TECHNICAL SCIENCES
WORKSHEET FOR TERM 2 WORK 2020
ELASTICITY (New topic in Grade 12)
ELASTICITY OVERVIEW OF CONCEPTS

<u>Define the following concepts</u>

Deforming force • Restoring force • Elasticity • Perfectly elastic body • Perfectly plastic body • Elastic limit • Stress • Strain and Hooke's law.

<u>Calculate and be able to use formulae given for the following concepts</u>

- **Stress**
- **Strain**
- **Hooke' Law**

Some definitions

A deforming force is a force that changes the **shape and size** of the object

A restoring force develops inside the body and tries to bring back the original **shape and size** of the object

Elasticity of a body is the property of the body by virtue of which the body regains its original **shape and size** when the deforming force is removed.

The internal restoring **force per unit area** of body is called stress.

Strain is the **ratio** of change in dimension to the original dimension.

Hooke's law states that, within the limit of elasticity, stress is directly proportional to strain.
Young's modulus: is a measure of the forces between the particles of the material. These forces resist a tensile force.

A spring constant is a measure of the force exerted by a spring as it tries to return to its equilibrium position.

The restoring force can be calculated using **:**

$$\vec{F} = -k\vec{x}$$

During deformation, two forces act on an object simultaneously – deforming force and restoring force

Deforming force and restoring force have equal magnitudes and opposite direction.

Deforming force is directly proportional to the restoring force

Perfectly plastic body:
A body which does not show a tendency to regain its original shape and size when the deforming force is removed is a perfectly plastic body

1. <u>Stress</u>

To calculate stress in a material, we use the equation:

$$\boldsymbol{\sigma = \frac{F}{A}}$$

SI unit of stress is the ***Pascal***

1 ***Pa*** = 1 ***N.m***$^{-2}$

Examples

A 15 m marble column of cross sectional area 3 m^2 support a mass of 15 000 kg. The column measures 14,7 m after the load is applied. Calculate the stress within the column.

$$\boldsymbol{\sigma} = \frac{F}{A}$$

$$F = mg$$

$$F = 15\,000 \times 9{,}8$$

F = 147 000 N

$$\sigma = \frac{147000}{3}$$

σ = 49 000 Pa

σ = 49 000 $N \cdot m^{-2}$

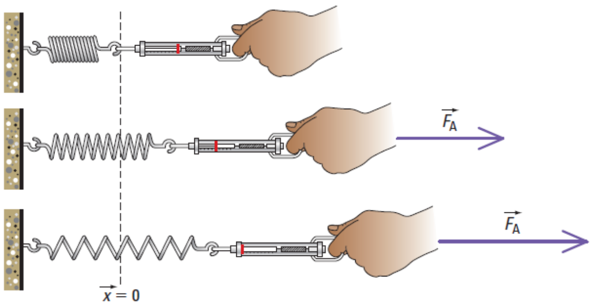
2. Strain

$$\boldsymbol{Strain = \frac{change\ in\ length}{original\ length}}$$

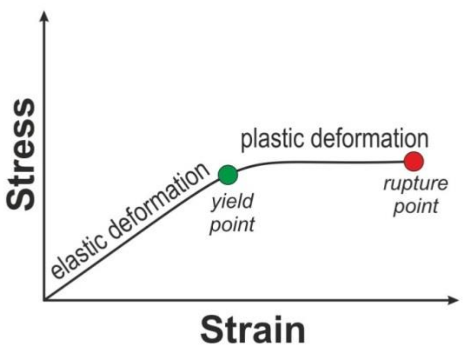
The equation

$$\boldsymbol{\varepsilon = \frac{\Delta l}{L_o}}$$, is used to calculate strain

strain has ***no units***, since it is a ratio of two same dimensions.



The graph below shows the relationship that exists between the restoring force of an object that is subjected to some degree of deformation.



The demonstration of a ruler can be used to explain the following observations:

- As the size of the deforming force increases, so does the size of the force that opposes deformation (stress).
- It is exactly when the ruler experiences elastic deformation.
- As the magnitude of the deforming force reaches the yield point, the size of the restoring force remains more or less constant until the ruler snaps (rapture point)
- Beyond the yield point, it is important to note that the ruler is now experiencing an irreversible from of deformation (plastic deformation).

3. Hooke's law page 44 CAPS Doc

Hooke's law states that, within the limit of elasticity, stress is directly proportional to strain. Stress and is given by

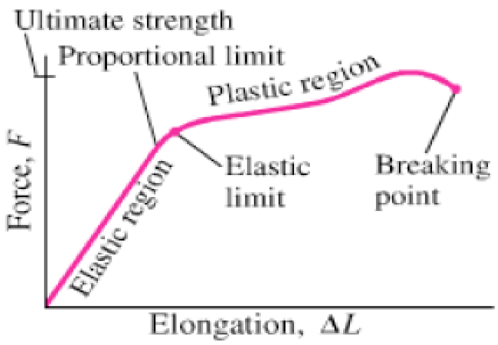
$$F = k\Delta L$$

Rearranging this to:

$$\Delta L = \frac{F}{K}$$

makes it clear that the ***deformation (ΔL) is proportional to the applied force (F)***. Hooke's law is applicable to most solids from iron to bone but is only valid up to a certain point.

The graph below shows a region where Hooke's law is obeyed and where it is not obeyed:



Youngs modulus, Y $= \dfrac{\text{tensile stress}}{\text{tensile strain}}$ represented mathematically as

$K = \frac{\sigma}{\varepsilon}$ **OR** $\gamma = \frac{\sigma}{\varepsilon}$

Unit: 1 ***N·m⁻²*** or 1 ***Pa***

Examples

A vertical steel beam in a building support a load of $6{,}0 \times 10^4 kg$. The length of the beam is 4,0 m and its cross-sectional area is $8{,}0 \times 10^{-3}\, m^2$. The elastic (young's) modulus is $2{,}0 \times 10^{11}\, Pa$ and the ultimate stress of steel is $5{,}0 \times 10^8\, Pa$.

1.1 Calculate the amount of compression in the beam.

1.2 Find the maximum load the beam can support.

Solution

1.1

$$\sigma = Y\varepsilon$$

$$\frac{F}{A} = Y\frac{\Delta l}{L_o}$$

$$\frac{6{,}0\times10^4}{8{,}0\times10^{-3}} = 2{,}0 \times 10^{11} \left(\frac{\Delta l}{4{,}0}\right)$$

$\Delta l = 1{,}5 \times 10^{-4}$ m

1.2 Set the compressive stress equal to the ultimate compressive strength.

$$\frac{F}{A} = \frac{F}{8.0 \times 10^{-3}} = 5.0 \times 10^{8}.$$
$$F = 4{,}0 \times 10^{6} \text{ N}$$

Example on Hooke's law

A 1,5 m long steel piano wire has a diameter of 0,2 cm. It is stretched 0,2 m when tightened.
1.1 State Hooke's law in words.
1.2 Calculate the tension in the wire**.**

Solutions

1.1 Hooke's law states that, within a limit of elasticity, stress is directly proportional to strain.

1.2

$$radius = \frac{diameter}{2}$$
$$radius = \frac{0{,}2 \times 10^{-2}}{2}$$
$$= 0{,}001\ m$$
$$A = \pi r^2$$
$$A = 3{,}14(0{,}001)^2$$
$$A = 3{,}1 \times 10^{-6}\ m^2$$
$$\sigma = Y\varepsilon$$

$\frac{F}{A} = Y\frac{\Delta l}{L_o}$

$\frac{F}{3.1\times10^{-6}} = 2.0 \times 10^{11} \left(\frac{0{,}2}{1.5}\right)$

$F = 8{,}27 \times 10^{4} N$

LEARNER ACTIVITY

Question 1: MULTIPLE CHOICE

1.1. Which one of the following is the correct regarding a deforming force?

A Changes the volume of object.

B Changes the mass of the object.

C Changes the size and shape of the object.

D Size and shape remain constant. **(2)**

1.2 Which one of the following is an example of a perfectly plastic body?

A Aluminium can

B Foam rubber

C Glass

D Tennis **(2)**

1.3 The SI unit of stress is equivalent to

A 1 $kg.m.s^{-1}$

B 1 $N.m^2$

C 1 J.s

D 1 W **(2)**

1.4 A force of 2 kN is exerted on a cubicle object, the stress experienced by the object is 500Pa. The area on which the force acts is:

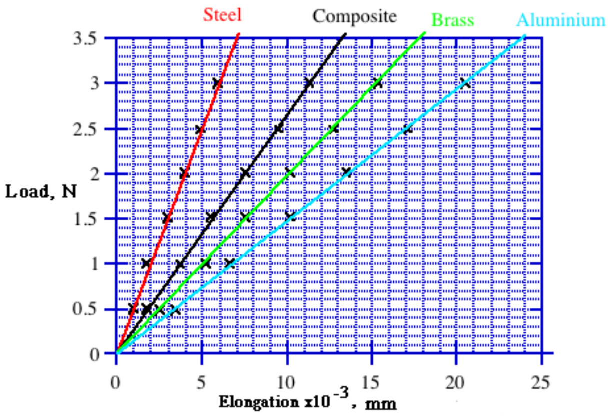
A 340 m^2

B 3,4 m^2

C 0,29 m^2

D 29 m^2 **(2)**

1.5 Consider the graph below showing the stress-strain for different materials:



Which one of the following materials has the greatest modulus of elasticity?

A Aluminium

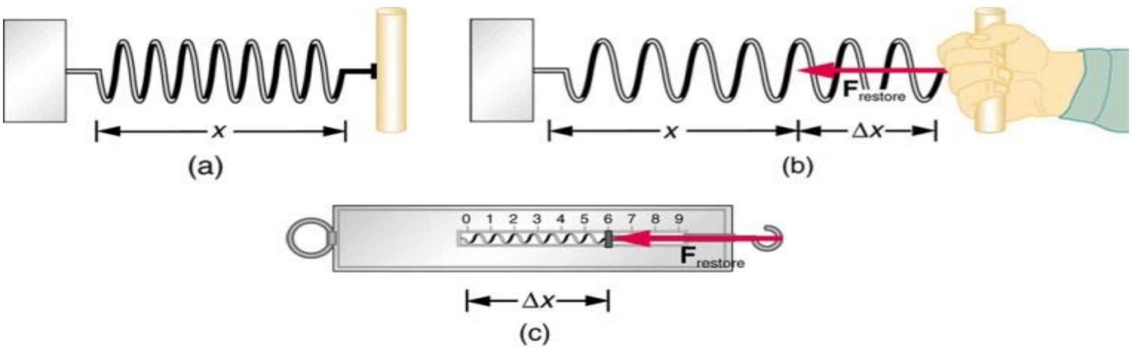
B Brass

C Composite

D Steel **(2)**

Question 2

Consider the situation below wherein a learner applies a force on a spring. The force meter measures the force applied



2.1 Define following terms:

2.1.1 Deforming force **(2)**

2.1.2 Restoring force **(2)**

2.2 What is the reading on the force meter which measures the applied force? **(1)**

2.3 How does the magnitude and direction of the applied force compare to that of the restoring force? Write down only LESS THAN, EQUAL TO or GREATER THAN. Explain briefly **(3)**

2.4 **2.4.1** In another demonstration, a learner applies a deforming force on a string with a constant of 40 $N.m^{-1}$ for a distance of 20 cm.

Calculate the magnitude of the restoring force? **(3)**

2.4.2 If the force applied to deform the spring is doubled, what will be the new restoring force. **(2)**
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Question 3

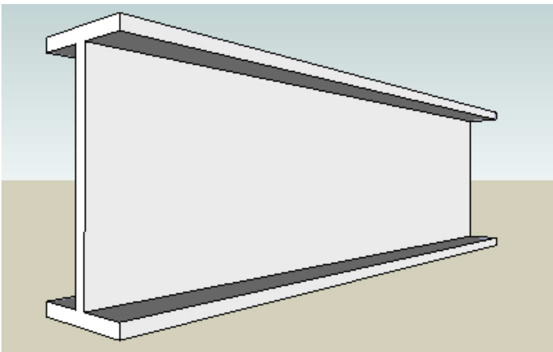
A metal strip has a diameter of 25 cm and is 2m long. A force of 12 N is applied on the circular cross-sectional area.

3.1 Assume that the material is elastic. Calculate the stress within the metal strip. **(3)**

3.2 A rectangular concrete beam carries a load of mass 400kg. The rectangular cross-sectional area of the beam has dimensions, 1,5 m x 2,5 m. Assume that the concrete is elastic. Determine the magnitude of stress in the beam. **(4)**

3.3 How much force would create a stress of 750 Pa on a cubic rubber material with a length of 200 mm? **(4)**

3.4 In most real-life applications, beams are usually I-shaped.



Give **TWO** reasons for this. **(2)**
[13]

Question 4

A metal strip has a diameter of 40mm and is 3 m long. A constant force acts on the strip and stretches it by 5mm.

4.1 Determine the strain in the strip **(3)**

4.2 A steel bar is 10 mm in diameter and 2 m long. It is stretched by a force of 20 kN and extends to 2,03 m. Calculate the strain of the bar **(3)**

4.3 A rod is 0,5 m long and 5 mm in diameter. The rod is stretched 0,06 mm by a force of 3 kN. Calculate the strain. **(3)**
[9]

Question 5

5.1 What is the magnitude of required to stretch a 20 cm long spring with a spring constant of 100 N/m to a length of 21 cm? **(3)**

5.2 What is the spring constant of a spring that needs a force of 3 N to be compressed from 40 cm to 35 cm? **(3)**

5.3 A spring stretches 5 cm when a load of 20 N is hung on it. If instead, we put a load of 30 N, how much will the spring stretch? What is the spring constant? **(3)**

5.4 A spring has a spring constant that is equal to 3.5. What force (in kilograms) will make it stretch 4 cm? **(4)**

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Question 6

A steel wire 10m long and 2mm in diameter is attached to the ceiling and a 200N weight is attached to its end.

6.1 What is the magnitude of the applied stress? **(3)**

6.2 It is noted that the wire stretches 3,08 mm due to the 200N load. Calculate the strain on the wire. **(3)**

6.3 The elastic limit of the steel wire is 2,48 x 10^8 Pa. Determine the maximum weight that can be supported without exceeding the elastic limit of the wire. **(3)**

6.4 Calculate the modulus of elasticity of the wire **(3)**

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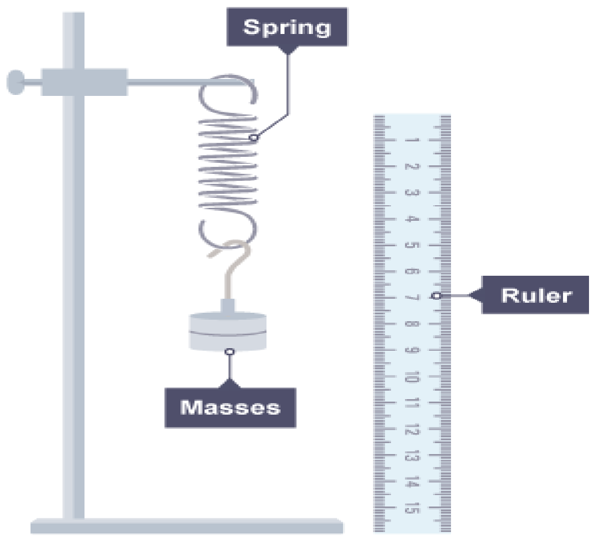
Question 7

The diagram below shows a setup of apparatus done by the teacher.



Different masses are suspended, and the corresponding elongations are obtained.

Table of results

Mass (g)	0	200	400	600	800	1000	1200
X (cm)	0	1	2	3	4	5	8

W (N)							

7.1 Copy and complete the table of results. **(7)**

7.2 What NAME is given to the force causing the spring to elongate? **(1)**

7.3 Using a graph paper, plot a graph of weight (N) versus elongation (cm). **(6)**

7.4 Briefly explain why the pattern changes from the fifth reading **(2)**
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