) = -2x^2 - 6x$ .



Determine the shaded area bounded by a curve defined by  $h(x) = -2x^2 - 6x$  and the  $x$ -axis. (5)  
 [9]

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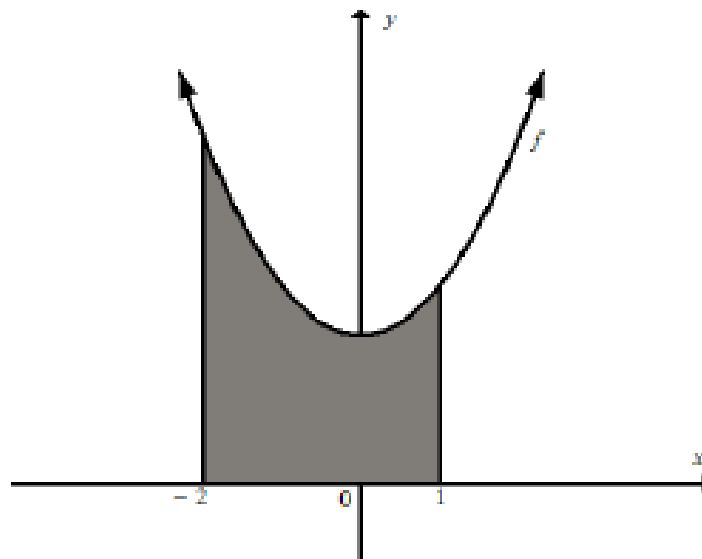
QUESTION 9

9.1 Determine the following integrals:

$$9.1.1 \quad \int \left( -\frac{6}{x} \right) dx \quad (2)$$

$$9.1.2 \quad \int (x-1)^2 dx \quad (4)$$

9.2 The sketch below represents the bounded area of the curve of the function defined by  $f(x) = x^2 + 3$



Determine the shaded area bounded by the curve and the  $x$ -axis between the points where  $x = -2$  and  $x = 1$

(6)  
[12]

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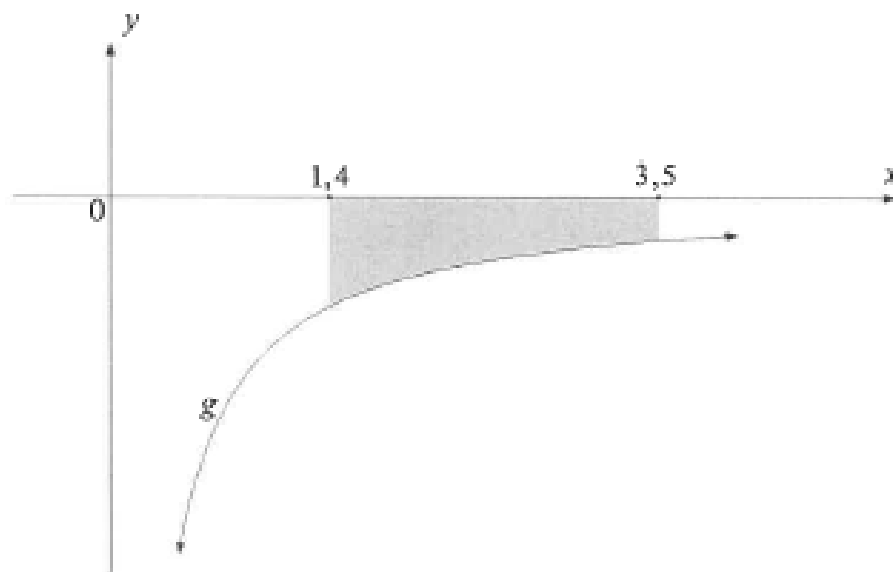
**QUESTION 9**

9.1 Determine the following integrals:

9.1.1  $\int mx^p dx$  where  $p \neq -1$  (2)

9.1.2  $\int \left( \frac{x^{-3} + x^2}{x^{-1}} - 2 \right) dx$  (4)

9.2 The sketch below shows the shaded bounded area of the curve of the function defined by  $g(x) = -\frac{4}{x}$ , where  $x > 0$ .



Determine (showing ALL calculations) the shaded area bounded by the curve and the  $x$ -axis between the points where  $x = 1,4$  and  $x = 3,5$ .

(6)  
[12]



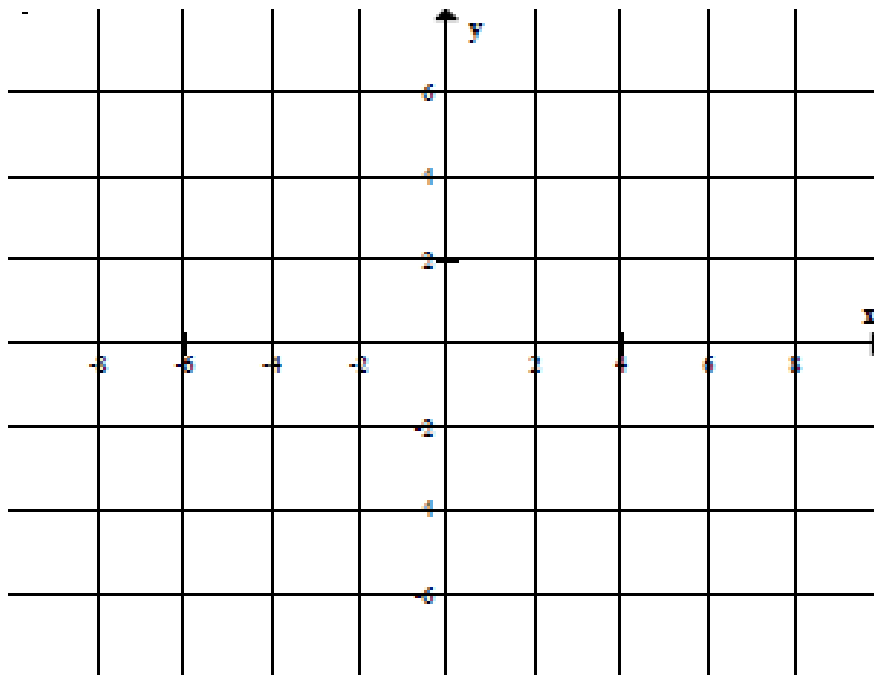
ANSWER SHEETS

EXEMPLAR 2018

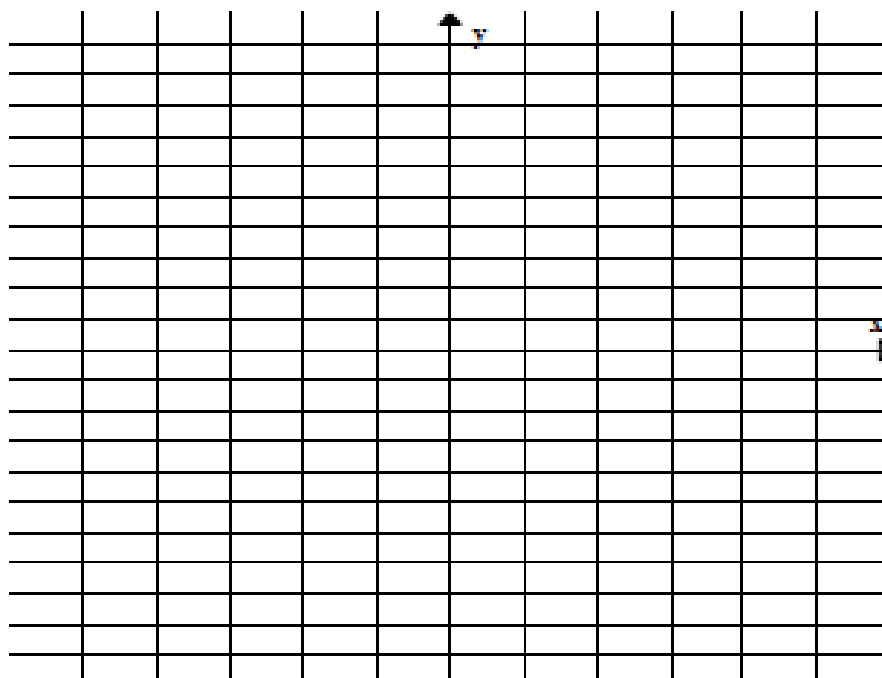
ANSWER SHEET

NAME: ..... SCHOOL: .....

QUESTION 4.2.4



QUESTION 7.4

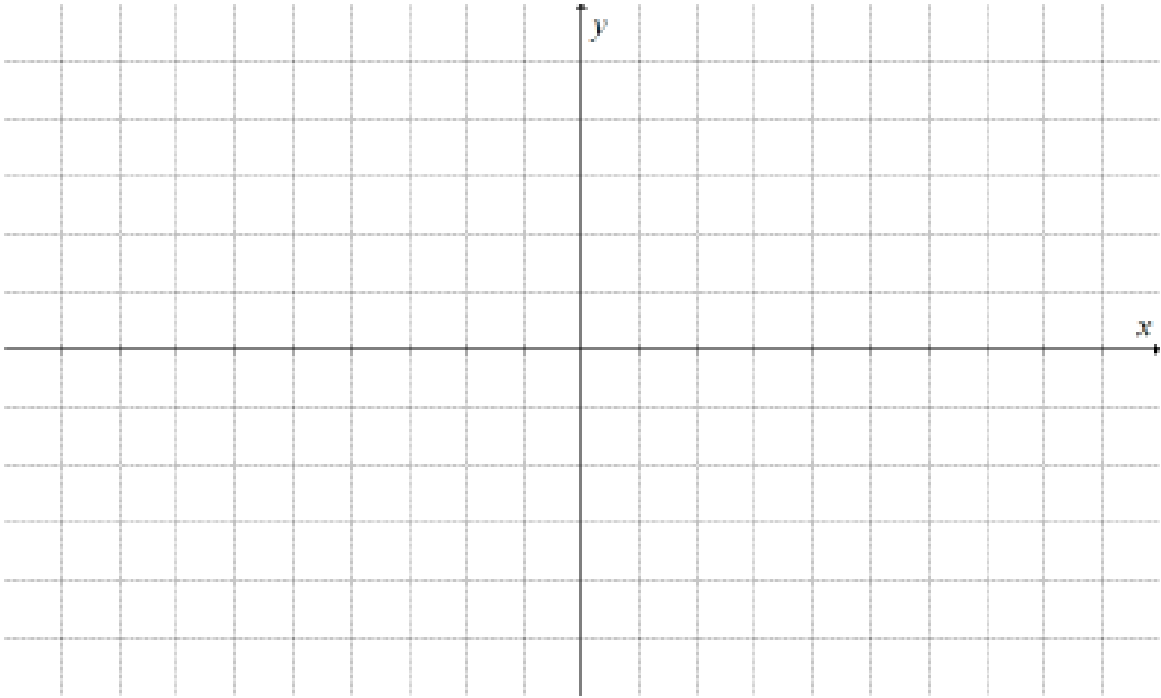


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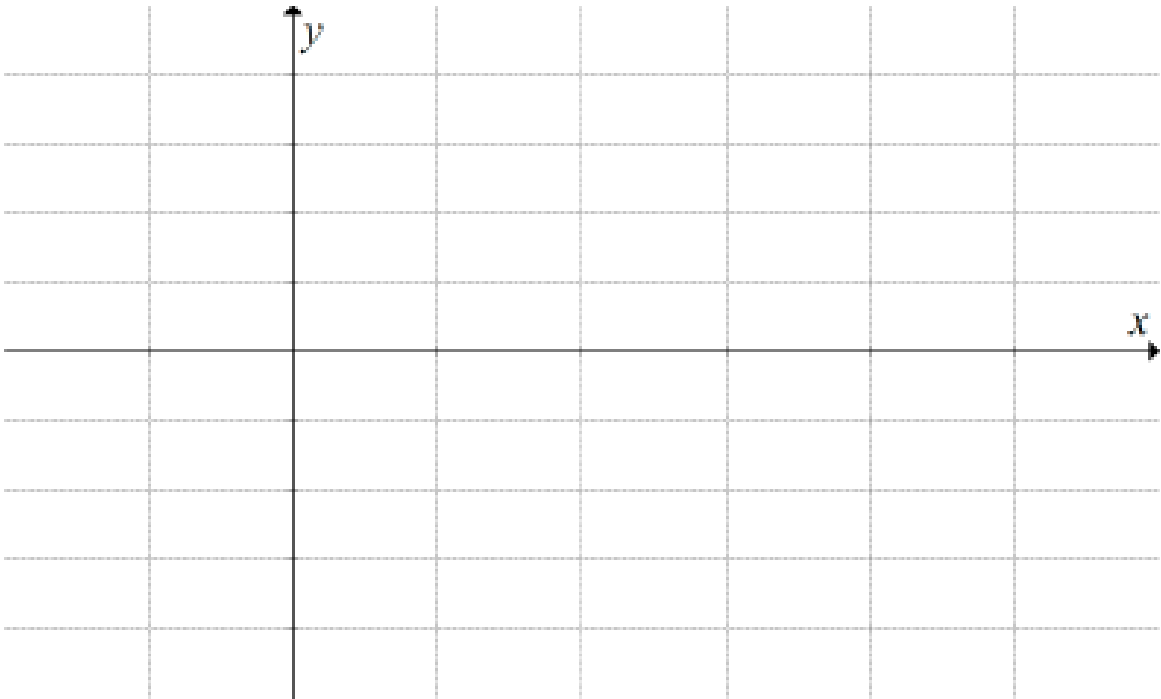
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QUESTION 4.1.3



QUESTION 7.5



A blank Cartesian coordinate system with a horizontal x-axis and a vertical y-axis. The origin is labeled with the number 0. The x-axis is labeled with 'x' at its right end, and the y-axis is labeled with 'y' at its top end. A grid of light gray lines is visible, with major grid lines every 1 unit and minor grid lines every 0.2 units. The grid extends from -10 to 10 on both the x and y axes.

# ANNEXURE A: INFORMATION SHEET

## INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a}$$

$$y = \frac{4ac - b^2}{4a}$$

$$a^x = b \Leftrightarrow x = \log_a b, \quad a > 0, a \neq 1 \text{ and } b > 0$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$\left(1 + \frac{i}{m}\right)^m - 1$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln x + C, \quad x > 0$$

$$\int a^x dx = \frac{a^x}{\ln a} + C, \quad a > 0$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area of } \triangle ABC = \frac{1}{2} ab \sin C$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$$

$$\pi \text{ rad} = 180^\circ$$

$$\text{Angular velocity} = \omega = 2\pi n = 360^\circ n \quad \text{where } n = \text{rotation frequency}$$

$$\text{Circumferential velocity} = v = \pi D n \quad \text{where } D = \text{diameter and } n = \text{rotation frequency}$$

$$s = r\theta \quad \text{where } r = \text{radius and } \theta = \text{central angle in radians}$$

$$\text{Area of a sector} = \frac{rs}{2} = \frac{r^2\theta}{2} \quad \text{where } r = \text{radius, } s = \text{arc length and}$$

$$\theta = \text{central angle in radians}$$

$$4h^2 - 4dh + x^2 = 0 \quad \text{where } h = \text{height of segment, } d = \text{diameter of circle and}$$

$$x = \text{length of chord}$$

$$A_T = a(m_1 + m_2 + m_3 + \dots + m_n) \quad \text{where } a = \text{equal parts, } m_1 = \frac{o_1 + o_2}{2}$$

$$\text{and } n = \text{number of ordinates}$$

OR

$$A_T = a \left( \frac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \dots + o_{n-1} \right) \quad \text{where } a = \text{equal parts, } o_i = i^{\text{th}} \text{ ordinate}$$

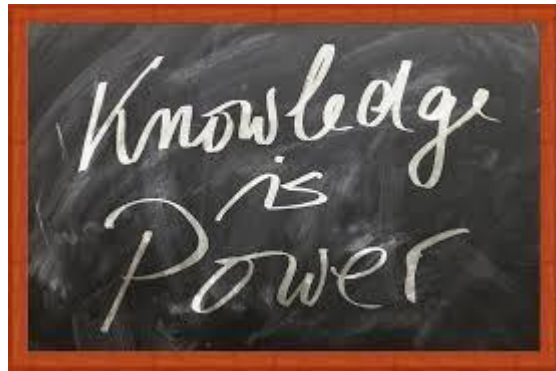
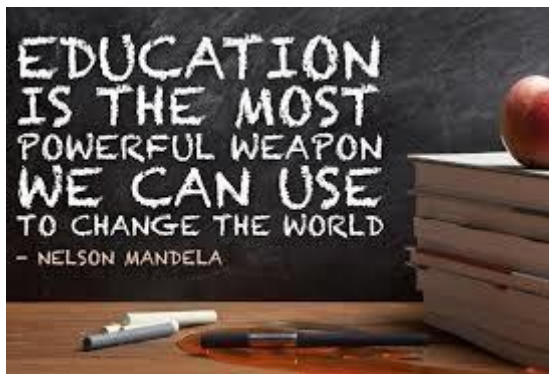
$$\text{and } n = \text{number of ordinates}$$






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# MEMORANDA GROUPED ACCORDING TO TOPICS

**ALGEBRA**  
**EXEMPLR 2018**
**QUESTION 1**

1.1.1	$x(x+2) = 0$ $\therefore x = 0$ or $x = -2$	$\checkmark x = 0$ $\checkmark x = -2$	(2)
1.1.2	$x(x+2) \geq 0$ $\therefore x \leq -2$ OR $x \geq 0$ 	$\checkmark x \leq -2$ $\checkmark x \geq 0$ $\checkmark$ OR $\checkmark$ Graphical representation	(4)
1.2	$5x^2 - 2 = x$ $5x^2 - x - 2 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(5)(-2)}}{2(5)} = \frac{1 \pm \sqrt{41}}{10}$ $\therefore x \approx 0,74$ or $x \approx -0,54$	$\checkmark$ Standard form $\checkmark$ Substitution into the quadratic formula $\checkmark x \approx 0,74$ $\checkmark x \approx -0,54$	(4)
1.3	$m - t - 1 = 0$ $m = t + 1$ $m^2 + t^2 = 5$ $(t+1)^2 + t^2 = 5$ $t^2 + 2t + 1 + t^2 - 5 = 0$ $2t^2 + 2t - 4 = 0$ $t^2 + t - 2 = 0$ $(t+2)(t-1) = 0$ $\therefore t = -2$ or $t = 1$ $m = -2 + 1 = -1$ or $m = 1 + 1 = 2$ OR $m - t - 1 = 0$ $t = m - 1$ $m^2 + t^2 = 9$ $m^2 + (m-1)^2 = 5$ $m^2 + m^2 - 2m + 1 - 5 = 0$ $2m^2 - 2m - 4 = 0$ $m^2 - m - 2 = 0$ $(m-2)(m+1) = 0$ $\therefore m = 2$ or $m = -1$ $t = 2 - 1 = 1$ or $t = -1 - 1 = -2$	$\checkmark$ Making $m$ the subject $\checkmark$ Substitution $\checkmark$ Simplification $\checkmark$ Factors $\checkmark$ Both values of $t$ $\checkmark$ Both values of $m$ OR $\checkmark$ Making $t$ the subject $\checkmark$ Substitution $\checkmark$ Simplification $\checkmark$ Factors $\checkmark$ Both values of $m$ $\checkmark$ Both values of $t$	(6)

1.4.1	$s = \frac{L_2 - L_1}{L_1}$ $\varepsilon L_1 = L_2 - L_1$ $\varepsilon L_1 + L_1 = L_2$ $L_1(\varepsilon + 1) = L_2$ $L_1 = \frac{L_2}{(\varepsilon + 1)}$	$s = \frac{L_2}{L_1} - 1$ $s + 1 = \frac{L_2}{L_1}$ $L_1(s + 1) = L_2$ $\therefore L_1 = \frac{L_2}{(s + 1)}$	$\checkmark$ multiply with LCD $\checkmark$ common factor $\checkmark$ divide by factor	(3)
1.4.2	$L_1 = \frac{L_2}{s + 1}$ $= \frac{18}{1 + 0,8}$ cm $= 10$ cm		$\checkmark$ Substitution $\checkmark$ Simplification	(2)
1.4.3	$10 = 8 + 2 = 2^3 + 2$ $= 1010_2$		$\checkmark 2^3 + 2$ $\checkmark 1010_2$	(2)
1.5	$12 \times 0,00361$ $= 0,43332$ $= 4,332 \times 10^{-2}$		$\checkmark 0,43332$ $\checkmark 4,332 \times 10^{-2}$	[25]

**QUESTION 2**

2.1.1	$p = -1$	$\checkmark p = -1$	(1)
2.1.2	$9 - 3p < 0$ $9 < 3p$ $\therefore p > 3$	$\checkmark 9 - 3p < 0$ $\checkmark p > 3$	(2)
2.1.3	$0$ OR $3$	$\checkmark 0$ OR $3$	(1)
2.2	$x^2 - 4x + (k-1) = 0$ For equal roots, $\Delta = b^2 - 4ac = 0$ $(-4)^2 - 4(1)(k-1) = 0$ $16 - 4k + 4 = 0$ $-4k = -20$ $\therefore k = 5$	$\checkmark$ For equal roots, $\Delta = 0$ $\checkmark$ Substitution $\checkmark$ Simplification $\checkmark$ Value of $k$	(4) [8]



QUESTION 3


3.1.1	$\frac{5 \times 2^{n-1} - 2^n}{2^n}$ $= \frac{2^n (5 \times 2^{-1} - 1)}{2^n}$ $= 5 \times \frac{1}{2} - 1 = \frac{3}{2}$ <p>OR</p> $\frac{5 \times 2^{n-1} - 2^n}{2^n}$ $= \frac{5 \times 2^{n-1}}{2^n} - \frac{2^n}{2^n} = 5 \times 2^{-1} - 1$ $= 2\frac{1}{2} - 1 = \frac{3}{2}$	<p>✓✓ Common factor</p> <p>✓ Simplification</p> <p>✓✓ Dividing each term by the denominator</p> <p>✓ Simplification</p>	(3)
3.1.2	$\sqrt{64+16} - \sqrt{20}$ $= \sqrt{80} - \sqrt{4 \times 5}$ $= 4\sqrt{5} - 2\sqrt{5}$ $= 2\sqrt{5}$	<p>✓ Addition</p> <p>✓ Simplified surd</p> <p>✓ Simplified surd</p> <p>✓ Simplification</p>	(4)
3.1.3	$\log_6 216 \times \log 0,001$ $= \log_6 6^3 \times \log \frac{1}{1000}$ $= \log_6 6^3 \times \log 10^{-3}$ $= 3 \log_6 6 \times (-3 \log 10)$ $= 3(1) \times (-3)(1)$ $= -9$	<p>✓ <math>\log_6 6^3</math> ✓ <math>\log 10^{-3}</math></p> <p>✓ <math>3 \log_6 6 = 3 \log 10</math></p> <p>✓ Simplification</p>	(4)
3.2.2	$\log(x+18) - \log x = 1$ $\log \frac{(x+18)}{x} = 1$ $\frac{(x+18)}{x} = 10$ $10x = x + 18$ $9x = 18$ $\therefore x = 2$	<p>✓ Apply log property</p> <p>✓ Change from log form to exp. Form</p> <p>✓ Simplification</p> <p>✓ Value of <math>x</math></p>	(4)

3.3	$z = 3 + \sqrt{3}i$ $ z  = r = \sqrt{x^2 + y^2}$ $= \sqrt{(3)^2 + (\sqrt{3})^2} = \sqrt{12}$ $\tan \theta = \frac{\sqrt{3}}{3}$ $\theta = 30^\circ$ $z = \sqrt{12} \operatorname{cis}(30^\circ) \text{ OR } z = \sqrt{12} [\cos 30^\circ + i \sin 30^\circ]$	<p>✓ Calculating the modulus</p> <p>✓ Simplification</p> <p>✓ <math>\tan \theta = \frac{\sqrt{3}}{3}</math></p> <p>✓ Argument</p> <p>✓ Correct polar form</p>	(5)
3.4	$x + yi = (3 + 5i)(2 - 7i)$ $x + yi = 6 - 11i - 35i^2$ $x + yi = 6 - 11i - 35(-1)$ $x + yi = 6 - 11i + 35$ $x + yi = 41 - 11i$ $\therefore x = 41 \text{ and } y = -11$	<p>✓ <math>6 - 11i - 35i^2</math></p> <p>✓ <math>i^2 = -1</math></p> <p>✓ <math>x = 41</math> ✓ <math>y = -11</math></p>	(4) [24]

NOV 2018

QUESTION/VRAG 1

1.1.1	$-2x(x+a)(3-x) = 0$ $x = 0$ or/of $x = -a$ or/of $x = 3$	$\checkmark x = 0$ A $\checkmark x = -a$ A $\checkmark x = 3$ A	(3)
1.1.2	$2x = 6 - x^2$ $x^2 + 2x - 6 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-2 \pm \sqrt{2^2 - 4(1)(-6)}}{2(1)}$ $= \frac{-2 \pm \sqrt{28}}{2}$ $\therefore x \approx 1,65$ or/of $x \approx -3,65$ OR/OR $2x = 6 - x^2$ $x^2 + 2x = 6$ $x^2 + 2x + 1 = 6 + 1$ $(x+1)^2 = 7$ $x+1 = \pm \sqrt{7}$ $x = -1 \pm \sqrt{7}$ $\therefore x \approx 1,65$ or/of $x \approx -3,65$	$\checkmark$ standard form/ <i>standaardvorm</i> $\checkmark$ SF CA $\checkmark$ S CA $\checkmark$ both values of/ <i>beide waardes van</i> x CA OR/OR $\checkmark$ completing square/ <i>kwadraatsvoltooiing</i> $\checkmark$ square as subject/ <i>vierkant as onderwerp</i> CA $\checkmark$ square root/ <i>vierkantswortel</i> CA $\checkmark$ both values of/ <i>beide waardes van</i> x CA NPR <div style="border: 1px solid black; padding: 5px; width: fit-content;">                         AO, full marks (exact values)/  <i>volpunte (ekskakte waardes)</i> </div>	(4)

1.1.3	$5x(x-3) \leq 0$ Critical values: 0 and 3 $\therefore 0 \leq x \leq 3$ OR/OR $x \in [0;3]$ OR/OR $x \geq 0$ and $x \leq 3$ 	$\checkmark$ both critical values/ <i>albei kritiese waardes</i> $\checkmark$ notation/notasie $\checkmark$ graphical representation/ <i>grafiese voorstelling</i> CA	(3)
1.2	$y - 2x = -7$ and $x^2 + xy + y^2 = 21$ $y = 2x - 7$ $x^2 + x(2x-7) + (2x-7)^2 = 21$ $x^2 + 2x^2 - 7x + 4x^2 - 28x + 49 - 21 = 0$ $7x^2 - 35x + 28 = 0$ $x^2 - 5x + 4 = 0$ OR/OR $(7x-28)(x-1) = 0$ $(x-4)(x-1) = 0$ $\therefore x = 4$ or $x = 1$ $y = 2(4) - 7$ or $y = 2(1) - 7$ $y = 1$ or $y = -5$ $\therefore y = 1$ and $x = 4$ OR/OR $y - 2x = -7$ $x = \frac{y+7}{2}$ $x^2 + xy + y^2 = 21$ $\left(\frac{y+7}{2}\right)^2 + y\left(\frac{y+7}{2}\right) + y^2 = 21$ $\frac{y^2 + 14y + 49}{4} + \frac{y^2}{2} + \frac{7y}{2} + y^2 = 21$ $y^2 + 14y + 49 + 2y^2 + 14y + 4y^2 = 84$ $7y^2 + 28y - 35 = 0$ $y^2 + 4y - 5 = 0$ $(y+5)(y-1) = 0$ OR/OR $(y+5)(y-7) = 0$ $\therefore y = -5$ or $y = 1$ $x = 1$ or/of $x = 4$ $\therefore y = 1$ and/en $x = 4$	$\checkmark$ y subject of formula/ <i>onderwerp van formule</i> $\checkmark$ SF CA $\checkmark$ S CA $\checkmark$ factors/ <i>faktore</i> CA $\checkmark$ x-values/-waardes CA $\checkmark$ y-values/-waardes CA $\checkmark$ y=1 and/en x=4 CA OR/OR $\checkmark$ x subject of formula/ <i>onderwerp van formule</i> $\checkmark$ substitution/ <i>vervanging</i> CA $\checkmark$ correct standard form/ <i>korrekte standaardvorm</i> CA $\checkmark$ factors/ <i>faktore</i> CA $\checkmark$ y-value/-waarde CA $\checkmark$ x-values/-waardes CA $\checkmark$ y=1 and/en x=4 CA <div style="border: 1px solid black; padding: 5px; width: fit-content;">                         If not squaring y or x after substitution,                          then a maximum of 6 marks,  <i>Indien geen kwadrering van y of x na</i>  <i>vervanging, dan 'n maksimum van 6 punte</i> </div>	(7)

1.3.1	$E = \frac{1}{12}ML^2$ $L^2 = \frac{12E}{M}$ $\therefore L = \sqrt{\frac{12E}{M}}$ OR / OF $2\sqrt{\frac{3E}{M}}$ OR / OF $\sqrt{\frac{E}{\frac{1}{12}M}}$	✓ $L^2$ subject/onderwerp ✓ $L$ subject/onderwerp CA AO, full marks / volpunte (2)
1.3.2	$L = \sqrt{\frac{12E}{M}}$ $= \sqrt{\frac{12(8,3 \times 10^{-2})}{1,6 \times 10^3}}$ $\therefore L \approx 0,02 \text{ m}$ OR / OF $L = 2\sqrt{\frac{3E}{M}}$ $= 2\sqrt{\frac{3(8,3 \times 10^{-2})}{1,6 \times 10^3}}$ $\therefore L \approx 0,02 \text{ m}$ OR / OF $E = \frac{1}{12}ML^2$ $8,3 \times 10^{-2} = \frac{1}{12}(1,6 \times 10^3)L^2$ $L = \sqrt{\frac{12(8,3 \times 10^{-2})}{1,6 \times 10^3}}$ $\therefore L \approx 0,02 \text{ m}$	CA from Question 1.3.1/ van Vraag 1.3.1 ✓ SF CA ✓ value of/waarde van $L$ CA OR / OF ✓ SF CA ✓ value of/waarde van $L$ CA OR / OF ✓ SF CA ✓ value of/waarde van $L$ CA NPU NPR (Accept scientific notation/Aanvaar wetenskaplike notasie) ✓ $32+4$ A ✓ $100100_2$ A AO: Full marks/ Volpunte (2)
1.4	$36 = 32 + 4$ $= 100100_2$	✓ $32+4$ A ✓ $100100_2$ A AO: Full marks/ Volpunte (2)

## QUESTION/VRAAG 2

2.1	$x = \frac{-8 \pm \sqrt{q-3}}{2}$	
2.1.1	Irrational / irrasionaal	✓ irrational / irrasionaal (Accept: real and unequal/ Aanvaar: reëel en ongelyk) (1)
2.1.2	Equal / gelyk	✓ equal (Accept: real OR rational/ Aanvaar: reël OF rasionaal) (1)
2.1.3	Non-real Nie-reël	✓ non-real/nie-reël (accept imaginary/ aanvaar imaginer) (1)
2.2	$3x^2 + 7x - 2x + p$ $3x^2 + 5x - p - 0$ $\Delta = b^2 - 4ac < 0$ $(5)^2 - 4(3)(-p) < 0$ $25 + 12p < 0$ $p < -\frac{25}{12}$	✓ standard form/ standaardvorm ✓ SF in $\Delta$ CA ✓ correct inequality/ korrekte ongelykheid ( $<0$ ) A ✓ values of/waardes van $p$ CA (4)

## QUESTION/VRAAG 3

3.1.1	$\left(2a^{\frac{7}{3}}\right)^3 - 2^3 \times \left(a^{\frac{7}{3}}\right)^3$ $= 8a^7$ <p style="text-align: center;">OR/OF</p> $\left(2a^{\frac{7}{3}}\right)^3 - \left(2a^{\frac{7}{3}}\right)\left(2a^{\frac{7}{3}}\right)\left(2a^{\frac{7}{3}}\right)$ $= 8a^7$	✓ exponent property/ eksponenteienskap A ✓ S CA <p style="text-align: center;">OR/OF</p> ✓ exponent property/eksponenteienskap A ✓ S CA AO: Full marks / Volpunte
3.1.2	$\log_p p + \log_m 1$ $= 1 + 0$ $= 1$	✓ 1 A ✓ 0 A AO: only one mark/ slegs een punt
3.1.3	$\frac{\sqrt{48} - \sqrt{12}}{2\sqrt{75}}$ $= \frac{\sqrt{3 \times 16} - \sqrt{3 \times 4}}{2\sqrt{3 \times 25}}$ $= \frac{4\sqrt{3} - 2\sqrt{3}}{2 \times 5\sqrt{3}} \text{ OR } \frac{\sqrt{3}(4 - 2)}{10\sqrt{3}}$ $= \frac{2\sqrt{3}}{10\sqrt{3}}$ $= \frac{1}{5}$ <p style="text-align: center;">OR/OF</p> $\frac{\sqrt{48} - \sqrt{12}}{2\sqrt{75}}$ $= \frac{\sqrt{48}}{2\sqrt{75}} - \frac{\sqrt{12}}{2\sqrt{75}}$ $= \frac{4\sqrt{3}}{10\sqrt{3}} - \frac{2\sqrt{3}}{10\sqrt{3}}$ $= \frac{2}{5} - \frac{1}{5} = \frac{1}{5}$	✓ $\frac{4\sqrt{3} - 2\sqrt{3}}{2.5\sqrt{3}}$ M ✓ S CA ✓ S CA <p style="text-align: center;">OR/OF</p> ✓ M ✓ S CA ✓ S CA AO: Only one mark / Slegs een punt

3.2	$\log_2(x+62) - \log_2 x = 5$ $\log_2\left(\frac{x+62}{x}\right) = 5$ $2^5 = \frac{x+62}{x}$ $32 = \frac{x+62}{x} \Rightarrow 32x = x+62$ $x = 2$ <p style="text-align: center;">OR/OF</p> $\log_2(x+62) - \log_2 x = 5$ $\log_2(x+62) - \log_2 x + \log_2 32$ $\log_2(x+62) - \log_2(32x)$ $x+62 = 32x$ $x = 2$ <p style="text-align: center;">OR/OF</p> $\log_2(x+62) - \log_2 x = 5$ $\log_2 \frac{x+62}{2^5} = \log_2 x$ $\frac{x+62}{32} = x$ $32x = x+62$ $x = 2$ <p style="text-align: center;">OR/OF</p> $\log_2(x+62) - \log_2 x = 5$ $\frac{\log(x+62)}{\log 2} - \frac{\log x}{\log 2} = 5$ $\log(x+62) - \log x = 5 \log 2$ $\log \frac{x+62}{x} = \log 2^5$ $\frac{x+62}{x} = 32 \Rightarrow x+62 = 32x$ $x = 2$ <p style="text-align: center;">OR/OF</p> $\log_2(x+62) - \log_2 x = 5$ $\log_2(x+62) - \log_2 x - \log_2 32 = 0$ $\log_2 \frac{x+62}{x(32)} = 0$ $\frac{x+62}{x(32)} = 1$ $31x = 62 \therefore x = 2$	✓ log property/ log-eienskap M ✓ exponential form/ eksponensiële vorm M ✓ S CA ✓ value of/ waarde van x CA <p style="text-align: center;">OR/OF</p> ✓ log property/ log-eienskap e M ✓ apply log property pas log-eienskap toe M ✓ S CA ✓ value of/ waarde van x CA <p style="text-align: center;">OR/OF</p> ✓ log property/ log-eienskap M ✓ log property/ log-eienskap M ✓ S CA ✓ value of/ waarde van x CA <p style="text-align: center;">OR/OF</p> ✓ log property/ log-eienskap M ✓ log property/ log-eienskap M ✓ S CA ✓ value of/ waarde van x CA <p style="text-align: center;">OR/OF</p> ✓ log property/ log-eienskap M ✓ log property/ log-eienskap M ✓ S CA ✓ value of/ waarde van x CA AO: only one mark / slegs een punt
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NOV 2019

QUESTION/PRAAG 1

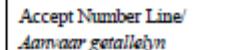
3.3	$z = -\sqrt{2} + \sqrt{2}i$ $ z  = r = \sqrt{x^2 + y^2}$ $= \sqrt{(-\sqrt{2})^2 + (\sqrt{2})^2} = \sqrt{4}$ $= 2$ $\theta = \tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$ OR any other trig. ratio to find $\theta$ OF enige ander trig. verh. of $\theta$ te bepaal $= 45^\circ$ OR/OF $\frac{\pi}{4}$ $\theta = 180^\circ - 45^\circ = 135^\circ$ OR/OF $\frac{3\pi}{4}$ $\therefore z = 2\text{cis}(135^\circ)$ OR/OF $z = 2\text{cis}\left(\frac{3\pi}{4}\right)$	$\checkmark$ calculating the modulus/ <i>bereken die modulus</i> M $\checkmark$ S CA $\checkmark \theta = \tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$ M $\checkmark$ ref. angle/ <i>verwysingshoek</i> CA $\checkmark$ correct quadrant/ <i>korrekte kwadrant</i> CA $\checkmark$ polar form/ <i>polêre vorm</i> CA Accept/Aanvaar: $z = 2[\cos 135^\circ + i \sin 135^\circ]$ $z = 2\left[\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right]$
3.4	$p + qi = (2 - 3i)^2$ $= 4 - 12i + 9i^2$ $= 4 - 12i + 9(-1)$ $= -5 - 12i$ $\therefore p = -5$ and/en $q = -12$	$\checkmark$ expansion/ <i>uitbreiding</i> $\checkmark i^2 = -1$ A $\checkmark p = -5$ $\checkmark q = -12$ CA

(6)

(4)

[21]

1.1.1	$p(x) = 2x^2 - \frac{8}{81}$ $= 2\left(x^2 - \frac{4}{81}\right)$ $= 2\left(x - \frac{2}{9}\right)\left(x + \frac{2}{9}\right)$ OR/OF $p(x) = \frac{162x^2 - 8}{81}$ $= \frac{2(81x^2 - 4)}{81}$ $= \frac{2(9x - 2)(9x + 2)}{81}$	$\checkmark$ common factor/ <i>gemene faktor</i> A $\checkmark$ both factors/ <i>beide faktore</i> CA OR/OF $\checkmark$ common factor/ <i>gemene faktor</i> A $\checkmark$ both factors/ <i>beide faktore</i> CA (2)
1.1.2	$p(x) = 2\left(x - \frac{2}{9}\right)\left(x + \frac{2}{9}\right) = 0$ $\therefore x - \frac{2}{9}$ or/of $x - \frac{2}{9}$ OR/OF $\therefore x = \pm \frac{2}{9}$ OR/OF $x \approx \pm 0,22$ OR/OF $x = \frac{-(0) \pm \sqrt{(0)^2 - 4(2)\left(-\frac{8}{81}\right)}}{2(2)}$ $\therefore x = \frac{2}{9}$ or/of $x = -\frac{2}{9}$ OR/OF $\therefore x = \pm \frac{2}{9}$ OR/OF $x \approx \pm 0,22$	$\checkmark$ both values of/ <i>beide waardes van</i> x CA OR/OF $\checkmark$ both values of/ <i>beide waardes van</i> x CA (1)

1.2.1	$(3x-5)(x+2) = -13$ $3x^2 + x - 10 + 13 = 0$ $3x^2 + x + 3 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(1) \pm \sqrt{(1)^2 - 4(3)(3)}}{2(3)}$ $x = \frac{-1 \pm \sqrt{-35}}{6}$ $x = \frac{-1 \pm \sqrt{35} i}{6} \quad \text{OR/OF} \quad x \approx \frac{-1 \pm 5,92i}{6}$ $\text{OR/OF}$ $x = -\frac{1}{6} + \frac{\sqrt{35}}{6}i \quad \text{or/of} \quad -\frac{1}{6} - \frac{\sqrt{35}}{6}i$ $\text{OR/OF}$ $x \approx -0,17 + 0,99i \quad \text{or/of} \quad x \approx -0,17 - 0,99i$ $\text{OR/OF}$ $x = -\frac{1}{6} \pm \frac{\sqrt{35}}{6}i \quad \text{OR/OF} \quad x \approx -0,17 \pm 0,99i$	<p>✓ standard form/standaard vorm A</p> <p>✓ SF CA</p> <p>✓ S CA</p> <p>✓ <math>\sqrt{-35} = \sqrt{35} i</math> CA          ✓ both x-values/beide x-waardes CA</p> <p>(5)</p>
1.2.2	$(4-x)(x+3) < 0 \quad \text{OR/OF} \quad (x-4)(x+3) > 0$ <p>CV: <math>x = -3</math> and <math>x = 4</math></p> $x < -3 \text{ or/of } x > 4 \quad \text{OR/OF}$ $x \in (-\infty; -3) \cup (4; \infty)$	<p>✓ both critical values/beide kritiese waardes A</p> <p>✓ correct notation for each interval/korrekte notasie vir elke interval CA</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;">             Accept Number Line/              Aanvaar getalrelyn         </div> <div style="text-align: center;">  </div> <p>(3)</p>

1.3	$y = 3x - 8$ $x^2 - xy + y^2 = 39$ $x^2 - x(3x-8) + (3x-8)^2 = 39$ $x^2 - 3x^2 + 8x + 9x^2 - 48x + 64 - 39 = 0$ $7x^2 - 40x + 25 = 0$  $(x-5)(7x-5) = 0 \text{ OR/OR}$ $x = \frac{-(-40) \pm \sqrt{(40)^2 - 4(7)(25)}}{2(7)}$ $\therefore x = 5 \text{ or/of } x = \frac{5}{7} \text{ OR / OF } 0,71$  $y = 3(5) - 8 = 7 \text{ or/of } y = 3\left(\frac{5}{7}\right) - 8 = -\frac{41}{7}$ OR / OF $y \approx -5,86$  OR/OR  $y = 3x - 8$ $x^2 - xy + y^2 = 39$ $x = \frac{y+8}{3}$ $x^2 - xy + y^2 = 39$ $\left(\frac{y+8}{3}\right)^2 - y\left(\frac{y+8}{3}\right) + y^2 = 39$ $\frac{y^2 + 16y + 64}{9} - \frac{y^2}{3} - \frac{8y}{3} + y^2 = 39$ $y^2 + 16y + 64 - 3y^2 - 24y + 9y^2 = 351$ $7y^2 - 8y - 287 = 0$ $(y-7)(7y+41) = 0 \text{ OR/OR}$ $x = \frac{-(-40) \pm \sqrt{(40)^2 - 4(7)(25)}}{2(7)}$ $\therefore y = 7 \text{ or/of } y = -\frac{41}{7} \text{ OR / OF } y \approx -5,86$  $x = \frac{7+8}{3} = 5 \text{ or/of } x = \frac{-\frac{41}{7}+8}{3} = \frac{5}{7}$ OR / OF $x \approx 0,71$	✓ substitution/ <i>vervanging</i> A  ✓ S CA ✓ standard form/ <i>standaard vorm</i> CA  ✓ factors or formula/ <i>faktore</i> CA  ✓ both x-values/ <i>beide x-waardes</i> CA  ✓ both y-values/ <i>beide y-waardes</i> CA  OR/OR
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1.4.1	$V = I \times Z$ $I = \frac{V}{Z}$ OR/OF $I = VZ^{-1}$	✓ dividing by/deel deur $Z$ A (1)	
1.4.2	$I = \frac{V}{Z}$ $= \frac{7i}{3-i}$ $= \frac{7i}{3-i} \times \frac{3+i}{3+i}$ $= \frac{21i + 7i^2}{9 - i^2}$ $= \frac{21i + 7(-1)}{9 - (-1)}$ $= \frac{-7 + 21i}{10}$ OR/OF $-0,7 + 2,1i$ OR/OF $-\frac{7}{10} + \frac{21i}{10}$	✓ substitution/vervanging CA  ✓ M multiply by/ vermenigvuldig met $\frac{3+i}{3+i}$ A ✓ S CA  ✓ value of/waarde van $i^2$ A ✓ value of current/waarde van stroom CA <div style="border: 1px solid black; padding: 2px; display: inline-block;">NPU</div> (5)	
1.5	$\begin{array}{r} 1 \quad 0 \quad 1_2 \\ \times \quad 1 \quad 1_2 \\ \hline 1 \quad 0 \quad 1_2 \\ + 1 \quad 0 \quad 1 \quad 0_2 \\ \hline 1 \quad 1 \quad 1 \quad 1_2 \end{array}$ OR/OF $101_2 \times 11_2 = 5 \times 3 = 15$ OR/OF $1111_2$	✓ M A ✓ $1111_2$ A  OR/OF ✓ both/beide 5 and/en 3 A ✓ $1111_2$ OR/OF 15 A <div style="border: 1px solid black; padding: 2px; display: inline-block;">AO: Full marks/Volpunte</div> (2) [25]	

## QUESTION/VRAAG 2

2.1.1	$G = \sqrt{\frac{p+1}{2p-1}}$ $2p-1=0$ $\therefore p = \frac{1}{2}$	✓ $p = \frac{1}{2}$ A (1)	
2.1.2	$p+1=0$ $p=-1$	✓ $p=-1$ A (1)	
2.2	$x^2 - k + 4 = 5x$ $x^2 - 5x - k + 4 = 0$ $b^2 - 4ac \geq 0$ $(-5)^2 - 4(1)(-k+4) \geq 0$ $25 + 4k - 16 \geq 0$ $4k + 9 \geq 0$ $k \geq -\frac{9}{4}$	✓ standard form/standard vorm A ✓ the discriminant/die diskriminant $\geq 0$ A ✓ SF A ✓ S CA  ✓ values of/waardes van $k$ CA (5) [7]	

QUESTION/VRAAG 3

3.1	$\left(-2\sqrt[4]{a^3}\right)^8$ $=(-2)^8 (a^3)^{\frac{8}{4}}$ $= 256 a^6$	$\checkmark 256$ $\checkmark a^6$	A A (2)
3.2	$\log_2(3x-2)+\log_2 0,5=3$ # $\log_2(3x-2)+\log_2 2^{-1}=3$ $\log_2(3x-2)-1=3$ $\log_2(3x-2)=4$ $(3x-2)=2^4$ $3x-2=16$ $x=6$  OR/OF $\log_2(3x-2)+\log_2 0,5=3$ $\log_2(0,5)(3x-2)=3$ $(0,5)(3x-2)=2^3$ $(0,5)(3x-2)=8$ $3x-2=16$ $x=6$  OR/OF $\log_2(3x-2)+\log_2 0,5=3$ $\log_2(3x-2)=3-\log_2 0,5$ $\log_2(3x-2)=3\log_2 2-\log_2 0,5$ $\log_2(3x-2)=\log_2 8-\log_2 0,5$ $\log_2(3x-2)=\log_2\left(\frac{8}{0,5}\right)$ $3x-2=16$ $x=6$  OR/OF	$\checkmark \log_2 0,5=-1$ $\checkmark 5$ $\checkmark$ exponential form/eksponent vorm  $\checkmark$ correct value/korrekte waarde  OR/OF  $\checkmark$ log property/eienskap $\checkmark$ exponential form/eksponent vorm  $\checkmark 5$ $\checkmark$ correct value/korrekte waarde  OR/OF  $\checkmark$ log property/eienskap  $\checkmark$ log property/eienskap  $\checkmark 5$ $\checkmark$ correct value/korrekte waarde	A CA CA CA  A A CA CA  A A CA CA  A A CA CA

$\log_2 0,5(3x-2) = \log_2 2^3$ $\log_2 0,5(3x-2) = \log_2 8$ $0,5(3x-2) = 8$ $3x-2 = 16$ $x = 6$	<p style="text-align: center;"><b>OR/OF</b></p> <p>✓ log property/eienskap      A</p> <p>✓ log property/eienskap      A</p> <p>✓ S      CA</p> <p>✓ correct value/korrekte waarde      CA</p>
<p style="text-align: center;"><b>OR/OF</b></p> $\log_2(3x-2) + \log_2 0,5 = 3$ $\log_2(3x-2) - \log_2 2 = 3$ $\log_2 \frac{(3x-2)}{2} = 3$ $\frac{(3x-2)}{2} = 2^3$ $3x-2 = 16$ $x = 6$	<p style="text-align: center;"><b>OR/OF</b></p> <p>✓ log property/eienskap      A</p> <p>✓ log property/eienskap      A</p> <p>✓ S      CA</p> <p>✓ correct value/korrekte waarde      CA</p> <p>(4)</p>



3.3	$\log \sqrt{0,6}$ $= \log (0,6)^{\frac{1}{2}} \text{ OR/OF } \log (0,3 \times 2)^{\frac{1}{2}} \text{ OR/OF } \log (0,2 \times 3)^{\frac{1}{2}}$ $= \frac{1}{2} \log \left( \frac{6}{10} \right) \text{ OR/OF } \frac{1}{2} \log (0,3 \times 2) \text{ OR/OF } \frac{1}{2} \log (0,2 \times 3)$ $= \frac{1}{2} \log \left( \frac{3 \times 2}{10} \right)$ $= \frac{1}{2} (\log 3 + \log 2 - \log 10)$ $= \frac{1}{2} (b + a - 1) \text{ OR/OF } \frac{1}{2} b + \frac{1}{2} a - \frac{1}{2}$  <p style="text-align: center;">OR/OF</p> $\log \sqrt{0,6} = \log \sqrt{\frac{3}{5}}$ $= \frac{1}{2} \log 3 - \frac{1}{2} \log 5$ $\therefore \log \sqrt{0,6} = \frac{1}{2} \log 3 + \frac{1}{2} (\log 2 - \log 10)$ $= \frac{1}{2} (b + a - 1) \text{ OR/OF } \frac{1}{2} b + \frac{1}{2} a - \frac{1}{2}$	✓ exp. form/eksp. vorm A ✓ conversion of / herleiding van 0,6 A  ✓ log property/eienskap A ✓ log10=1 CA ✓ substitution/verwagting CA  <p style="text-align: center;">OR/OF</p> ✓ conversion of / herleiding van 0,6 A ✓ log property/eienskap A ✓ conversion of log 5 A ✓ log10=1 CA ✓ substitution/verwagting CA (5)	
3.4.1	$V = 2(\cos 240^\circ + i \sin 240^\circ)$ <p style="text-align: center;">OR/OF</p> $V = 2\left(\cos \frac{3}{4}\pi + i \sin \frac{3}{4}\pi\right)$	✓ value of/ waarde van V A (1)	

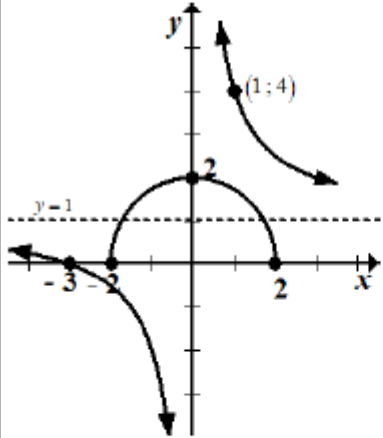
3.4.2	$V = 2(\cos 240^\circ + i \sin 240^\circ) \text{ OR/OF}$ $V = 2\left(\cos \frac{3}{4}\pi + i \sin \frac{3}{4}\pi\right)$ $V = 2\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$  $\therefore V = -1 - \sqrt{3}i$  <p style="text-align: center;">OR/OF</p> $\theta = 60^\circ$ [ref angle/ verw. hoek] $\tan 60^\circ = \sqrt{3} = \frac{-\sqrt{3}}{-1}$ $V = -1 - \sqrt{3}i$	CA from/vanaf Q/V3.4.1 ✓ value of/waarde van $\cos 240^\circ \text{ OR/OF } \cos \frac{3}{4}\pi$ CA ✓ value of/waarde van $\sin 240^\circ \text{ OR/OF } \sin \frac{3}{4}\pi$ CA ✓ V in rect. form/ in regh.vorm CA  <p style="text-align: center;">OR/OF</p> ✓ ref. angle/verw. hoek A ✓ tan ratio/verhouding A  ✓ V in rect form/ in regh.vorm CA AO: Full marks/Volpunte (3)	
3.5	$m + ni = 2(6 - 4i) - (-7i)$ $= 12 - 8i + 7i = 12 - i$ $m = 12 \text{ and/ en } n = -1$  <p style="text-align: center;">OR/OF</p> $m = 2(6) = 12 \text{ and/ en } n = 2(-4) - (-7) = -1$	✓ value of m/ waarde van m A ✓ value of n/ waarde van n A  <p style="text-align: center;">OR/OF</p> ✓ value of m/ Waarde van m A ✓ value of n/ Waarde van n A (2) AO: Full marks/Volpunte [17]	

## FUNCTIONS AND GRAPHS

## EXEMPLAR 2018

## QUESTION 4

4.1.1	$x$ -intercepts, $f(x) = 0$ $2x^2 + 4x - 6 = 0$ $2(x+3)(x-1) = 0$ OR $(x+3)(2x-2) = 0$ $\therefore x = -3$ or $x = 1$ $\therefore B(1; 0)$	✓ Finding other factor ✓ Coordinates of B. (2)
4.1.2	$f(x) = 2x^2 + 4x - 6$ $\left(\frac{-b}{2a}; \frac{4ac - b^2}{4a}\right) = \left(\frac{-4}{2(2)}; \frac{4(2)(-6) - (4)^2}{4(2)}\right)$ $\therefore D(-1; -8)$ OR $x = \frac{-b}{2a} = \frac{-4}{2(2)}$ $\therefore x = -1$ $f(-1) = 2(-1)^2 + 4(-1) - 6 = -8$ $\therefore D(-1; -8)$ OR $x_D = \frac{-3+1}{2} = -1$ $f(-1) = 2(-1)^2 + 4(-1) - 6 = -8$ $\therefore D(-1; -8)$ OR $f(x) = 2x^2 + 4x - 6$ $f'(x) = 4x + 4 = 0$ $\therefore x = -1$ $f(-1) = 2(-1)^2 + 4(-1) - 6 = -8$ $\therefore D(-1; -8)$	✓ Substitution in formula ✓ Coordinates of D OR ✓ Substitution in formula ✓ Substitution to find y ✓ Coordinates of D OR ✓ Using x-intercepts ✓ Substitution to find y ✓ Coordinates of D OR ✓ Using the derivative ✓ Substitution to find y ✓ Coordinates of D (3)
4.1.3	$g(x) = k^2 + q$ $10 = k^2 + 6$ $k^2 = 4$ $\therefore k = 2$	✓ Substituting coordinates of Q ✓ Simplified equation ✓ Correct value of k. (3)
4.1.4	$y = 6$	✓ $y = 6$ (1)
4.1.5	$-3 < x < 1$	✓ Correct critical values ✓ Correct notation (2)

4.2.1	$x = 0$ and $y = 1$	✓ $x = 0$ ✓ $y = 1$ (2)
4.2.2	$h(x) = \frac{3}{x} + 1$ $0 = \frac{3}{x} + 1$ $-1 = \frac{3}{x}$ $\therefore x = -3$	✓ Substituting coordinates of Q ✓ Value of x (2)
4.2.3	$r = 2$	✓ $r = 2$ (1)
4.2.4		✓ Shape of h ✓ Asymptote ✓ x-intercept ✓ Any other point on the graph of h ✓ Shape of g ✓ x-intercepts of g ✓ y-intercept of g (7)
4.2.5	$0 \leq y \leq 2$	✓ $0 \leq y$ ✓ $y \leq 2$ (2) [25]

NOV 2018

QUESTION/VRAAG 4

	<b>Q4.1.2: Penalty of ONE mark if intercepts are not given as coordinates (refer also to Question 7.1)</b> <b>V4.1.2: Penaliseer EEN punt indien afsnitte nie as ko-ördinate gegee (verwys ook na Vraag 7.1)</b>	
4.1.1	$x = 0$ and/en $y = -1$ ✓vertical asymptote/ vertikale asimptoot A ✓horizontal asymptote/ horisontale asimptoot A (2)	
4.1.2	$h(x) = -\frac{6}{x} - 1$ $0 = -\frac{6}{x} - 1$ $\therefore x = -6$ $(-6; 0)$ ✓ $0 = -\frac{6}{x} - 1$ M ✓ $-6$ A (2)	
4.1.3	 ✓horizontal asymptote/ horisontale asimptoot CA from /van Q/V4.1.1 g: ✓intercept/afsnit $(0; 0)$ A ✓shape/vorm A h: ✓x-intercept/afsnit CA ✓both curves/beide kromme A (5)	
4.1.4	$g(-2) = 2^{-(-2)} - 1 = 3$ OR/OF $3 = 2^{-x} - 1$ $2^2 = 2^{-x}$ $\therefore x = -2$ ✓SF A OR/OF ✓SF A (1)	
4.1.5	$y > -1$ OR/OF $y \in (-1; \infty)$ ✓ $y > -1$ A (1)	
4.1.6	$x = 0$ OR/OF $x \in (-\infty; 0) \cup (0; \infty)$ OR/OF $x \in \mathbb{R}; x \neq 0$ OR/OF $x < 0$ or $x > 0$ OR/OF $x \in \mathbb{R} - \{0\}$ ✓ A (1)	

4.2.1	$M(1; 0)$ ✓ $(1; 0)$ A (1)	
4.2.2	$MT = 8$ $MR = g(1) = \sqrt{36 - (1)^2} = \sqrt{35}$ $\therefore TR = MT - MR = 8 - \sqrt{35}$ ✓length of/lengte van MT A ✓length of/lengte van MK SF ✓length of/lengte van TR CA (3)	
4.2.3	$g(0) = \sqrt{36 - (0)^2} = 6$ $\therefore L(0; 6)$ OR/OF $r = \sqrt{36} = 6$ $L(0; 6)$ ✓SF A OR/OF ✓calculating radius/bereken radius A (1)	
4.2.4	$f(x) = y = a(x + p)^2 + q$ $y = a(x - 1)^2 + 8$ $6 = a(0 - 1)^2 + 8$ $a = -2$ $\therefore f(x) = -2(x - 1)^2 + 8$ $= -2(x^2 - 2x + 1) + 8$ $= -2x^2 + 4x + 6$ $= -2(x^2 - 2x - 3)$ $\therefore f(x) = -2(x + 1)(x - 3)$ ✓SF $(1; 8)$ ✓ $a = -2$ CA ✓S CA ✓common factor/gemene faktor M (4)	
4.2.5	$\therefore K(-1; 0)$ ✓coordinates of/koördinate van K A (1)	
4.2.6	$x \in (-1; 0)$ OR/OF $-1 < x < 0$ CA from /van Q4.2.5 ✓critical values/kritiese waardes CA OR/OF ✓notation/notasie CA (2) [24]	

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QUESTION/VRAAG 4

4.1.1	$(x-5)(x+3)=0$ $x=5$ or/of $x=-3$ OR/OF $(5;0)$ and/en $(-3;0)$	✓ both values of/ beide waardes van $x$ A (1)	
4.1.2	$q(x) = \frac{12}{x} - 2$ $\frac{12}{x} - 2 = 0$ $2x = 12$ $x = 6$ OR/OF $(6;0)$  OR/OF $q(x) = \frac{a}{x} + p$ $x = \frac{a}{-p}$ $[q(x) = 0]$ $= \frac{12}{-(-2)}$ $= 6$ OR/OF $(6;0)$	✓ $y = 0$ A  ✓ $x$ -value/waarde CA  OR/OF  ✓ $y = 0$ A  ✓ $x$ -value/waarde CA  (2)	AO: Full marks /Volpunte

4.1.3	$x = \frac{5-3}{2} = 1$ $k(1) = (1-5)(1+3) = -16$ $\therefore$ TP/DP $(1; -16)$  OR/OF $(x-5)(x+3) = 0$ $x^2 - 2x - 15 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-15)}}{2(1)} = 1$ $k(1) = (1)^2 - 2(1) - 15 = -16$ OR/OF $y = \frac{4ac - b^2}{4a} = \frac{4(1)(-15) - (-2)^2}{4(1)} = -16$ $\therefore$ TP/DP $(1; -16)$  OR/OF $k'(x) = 2x - 2$ $2x - 2 = 0$ $\therefore x = 1$ $k(1) = (1)^2 - 2(1) - 15 = -16$ $\therefore$ TP/DP $(1; -16)$	✓ M CA ✓ $x = 1$ A ✓ $y = -16$ CA  OR/OF  ✓ M CA ✓ $x = 1$ A  ✓ $y = -16$ A  OR/OF  ✓ M A ✓ $x = 1$ A ✓ $y = -16$ CA (3)	
4.1.4	$y = -2$ $x = 0$	✓ $y = -2$ A ✓ $x = 0$ A (2)	
4.1.5		<u>Graph / grafiek k:</u> ✓ x-intercepts/aftsnitte CA ✓ y-intercept/aftsnitte CA ✓ shape/vorm A ✓ turning point/draaipunt CA  <u>Graph / grafiek q:</u> ✓ hor. asympt/asimpt A ✓ x-intercepts/aftsnit CA ✓ shape/vorm A (7)	

4.2.1	$d = -4$	✓ value of/waarde van $d$ A Accept/ Aanvaar $y = -4$ (1)	
4.2.2	$h(x) = a^x - 4$ $-2 = a^{-1} - 4$ $a = \frac{1}{2}$ $\therefore h(x) = \left(\frac{1}{2}\right)^x - 4$	✓ substitution/vervanging CA  ✓ correct value of/ korrekte waarde van $a$ CA (2)	
4.2.3	$h(0) = a^0 - 4$ OR / OF $h(0) = (0,5)^0 - 4$ $T(0; -3)$	✓ $x = 0$ A  ✓ $y = -3$ CA (2)	
4.2.4	$y \in [-3; 0]$ OR / OF $-3 \leq y \leq 0$	✓ range/ waardeversm(terrein) CA (1)	
4.2.5	$p(x) = -\sqrt{9-x^2}$ $\therefore w(x) = \sqrt{9-x^2}$ OR / OF $\therefore w(x) = \sqrt{r^2 - x^2}$	✓ equation of/ vergelyking van $p$ CA  ✓ equation of/ vergelyking van $w$ CA (2)  AO: Full marks/Volpunte	
4.2.6	$0 < x \leq 3$ OR / OF $x \in (0; 3]$	✓ endpoints/eindpunte A ✓ correct notation/ korrekte notasie A (2) [25]	

# FINANCE, GROWTH AND DECAY EXEMPLAR 2018

## QUESTION 5

5.1	$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$ $i_{eff} = \left(1 + \frac{0,072}{2}\right)^2 - 1$ $\approx 0,073296$ $\therefore \text{annual effective interest rate is } 7,33\%$	✓ Correct substitution ✓ Simplification ✓ Effective rate as % (3)
5.2	$A = P(1-i)^n$ $70 = 220(1-0,08)^n$ $\frac{7}{22} = (0,92)^n$ $n = \log_{0,92} \frac{7}{22}$ $\therefore n \approx 13,73363166$ It will take approximately 14 minutes.	✓ Correct formula ✓ Correct substitution ✓ Simplified power form  ✓ Using logarithms ✓ Nearest minute (5)
5.2.2	Value of $A$ after 3 years: $A = P(1+i)^n$ $A = R150000 \left(1 + \frac{10,5\%}{4}\right)^{3 \times 4}$ $= R204705,40$ Value of $P$ after withdrawal: $P = R204705,40 - R30000 = R174705,40$ Amount received at the end of the investment period: $A = R174705,40 \left(1 + \frac{10,5\%}{4}\right)^{2 \times 4}$ $\therefore A = R214947,15$	✓ Correct formula ✓ Correct substitution ✓ R204705,40  ✓ $P = R174705,40$  ✓ Correct substitution ✓ Final amount (6) [14]

## NOV 2018

## QUESTION/VRAAG 5

5.1	$i_{eff} = \left(1 + \frac{i_{nom}}{m}\right)^m - 1$ $0,067 = \left(1 + \frac{i_{nom}}{12}\right)^{12} - 1$ $1,067 = \left(1 + \frac{i_{nom}}{12}\right)^{12}$ $\sqrt[12]{1,067} = \left(1 + \frac{i_{nom}}{12}\right)$ $i_{nom} = 12 \left(\sqrt[12]{1,067} - 1\right) \approx 0,065$ $\therefore \text{nominal interest rate is/nominale rentekoers is } 6,5\%$	✓ SF  $\sqrt[12]{1,067} = \left(1 + \frac{i_{nom}}{12}\right) \quad \text{CA}$ $\sqrt[12]{1,067} - 1 = \frac{i_{nom}}{12} \quad \text{CA}$ $i_{nom} = 12 \left(\sqrt[12]{1,067} - 1\right) \approx 0,065$ ✓ 6,5 % CA (Accept/aanvaar 0,065) (4)
5.2.1	R120 000	✓ R120 000 A (1)
5.2.2	Reducing balance method/verminderde balans-metode: $A = P(1-i)^n$ $120000 = 240000(1-16\%)^n$ $0,5 = (0,84)^n$ $n = \log_{0,84} 0,5$ $\therefore n \approx 3,98$ $\therefore \text{it will take 4 years/dit sal 4 jaar neem}$ OR/OF Straight line - method/Re gelykmetode: $A = P(1-i \times n)$ $120000 = 240000(1-16\% \times n)$ $-0,5 = (-16\% \times n)$ $n = \frac{-0,5}{-16\%}$ $\therefore n \approx 3,125$ $\therefore \text{it will take 3 years/dit sal 3 jaar neem}$	✓ SF $0,5 = (0,84)^n \quad \text{CA}$ $\log 0,5 = n \log 0,84 \quad \text{CA}$ $n = \frac{\log 0,5}{\log 0,84} \quad \text{CA}$ $\therefore n \approx 3,98 \quad \text{CA}$ ✓ R OR/OF  ✓ SF $-0,5 = (-16\% \times n) \quad \text{CA}$ $n = \frac{-0,5}{-16\%} \quad \text{CA}$ $\therefore n \approx 3,125 \quad \text{CA}$ ✓ R (5)

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QUESTION/VRAAG 5

5.3	<p>For the first 4 years/<i>Vir eerste 4 jaar</i> :</p> $A = P(1+i)^n = 40\,000 \left(1 + \frac{11,2\%}{4}\right)^{4 \times 4}$ <p><math>\therefore A \approx R\,62\,222,83...</math></p> <p>For the last 3 years/<i>Vir laaste 3 jaar</i> :</p> $A = 62\,222,83... (1 + 13\%)^3$ $\approx R\,89\,781,15$ <p style="text-align: center;"><b>OR/OF</b></p> $A = P(1+i)^n \cdot (1+i)^n$ $= 40\,000 \left(1 + \frac{11,2\%}{4}\right)^{4 \times 4} \times (1 + 13\%)^3$ $\approx R\,89\,781,15$	<p>✓ value of/waarde van <math>i</math> and/en <math>n</math> A</p> <p>✓ SF CA</p> <p>✓ 62 222,83... CA</p> <p>✓ SF CA</p> <p>✓ 89 781,15 CA</p> <p style="text-align: center;"><b>OR/OF</b></p> <p>✓ <math>A = P(1+i)^n \cdot (1+i)^n</math> M</p> <p>✓ value of/waarde van <math>i</math> and/en <math>n</math> A</p> <p>✓ value of/waarde van <math>i</math> and/en <math>n</math> A</p> <p>✓ SF CA</p> <p>✓ 89 781,15 CA</p> <p style="text-align: right;">(5)</p>
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[15]

5.1.1	R11 000	✓ new value/ nuwe waarde A (1)	
5.1.2	<p><math>A = R5\,500</math> <math>P = R11\,000</math> <math>n = 5</math> <math>i = ?</math></p> <p><math>A = P(1-in)</math></p> <p><math>R5\,500 = R11\,000(1-i \times (5))</math></p> $i = \frac{\frac{R5\,500}{R11\,000} - 1}{-5}$ <p><math>= 0,10</math></p> <p><math>= 10\%</math></p>	<p>✓ SF CA</p> <p>✓ making <math>i</math> the subject/ maak <math>i</math> die onderwerp CA</p> <p>✓ interest rate/ rentekoers CA</p> <p style="border: 1px solid black; padding: 2px;">AO: Full marks/Volpunte</p> <p style="text-align: right;">(3)</p>	
5.2	<p><math>A = P(1+i)^n</math></p> <p># <math>273 = 200(1 + 3,5\%)^n</math></p> $\frac{273}{200} = (1,035)^n$ $n = \frac{\log \frac{273}{200}}{\log (1,035)} \text{ OR / OF } n = \log_{1,035} \frac{273}{200}$ <p><math>\approx 9,04 \text{ years/ jaar}</math></p> <p><math>\therefore \text{ the year 2018 / die jaar 2018}</math></p> <p style="text-align: center;"><b>OR/OF</b></p> <p><math>A_1 = R200(1 + 3,5\%)^1 \approx R207</math></p> <p><math>A_2 = R200(1 + 3,5\%)^2 \approx R214,25</math></p> <p>.</p> <p><math>A_9 = R200(1 + 3,5\%)^9 \approx R272,58</math></p> <p><math>A_{10} = R200(1 + 3,5\%)^{10} \approx R282,12</math></p> <p><math>\therefore \text{ the year 2018 / die jaar 2018}</math></p>	<p>✓ SF A</p> <p>✓ S CA</p> <p>✓ log property/ eienskap CA</p> <p>✓ value of/waarde van <math>n</math> CA</p> <p>✓ correct year/ korrekte jaar CA</p> <p style="text-align: center;"><b>OR/OF</b></p> <p>✓ SF A</p> <p>✓ S CA</p> <p>✓ correct year/ korrekte jaar CA</p> <p style="border: 1px solid black; padding: 2px;">NOTE/ NOTA: Accept/Aanvaar 2019</p> <p style="border: 1px solid black; padding: 2px;">NPR</p> <p style="text-align: right;">(5)</p>	

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5.3	Value of the investment after initial deposit/ waarde van die belegging na die aanvangs deposito:		
#	$A = P(1+i)^n$ $= R293\,000 \left(1 + \frac{6,7\%}{4}\right)^{2 \times 4}$ $\approx R334\,642,4791$	✓SF ✓S	A CA
	$R334\,642,4791 + R95\,000 = R429\,642,4791$	✓S	CA
	Value of the investment after adding/waarde van die belegging na byvoeging van R95 000 :		
	$= 429\,642,4791 \left(1 + \frac{6,7\%}{4}\right)^{2 \times 4}$ $\approx R490\,705,2026$	✓SF ✓S	A CA
	Value of the investment after change in interest rate/waarde van die belegging na verandering van rentekoers :		
	$= R490\,705,2026 \left(1 + \frac{7,5\%}{12}\right)^{4 \times 12}$ $\approx R661\,764,62 > R660\,580$	✓ value of $i$ and $n$ / waarde van $i$ en $n$ ✓S	A CA
	∴ Yes she has accumulated sufficient funds./Ja, sy het genoeg fondse geakkumuleer	✓ conclusion/ gevolgtrekking	CA
	OR/OF	OR/OF	
	$A = \left[ 293\,000 \left(1 + \frac{6,7\%}{4}\right)^{2 \times 4} + 95\,000 \right] \times \left(1 + \frac{6,7\%}{4}\right)^{2 \times 4} \left(1 + \frac{7,5\%}{12}\right)^{4 \times 12}$ $\approx R661\,764,62 > R660\,580$	✓ M ✓✓SF ✓ value of $i$ and $n$ / waarde van $i$ en $n$ ✓ adding/byvoeging R95 000 ✓ change of rate/ verandering van koers ✓S	A A A A CA
	∴ Yes she has accumulated sufficient funds./Ja, sy het genoeg fondse geakkumuleer	✓ conclusion/ gevolgtrekking	CA
	OR/OF	OR/OF	

$A = R293\,000 \left(1 + \frac{6,7\%}{4}\right)^{4 \times 4} \cdot \left(1 + \frac{7,5\%}{12}\right)^{4 \times 12}$ $+ R95\,000 \left(1 + \frac{6,7\%}{4}\right)^{2 \times 4} \left(1 + \frac{7,5\%}{12}\right)^{4 \times 12}$	✓ M ✓✓SF ✓ value of $i$ and $n$ / waarde van $i$ en $n$ ✓ adding/byvoeging R95 000 with interests/met rente	A A A A
$\approx R661\,764,62 > R660\,580$ ∴ Yes she has accumulated sufficient funds./Ja, sy het genoeg fondse geakkumuleer	✓ change of rate/ verandering van koers ✓S ✓ conclusion/ gevolgtrekking	A CA CA
OR/OF	OR/OF	
$P = R660\,580 \left(1 + \frac{6,7\%}{4}\right)^{-4 \times 2} \cdot \left(1 + \frac{7,5\%}{12}\right)^{-4 \times 12}$ $- R95\,000 \left(1 + \frac{6,7\%}{4}\right)^{-2 \times 4} \left(1 + \frac{7,5\%}{12}\right)^{-4 \times 12}$	✓ M ✓✓SF ✓ value of $i$ and $n$ / waarde van $i$ en $n$ ✓ subtracting R95 000 with interests ✓ change of rate/ verandering van koers ✓S ✓ conclusion/ gevolgtrekking	A A A A A CA CA
$\approx R282\,547,91 < R293\,000$ Yes she has accumulated sufficient funds./Ja, sy het genoeg fondse geakkumuleer	OR/OF	
$1 + i_{eff} = \left(1 + \frac{i_{nom}}{m}\right)^m = \left(1 + \frac{6,7\%}{4}\right)^4 \approx 0,068...$		
2 years/jaar: $A = P(1+i)^n = R293\,000(1+0,068...) + R95\,000$ $\approx R429\,642,48$	✓SF ✓S	A CA
4th year/jaar: $A = R429\,642,48(1+0,068...) \approx R490\,705,20$	✓SF ✓S	CA CA
5 – 8 years/jaar: $A = R490\,705,20 \left(1 + \frac{7,5\%}{12}\right)^{4 \times 12} \approx R661\,764,62 > R660\,580$	✓S	CA
∴ Yes she has accumulated sufficient funds./Ja, sy het genoeg fondse geakkumuleer	✓ value of $i$ and $n$ / waarde van $i$ en $n$ ✓S	A CA
OR/OF	OR/OF	



	$A = P(1+i)^n$ $A_0 = R293\,000 \left(1 + \frac{0,067}{4}\right)^{4 \times 4}$ $\approx R382\,203,3749$ $A_1 = R382\,203,3749 \left(1 + \frac{0,075}{12}\right)^{4 \times 12}$ $\approx R515\,439,147$ $A_2 = R95\,000 \left(1 + \frac{0,067}{4}\right)^{2 \times 4}$ $\approx R108\,501,8277$ $A_3 = R108\,501,8277 \left(1 + \frac{0,075}{12}\right)^{4 \times 12}$ $\approx R146\,325,4728$ $A_1 + A_3 \approx R661\,764,62 > R660\,580$ $\therefore$ Yes she has accumulated sufficient funds./ <i>Ja, sy het genoeg fondse geakkumuleer</i>	✓ conclusion/ <i>gevolgtrekking CA</i> OR/OF  ✓ SF            A ✓ S              CA ✓ SF            A ✓ S              CA ✓ S              CA ✓ value of <i>i</i> and <i>n</i> / <i>waarde van i en n</i> A ✓ S              CA	
	NOTE/NOTA: 1) Max. 4 marks if Simple Interest is used./ <i>Maks 4 punte indien enkelvoudige rente</i> 2) Max. 2 marks if any Depreciation is used./ <i>Maks 2 punte indien enkelvoudige rente</i>		
			(8) [17]

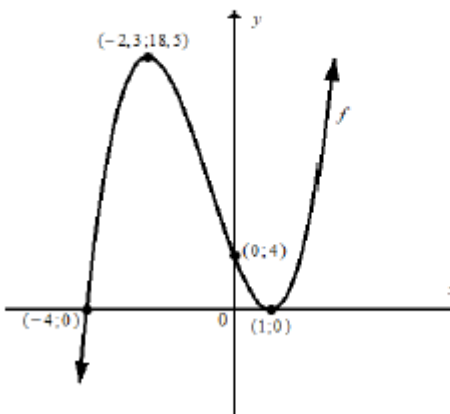
## DIFFERENTIAL CALCULUS

## EXEMPLAR 2018

## QUESTION 6

6.1	$f(x) = 2x^2 - 3$ Average gradient = $\frac{f(x_2) - f(x_1)}{x_2 - x_1}$ $= \frac{[2(1)^2 - 3] - [2(-2)^2 - 3]}{1 - (-2)}$ $= \frac{-1 - 5}{3}$ $= -2$	✓ Corresponding y-value ✓ Corresponding y-value ✓ Substitution in formula ✓ Simplification (4)
6.2	$f(x) = 4 - 3x$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{[4 - 3(x+h)] - (4 - 3x)}{h}$ $= \lim_{h \rightarrow 0} \frac{4 - 3x - 3h - 4 + 3x}{h}$ $= \lim_{h \rightarrow 0} \frac{-3h}{h}$ $= \lim_{h \rightarrow 0} (-3)$ $= -3$	✓ Definition ✓ Substitution in the definition ✓ Simplification (removing brackets) ✓ Simplification (division) ✓ Simplification (5)
6.3	$y = \frac{2}{x^3} + \sqrt{x}$ $y = 2x^{-3} + x^{\frac{1}{2}}$ $\frac{dy}{dx} = -6x^{-4} + \frac{1}{2}x^{-\frac{1}{2}}$	✓ $2x^{-3}$ ✓ $x^{\frac{1}{2}}$ ✓ $-6x^{-4}$ ✓ $\frac{1}{2}x^{-\frac{1}{2}}$ (4)
6.4	$g(x) = -x^2 - x$ $g(2) = -(2)^2 - 2 = -6$ The point of contact is (2; -6) $g'(x) = -2x - 1$ $\therefore m_{\text{tan}} = g'(2) = -2(2) - 1 = -5$ $y = mx + c$ OR $y - y_1 = m(x - x_1)$ $-6 = -5(2) + c$ OR $y - (-6) = -5(x - 2)$ $c = 4$ OR $y + 6 = -5x + 10$ $\therefore y = -5x + 4$	✓ value of y ✓ $m_{\text{tan}} = -5$ ✓ Correct substitution ✓ Value of c (simplification) ✓ Equation (any form) (5) [18]

## QUESTION 7

7.1	$f(x) = x^3 + 2x^2 - 7x + 4$ $f(1) = (1)^3 + 2(1)^2 - 7(1) + 4$ $\therefore f(1) = 0$ $\therefore x - 1$ is a factor of $f$	✓ Substitution ✓ 0 (2)
7.2	x-intercepts: $f(x) = 0$ $x^3 + 2x^2 - 7x + 4 = 0$ $(x-1)(x^2 + 3x - 4) = 0$ $(x-1)(x-1)(x+4) = 0$ $x = 1$ or $x = -4$	✓ $(x^2 + 3x - 4)$ (quadratic) ✓ $(x-1)(x-1)(x+4)$ (linear) ✓ x-intercepts (3)
7.3	$f(x) = x^3 + 2x^2 - 7x + 4$ $f'(x) = 3x^2 + 4x - 7$ $f'(x) = 0$ $\therefore 3x^2 + 4x - 7 = 0$ $(3x+7)(x-1) = 0$ $\therefore x = -\frac{7}{3}$ or $x = 1$ $(-2, 3; 18,5)$ and $(1;0)$	✓ Derivative ✓ $f'(x) = 0$ ✓ Factorisation ✓ Both values of x ✓ Coordinates of the turnings (5)
7.4		✓ Shape ✓ Intercepts with x-axis ✓ y-intercept ✓ Turning points (4) [14]

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QUESTION/VRAAG 6

QUESTION 8

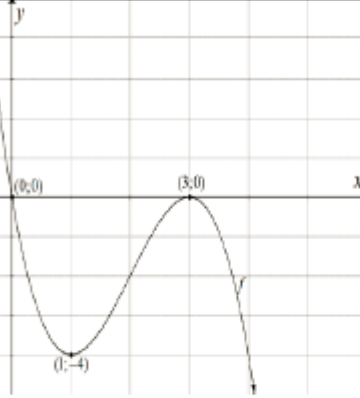
8.1.1	After 2 hrs $D(2) = 4 + 0,5(2)^2 - 0,25(2)^3$ m = 4m	✓ Substituting 2 ✓ Simplification (2)
8.1.2	$D = 4 + 0,5t^2 - 0,25t^3$ $D'(t) = t - 0,75t^2$ At 12:00 (3 hours later): $D'(3) = (3) - 0,75(3)^2$ = -3,75 m.h <sup>-1</sup> ∴	✓ Derivative  ✓ Substitution of 3  ✓ Simplified rate (3)
8.2.1	$P = -3v^2 + 30v$ Neither profit nor loss at $P = 0$ $-3v^2 + 30v = 0$ $-3v(v-10) = 0$ ∴ $v = 0$ or $v = 10$ $v = 10 \text{ km.h}^{-1}$	✓ $P = 0$  ✓ Factors  ✓ Correct value of v (3)
8.2.2	$P = -3v^2 + 30v$ $\frac{dP}{dv} = -6v + 30 = 0$ ∴ $v = 5 \text{ km.h}^{-1}$	✓ Derivative  ✓ Equating to 0  ✓ Value of v (3)
8.2.3	$P_{\max} (\text{in R1000}) = -3(5)^2 + 30(5) = 75$ OR R75 000	✓ Substitution ✓ Profit in R1 000 (2) [13]

6.1	$f(x) = 7x - 2$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{[7(x+h) - 2] - (7x - 2)}{h}$ $= \lim_{h \rightarrow 0} \frac{7x + 7h - 2 - 7x + 2}{h}$ $= \lim_{h \rightarrow 0} \frac{7h}{h}$ $= \lim_{h \rightarrow 0} (7)$ ∴ $f'(x) = 7$	✓ definition/definisie A ✓ SF ✓ S CA ✓ S CA ✓ 7 CA  AO only one mark/slegs een punt  Penalty of one mark incorrect notation  Penaliseer een punt indien notasie foutief is. (5)
Penalty of only one mark only once for incorrect notation (last step) for sub-questions 6.2.1, 6.2.2 & 6.2.3 Penaliseer slegs een punt vir foutiewe notasie (laaste stap) vir subvrae 6.2.1, 6.2.2 & 6.2.3		
6.2.1	$\frac{d}{dx}(\pi^2) = 0$	✓ 0 A (1)
6.2.2	$D_x(x^4 - \sqrt[3]{x})$ $= D_x\left(x^4 - x^{\frac{1}{3}}\right)$ $= 4x^3 - \frac{1}{3}x^{-\frac{2}{3}}$	✓ power form/magvorm ✓ $4x^3$ A ✓ $\frac{1}{3}x^{-\frac{2}{3}}$ CA (3)
6.2.3	$y = \frac{x^5 + 2}{x^2}$ $= x^3 + 2x^{-2}$ $\frac{dy}{dx} = 3x^2 - 4x^{-3}$	✓ $x^3$ A ✓ $2x^{-2}$ A ✓ $3x^2$ CA ✓ $-4x^{-3}$ CA (4)

6.3.1	$p(x) = x^3 + 1$ $p(2) = (2)^3 + 1 = k$ $\therefore k = 9$	✓ S ✓ 9 A (2)
6.3.2	$p(x) = x^3 + 1$ $p'(x) = 3x^2$	✓ derivative/afgeleide A (1)
6.3.3	$p'(x) = 3x^2$ $m = 3(2)^2$ $= 12$ $y - 9 = 12(x - 2)$ OR / OF $9 = 12(2) + c$ $y = 12x - 24 + 9$ $c = -15$ $\therefore y = 12x - 15$ $\therefore y = 12x - 15$ OR/OF $y - 12x + 15 = 0$	✓ gradient/gradient CA ✓ SF CA  ✓ $y = 12x - 15$ CA (3) [19]

QUESTION/VRAAG 7

	<b>Q7.1: Penalty of ONE mark if intercepts not given as coordinates (refer also to Question 4)</b> <b>V7.1: Penaliseer EEN punt indien afsnitte nie as koördinate gegee (verwys ook na Vraag 4)</b>	
7.1	$f(x) = -x(x-3)(x-3)$ $x = 0$ and/en $x = 3$ $(0;0)$ and/en $(3;0)$	✓ $(0;0)$ A ✓ $(3;0)$ A (2)
7.2	$f(x) = -x(x-3)(x-3)$ $y = f(0) = -(0)(0-3)(0-3) = 0$	✓ 0 A Accept/Aanvaar. $(0;0)$ (1)
7.3	$f(x) = -x(x-3)(x-3)$ $= -x(x^2 - 6x + 9)$ OR/OF $= -(x^2 + 3x)(x - 3)$ OR/OF $= (x^2 - 3x)(-x + 3)$ $\therefore f(x) = -x^3 + 6x^2 - 9x$	✓ ✓ $-x(x^2 - 6x + 9)$ M A OR/OF ✓ ✓ $(-x^2 + 3x)(x - 3)$ M A OR/OF ✓ ✓ $(x^2 - 3x)(-x + 3)$ M A (2)
7.4	$f(x) = -x^3 + 6x^2 - 9x$ $f'(x) = -3x^2 + 12x - 9$ $-3x^2 + 12x - 9 = 0$ $x^2 - 4x + 3 = 0$ $(x-3)(x-1) = 0$ $\therefore x = 3$ or/of $x = 1$ $f(1) = -(1)^3 + 6(1)^2 - 9(1) = -4$ $(3;0)$ and/en $(1;-4)$	✓ derivative/afgeleide M ✓ equating derivative to 0/ stel afgeleide gelyk aan 0 M  ✓ factors/formula/faktore CA ✓ both values of /beide waardes van x CA  ✓ both values of /beide waardes van y CA AO: Full marks/Volpunte  Coordinates of one turning point only: two marks/ Koördinate van een draaipunt slegs twee punte (5)

7.5		✓ correct shape/ <i>korrekte vorm</i> A ✓ y-intercept/ <i>afsnit</i> CA ✓ x-intercepts/ <i>afsnitte</i> CA ✓ turning points/ <i>draaipunte</i> CA (4)
7.6	$1 < x < 3$ OR/OF $x \in (1;3)$	✓ critical values/ <i>kritieke waardes</i> : 1 and/en 3 CA ✓ notation/ <i>notasie</i> (2) [16]

QUESTION/VRAAG 8

8.1.1	$V = l \times b \times h$ $= 3x(1,5)(1-x)$	OR/OF $V = 4,5x - 4,5x^2$ ✓ formula/ <i>formule</i> ✓ ✓ SF (3)
8.1.2	$V = 4,5x - 4,5x^2$ $\frac{dV}{dx} = 4,5 - 9x$ $4,5 - 9x = 0$ $9x = 4,5$ $\therefore x = 0,5$  OR/OF $x = -\frac{b}{2a}$ $= -\frac{4,5}{2(-4,5)}$ $= 0,5$	CA from Question/ Vraag 8.1.1 ✓ derivative/ <i>afgeleide</i> M ✓ equating to 0/ <i>stel gelyk aan 0</i> M ✓ value of/ <i>waarde van</i> x CA  OR/OF  ✓ using a formula/ <i>gebruik 'n formule</i> ✓ S ✓ value of/ <i>waarde van</i> x CA (3)
8.2.1	$v(0) = 8 + 4(0) - (0)^2 \text{ m/s} = 8 \text{ m/s}$ $\therefore$ the initial velocity of the car/ die aanvanklike snelheid van die motor 8 m/s	✓ 8 m/s A NPU (1)
8.2.2	$v(t) = 8 + 4t - t^2$ $v(0,2) = 8 + 4(0,2) - (0,2)^2 \text{ m/s}$ $= 8,76 \text{ m/s}$ $\therefore$ the velocity of the car when 0,2 seconds will be 8,76 m/s die snelheid van die motor wanneer 0,2 sekondes sal 8,76 m/s wees	✓ S ✓ 8,76 m/s A NPU (2)
8.2.3	$v(t) = 8 + 4t - t^2$ $v'(t) = 4 - 2t$ $v'(1,2) = 4 - 2(1,2) \text{ m/s}^2$ $= 1,6 \text{ m/s}^2$	✓ 4 A ✓ $-2t$ A ✓ SF into a derivative/ <i>in 'n afgeleide</i> CA  ✓ $1,6 \text{ m/s}^2$ CA NPU (4) [13]

NOV 2019

## QUESTION/PRAAG 6

6.1	$f(x) = 5 - \frac{1}{2}x$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{\left[5 - \frac{1}{2}(x+h)\right] - \left[5 - \frac{1}{2}x\right]}{h}$ $= \lim_{h \rightarrow 0} \frac{5 - \frac{1}{2}x - \frac{1}{2}h - 5 + \frac{1}{2}x}{h}$ $= \lim_{h \rightarrow 0} \frac{-\frac{1}{2}h}{h}$ $= \lim_{h \rightarrow 0} \left(-\frac{1}{2}\right)$ $\therefore f'(x) = -\frac{1}{2}$	✓ definition/definisie A ✓ SF A ✓ S CA ✓ $\lim_{h \rightarrow 0} \frac{-\frac{1}{2}h}{h}$ CA ✓ $-\frac{1}{2}$ CA AO: ONLY 1 mark/ SLEGS 1 punt Penalty of 1 mark for incorrect notation/ penaliseer 1 punte vir verkeerde notasie (5)
No penalty for notation in the remaining questions/ Geen penaliseering vir notasie in die		
6.2.1	$f(x) = a^3 - 0,5x^2 - x^{-1}$ $f'(x) = 0 - 1,5x^2 + x^{-2}$ OR / OF $-1,5x^2 + x^{-2}$	✓ derivative of/ afgeleide van $a^3$ A ✓ $-1,5x^2$ A ✓ $x^{-2}$ A (3)
6.2.2	$D_x \left[ x(\sqrt{x} + 2) \right]$ $= D_x \left[ x \left( x^{\frac{1}{2}} + 2 \right) \right]$ $= D_x \left[ x^{\frac{3}{2}} + 2x \right]$ $= \frac{3}{2}x^{\frac{1}{2}} + 2$	✓ $x^{\frac{1}{2}}$ A ✓ product/produk CA ✓ $\frac{3}{2}x^{\frac{1}{2}}$ CA ✓ 2 CA (4)

6.3.1	$xy + 2x^3 = 7x^6$ $y = \frac{7x^6 - 2x^3}{x}$ OR / OF $y = 7x^5 - 2x^2$	✓ M A ✓ making y the subject/maak y die onderwerp A (2)
6.3.2	$y = 7x^5 - 2x^2$ $\frac{dy}{dx} = 35x^4 - 4x$	✓ $35x^4$ CA ✓ $-4x$ CA (2)
6.4.1	$P(300) = 0,8(300)^2 - 200(300)$ $= R12\ 000$	✓ profit/wins A (1)
6.4.2	$P(x) = 0,8x^2 - 200x$ $0,8x^2 - 200x = 0$ $x(0,8x - 200) = 0$ $x = 0$ or $x = 250$ $\therefore 250$ light bulbs/gloeilampe OR/OF $0,8x^2 - 200x = 0$ $0,8x^2 = 200x$ $x = \frac{200x}{0,8x}$ ; $x \neq 0$ $\therefore x = 250$ $\therefore 250$ light bulbs/gloeilampe	✓ factors/faktore M A ✓ correct value of/korrekte waarde van x CA OR/OF ✓ isolating/ isoleer x M A ✓ correct value of/korrekte waarde van x CA AO: Full marks/Volpunte (2)
6.4.3	$P(x) = 0,8x^2 - 200x$ $P'(x) = 1,6x - 200$ $P'(200) = 1,6(200) - 200$ $= 120$ bulbs per day / gloeilampe per dag	✓ derivative/afgeleide A ✓ substitution/vervanging CA ✓ rate/tempo CA NPU (3) [22]

## QUESTION/VRAAG 7

7.1	$g(x) = 9x + 18$ x-intercept/afsnit, $g(x) = 0$ $0 = 9x + 18$ $-9x = 18$ $x = -2$ $Q(-2; 0)$ y-intercept/afsnit, $x = 0$ $T(0; 18)$  <b>OR / OF</b> $(x+2)(x-3)(x-3) = 0$ $Q(-2; 0)$ y-intercept/afsnit, $x = 0$ $T(0; 18)$	$\checkmark 0 = 9x + 18$ A  $\checkmark$ coordinates of/koördinate van Q A $\checkmark$ coordinates of/koördinate van T A  <b>OR/OF</b> $\checkmark M$ A $\checkmark$ coordinates of/koördinate van Q A  $\checkmark$ coordinates of/koördinate van T A (3) <b>AO: Full marks/Volpunte</b>	
7.2	$f(x) = (x+2)(x-3)(x-3)$ $= (x+2)(x^2 - 6x + 9)$ <b>OR/OF</b> $(x-3)(x^2 - x - 6)$ $f(x) = x^3 - 4x^2 - 3x + 18$ $b = -4$ $c = -3$ $d = 18$  <b>OR/OF</b>  $f(x) = x^3 + bx^2 + cx + d$ $f(x) = x^3 + bx^2 + cx + 18$ $f(-2) = 0: 2b - c = -5 \dots (1)$ $f(3) = 0: 3b + c = -15 \dots (2)$ $(1) + (2): 5b = -20$ $\therefore b = -4$ $2(-4) - c = -5$ $c = -3$	$\checkmark$ repeated factor/herhalde faktor A $\checkmark$ quadratic factor/kwadratische faktor A $\checkmark$ expanding/uitbrei A  <b>OR/OF</b>  $\checkmark d = 18$ A $\checkmark S$ A $\checkmark S$ A	
<b>OR/OF</b>		<b>OR/OF</b>	

	$f(x) = x^3 + bx^2 + cx + d$ $f(x) = x^3 + bx^2 + cx + 18$ $0 = (3)^3 + b(3)^2 + c(3) + 18$ $0 = 45 + 9b + 3c$ $c = -15 - 3b$ $f'(x) = 3x^2 + 2bx + c$ $f'(3) = 3(3)^2 + 2b(3) + c$ $c = -27 - 6b$ $-15 - 3b = -27 - 6b$ $\therefore b = -4$ $c = -27 - 6(-4)$ $\therefore c = -3$	$\checkmark d = 18$ A  $\checkmark S$ A  $\checkmark S$ A  (3)	
7.3	$f(x) = x^3 - 4x^2 - 3x + 18$ $f'(x) = 3x^2 - 8x - 3 = 0$ $3x^2 - 8x - 3 = 0$ $(3x+1)(x-3) = 0$ <b>OR/OF</b> $x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(-3)}}{2(3)}$ $x = -\frac{1}{3}$ or/af $x = 3$ $y = f\left(-\frac{1}{3}\right) = \left(-\frac{1}{3}\right)^3 - 4\left(-\frac{1}{3}\right)^2 - 3\left(-\frac{1}{3}\right) + 18 = \frac{500}{27}$ $\therefore R\left(-\frac{1}{3}; \frac{500}{27}\right)$ <b>OR/OF</b> $R(-0,33; 18,52)$	$\checkmark f'(x)$ CA $\checkmark f'(x) = 0$ CA $\checkmark$ factors/formula/faktore/form CA $\checkmark$ correct value of/korrekte waarde van $x$ CA $\checkmark R$ coordinates/koördinate CA (5)	
7.4.1	$y = \frac{500}{27}$ <b>OR/OF</b> $y \approx 18,52$	$\checkmark$ equation/vergelijking CA (1)	
7.4.2	$x > -2$ <b>OR/OF</b> $x \in (-2; \infty)$	$\checkmark$ correct inequality/korrekte ongelykheid A (1)	
7.4.3	$-\frac{1}{3} < x < 3$ <b>OR/OF</b> $x \in \left(-\frac{1}{3}; 3\right)$	$\checkmark$ critical values/kritiese waardes CA $\checkmark$ correct notation/korrekte notasie CA (2)	[15]

## QUESTION/VRAAG 8

8.1	$h = (66 - 2x - x) \text{ cm}$ $= (66 - 3x) \text{ cm}$	✓ height/hoogte A (1) <div style="border: 1px solid black; padding: 2px; display: inline-block;">NPU</div>	
8.2	$V(\text{in cm}^3) = \pi r^2 h + \frac{1}{2} \left( \frac{4}{3} \pi r^3 \right)$ $= \pi x^2 (66 - 3x) + \frac{1}{2} \left( \frac{4}{3} \pi x^3 \right)$ $= 66\pi x^2 - 3\pi x^3 + \frac{2}{3} \pi x^3$ $= 66\pi x^2 - \frac{7}{3} \pi x^3$	✓ F A  ✓ SF CA ✓ S CA <div style="border: 1px solid black; padding: 2px; display: inline-block;">NPU</div> (3)	
8.3	$V(\text{in cm}^3) = 66\pi x^2 - \frac{7}{3} \pi x^3$ $\frac{dV}{dx} = -7\pi x^2 + 132\pi x$ $0 = -7\pi x^2 + 132\pi x$ $0 = -\pi x(7x - 132) \text{ OR / OF } x = \frac{-(132\pi) \pm \sqrt{(132\pi)^2 - 4(-7\pi)(0)}}{2(-7\pi)}$ $-\pi x = 0 \text{ or/of } x = \frac{132}{7}$ $x \neq 0$ $\therefore x = \frac{132}{7}$	✓ derivative/afgeleide CA ✓ equating $f'(x)$ to/stel gelyk aan 0 A ✓ factors/ formula faktore/form CA  ✓ correct value of/ korrekte waarde van x CA (4)	
8.4	$V(\text{in cm}^3) = 66\pi x^2 - \frac{7}{3} \pi x^3$ $= 66\pi \left( \frac{132}{7} \right)^2 - \frac{7}{3} \pi \left( \frac{132}{7} \right)^3$ $\approx 24\,576,74 \text{ OR/OF } 7823,02 \pi$	✓ SF CA ✓ max/maks volume CA <div style="border: 1px solid black; padding: 2px; display: inline-block;">NPR</div> (2) [10]	



# INTEGRATION EXEMPLAR 2018

## QUESTION 9

9.1	$\int \left( x^{-4} + \frac{7}{x} - 1 \right) dx$ $= \int x^{-4} dx + 7 \int \frac{1}{x} dx - \int dx$ $= \frac{x^{-3}}{-3} + 7 \ln x - x + C$	$\checkmark \frac{x^{-3}}{-3}$ $\checkmark 7 \ln x$ $\checkmark -x$ $\checkmark C$	(4)
9.2	$h(x) = -2x^2 - 6x$ $\int_{-3}^0 (-2x^2 - 6x) dx$ $= \left[ -\frac{2x^3}{3} - 3x^2 \right]_{-3}^0$ $= \left[ \left( -\frac{2(0)^3}{3} - 3(0)^2 \right) - \left( -\frac{2(-3)^3}{3} - 3(-3)^2 \right) \right]$ $= -18 + 27$ $= 9 \text{ units square}$	$\checkmark -\frac{2x^3}{3}; \quad \checkmark -3x^2$ $\checkmark \text{Substituting } 0$ $\checkmark \text{Substituting } -3$ $\checkmark \text{Simplification}$	(5) [9]

## NOV 2018

## QUESTION/VRAAG 9

	Penalty of one mark once only if the constant in Questions 9.1.1 & 9.1.2 is omitted/ <i>Penalisering met slegs een punt, indien konstante in Vrae 9.1.1 &amp; 9.1.2 weggelaat is.</i>		
9.1.1	$\int -\frac{6}{x} dx$ $= -6 \int \frac{1}{x} dx$ $= -6 \ln x + C \quad \text{OR} \quad \ln \frac{1}{x^6} + C \quad \text{OR} \quad -6 \log_e x + C$	$\checkmark -6 \ln x$ OR/OF $\ln \frac{1}{x^6}$ OR/OF $-6 \log_e x$  $\checkmark C$	(2)
9.1.2	$\int (x-1)^2 dx$ $= \int (x^2 - 2x + 1) dx$ $= \frac{x^3}{3} - x^2 + x + C$	$\checkmark$ product / product M $\checkmark \frac{x^3}{3}$ CA $\checkmark -x^2$ CA $\checkmark x + C$ CA	(4)
9.2		$\checkmark$ Area notation using integrals/ <i>Area-notasie met gebruik van integrale</i> M  $\checkmark \frac{x^3}{3}$ A $\checkmark 3x$ A $\checkmark \checkmark$ SF CA  $\checkmark$ bounded area/ <i>begrensde oppervakke</i> CA NPU <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> AO: 3 marks/punte </div>	(6) [12]

NOV 2019

QUESTION/VRAG 9

9.1.1	$\int m x^p dx$ $= m \left( \frac{x^{p+1}}{p+1} \right) + C \quad p \neq -1$  OR/OF $= \frac{mx^{p+1}}{p+1} + C \quad p \neq -1$	$\checkmark m \left( \frac{x^{p+1}}{p+1} \right)$ $\checkmark C$  OR/OF $\checkmark \frac{mx^{p+1}}{p+1}$ $\checkmark C$	A A  A A (2)
9.1.2	$\int \left( \frac{x^{-3} + x^2}{x^{-1}} - 2 \right) dx$ $= \int (x^{-2} + x^3 - 2x^0) dx$ $= -x^{-1} + \frac{x^4}{4} - 2x^1 + C$	$\checkmark S$ $\checkmark -1x^{-1}$ $\checkmark \frac{x^4}{4}$ $\checkmark -2x^1 + C$	A CA CA CA (4)

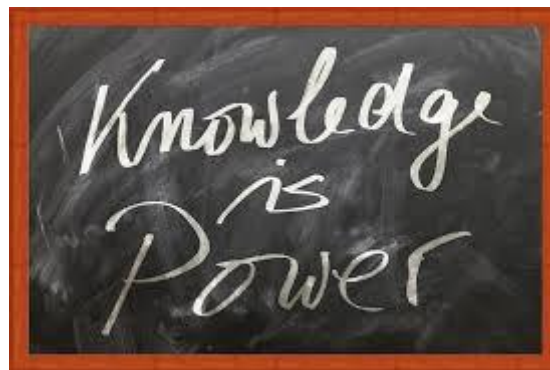
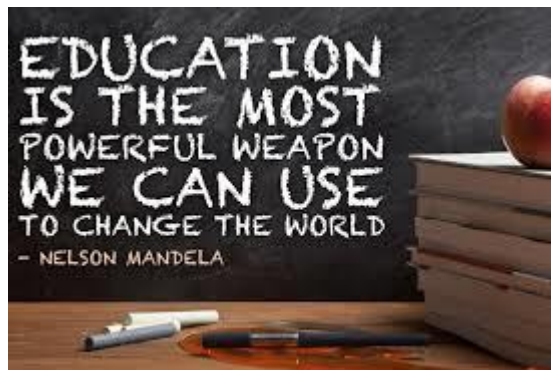
9.2	$A = \int_{1,4}^{3,5} \left( -\frac{4}{x} \right) dx$ $= [-4 \ln x]_{1,4}^{3,5}$ $= [-4 \ln(3,5)] - [-4 \ln(1,4)]$ $\approx -3,67$ $\therefore 3,67$ square units <i>vierkante eenhede</i>	$\checkmark$ area notation using integral/ <i>oppervlakte notasie deur gebruik van integrasie</i> A $\checkmark$ integral/integraal A $\checkmark$ substitution/vervanging CA $\checkmark$ substitution/vervanging CA $\checkmark S$ CA $\checkmark$ correct positive value of the bounded area/ <i>korrekte positiewe waarde van die begrensde oppervlakte</i> CA
	OR/OF $A = - \int_{1,4}^{3,5} \left( -\frac{4}{x} \right) dx$ $= -[-4 \ln x]_{1,4}^{3,5}$ $= [4 \ln(3,5)] - [4 \ln(1,4)]$ $\approx 3,67$ $\therefore 3,67$ square units/ <i>vierkante eenhede</i>	$\checkmark$ area notation using integral/ <i>oppervlakte notasie deur gebruik van integrasie</i> A $\checkmark$ integral/integraal A $\checkmark$ substitution/vervanging CA $\checkmark$ substitution/vervanging CA $\checkmark S$ CA $\checkmark$ correct positive value of the bounded area/ <i>korrekte positiewe waarde van die begrensde oppervlakte</i> CA $\checkmark$ correct positive value of the
	OR/OF $A = \int_{1,4}^{3,5} \left[ 0 - \left( -\frac{4}{x} \right) \right] dx$ $= \int_{1,4}^{3,5} 0 dx + \int_{1,4}^{3,5} \left( \frac{4}{x} \right) dx$ $= [4 \ln x]_{1,4}^{3,5}$ $= [4 \ln(3,5)] - [4 \ln(1,4)]$ $\approx 3,67$ $\therefore 3,67$ square units/ <i>vierkante eenhede</i>	$\checkmark$ area notation using integral/ <i>oppervlakte notasie deur gebruik van integrasie</i> A $\checkmark$ integral/integraal A $\checkmark$ substitution/vervanging CA $\checkmark$ substitution/vervanging CA $\checkmark S$ CA $\checkmark$ correct positive value of the bounded area/ <i>korrekte positiewe waarde van die begrensde oppervlakte</i> CA
		<div style="border: 1px solid black; padding: 2px;">NPR &amp; NPU</div> <div style="text-align: right;">(6) [12]</div>





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