Province of the
EASTERN CAPE
EDUCATION

Developing Fluency in Multiplication and Division in the Foundation Phase

July 2013

FOREWORD

The Foundation Phase Curriculum directorate has developed the following documents to strengthen and support the planning, teaching and assessment of mathematics in the classroom:

- Teaching number concepts and numeration skills with the use of Flard Cards (2010).
- Development of a strong sense of number and the role of problems in teaching mathematics (2011).
- Developing fluency in Multiplication and Division in the Foundation Phase (2013).

The results of the 2012 ANA results indicate that learners generally have performed poorly in basic number operations which has the most content as compared with the other content areas and the heaviest weighting (Grade 1 : 65%, Grade 2: 60% and Grade 3: 58% in Mathematics. In view of this we have developed a document to support teachers in the teaching of multiplication and division in the Foundation Phase. It is hoped that this document will provide clarity to teachers on teaching the basic number operations, focusing on multiplication and division in a meaningful and fun filled–manner so as to strengthen the numeracy skills of every learner in the classroom.

It is in this phase that the foundational skills are taught and if the skills are firmly entrenched then the learners will be able to make decisions and solve more complex problems. The activities in the document are adapted to suit the developmental needs of all learners.

It is hoped that both SMTs and teachers will benefit from using this resource material which addresses the pertinent aspects that poses a challenge to teachers in teaching and assessing multiplication and division in the Foundation Phase.

Dr T Reddy

CES: ECD/Foundation Phase Curriculum

CONTENT PAGE

INTRODUCTION:

Performance by South African children in both regional (SACMEQ) and international (TIMSS) studies suggests that they are not thinking mathematically. All children must learn to think mathematically and to learn mathematics.

An analysis of the 2012 ANA revealed that learners achieved at 37%. Many learners still struggle with understanding the basic Number Operations. Multiplication and division has been identified as the challenging area in mathematics. South African learners performed poorly in these areas.

It is critical for children to be able to perform the basic operations, know their basic number facts and perform mental mathematics with confidence. Children who leave the Foundation Phase with a poorly developed sense of number are almost certainly unable to ever make sense of mathematics.

Learners need to know, understand and learn mathematics. It should be taught in flexible and meaningful ways. Learners must engage with it and use it with confidence to make sense of the world – to solve problems, or else they will demonstrate their inability to do the basic operations of subtraction, multiplication and division that involve whole numbers.

SECTION 1

Developmental trajectories for multiplication and division appropriate in the Foundation Phase

In the lower grades, learners have many opportunities to use mental mathematical strategies for addition such as, using doubles and using addition to subtract. This knowledge will provide a fundamental basis on which to build, when multiplication is introduced. In Grade 3, the emphasis is on beginning to build learners' conceptual understanding of the multiplication operation.

Learners should focus on the meanings of, and relationship between, multiplication and division. Learners should think about multiplication numerically as repeated addition of the same quantities or equal groups, and geometrically as rows and columns in rectangular arrays. Likewise, learners should think about division numerically as repeated subtraction, equal sharing, and equal grouping. The table below indicates progression on multiplication:

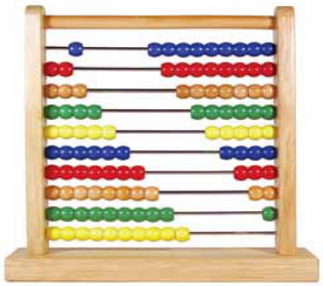
Grade R	Grade 1	Grade 2	Grade 3
	Can perform calculations, using appropriate symbols, to solve problems involving: • repeated addition with whole numbers and with solutions to at least 34 • estimation	Can perform calculations, using appropriate symbols, to solve problems involving: • multiplication of whole 1-digit by 1-digit numbers with solutions to at least 50 • estimation	Can perform calculations, using appropriate symbols, to solve problems involving: • multiplication of at least whole 2-digit by 1-digit numbers • division of at least whole 2- digit by 1-digit numbers • estimation
		Performs mental calculations involving: • multiplication of whole numbers with solutions to at least 20	Performs mental calculations involving: • multiplication of whole numbers • with solutions to at least 50

Uses the following techniques: • building up and breaking down numbers to at least 10 • doubling and halving to at least 10 • using concrete • apparatus e.g. counters	Uses the following techniques: • building up and breaking down numbers • doubling and halving • using concrete apparatus e.g. counters • number-lines	Uses the following techniques: • building up and breaking down numbers • doubling and halving • using concrete • apparatus e.g. • counters • number-lines	Uses the following techniques: • building up and breaking down numbers • doubling and halving • number-lines • rounding off in tens
Explains own solutions to problems	Explains own solutions to problems	Explains own solutions to problems	Explains own solutions to problems

It is essential that learners be given experiences where they see how multiplication and division can be used on a daily basis and how it relates to the world around them. Learners need to be able to interpret a variety of language patterns representing multiplication experiences. Sometimes learners learn multiplication facts with little understanding. Initially, learners need to understand that multiplication is the process of counting objects by equal groups rather than as single objects. Help learners recognize equal groups and help them develop the language of multiplication experiences.

Through a variety of teacher-modelled activities, the teacher will demonstrate to learners how multiplication can represent equal groups that can be displayed as 'rows of..', 'stacks of...', 'piles of...', etc. It is important for learners to understand the following meanings of multiplication:

1. Repeated addition
2. Equal groups or sets
3. An Array



Teachers must challenge learners to think of real-life objects that come in equal groups, such as wheels on bicycles/tricycles, legs on chairs, legs on stools, 3-leaf clovers, animal legs, pairs of shoes, cookies on a tray, people's eyes or fingers on hands. Provide a real-life multiplication problem for learners to solve. For example, ask learners to find out how many eyes the children in the class have altogether. A desk has four legs, so how many legs will there be in four desks? When learners have an answer, ask them if there is a faster way to solve the problem besides adding. Record all suggestions and introduce multiplication as a faster way of adding equal groups.

1.2 MATHEMATICS TEACHING

Whole class activities will involve all learners to:

- Improve counting
- Develop number sense
- Play the games

When teaching in Small Groups

- Group according to the abilities
- Cover similar concepts at different levels
- Engage in one-on-one activities

Supervise directly and individuality

How can we make group work effective?

- Plan well
- Be prepared
- Keep sessions interactive
- Use manipulatives
- Encourage learners to explain their strategies

1.3 UNDERSTANDING MULTIPLICATION AND DIVISION

Explain your understanding of multiplication and division.

MULTIPLICATIVE REASONING

- From the literature some researchers have considered **multiplication as repeated addition** (Clark & Kami: 1996).
- **Multiplication is an operation that requires higher-order multiplicative thinking** that children construct out of their ability to think additively
- Multiplication is a more complex operation that is **constructed out of addition at a higher level of abstraction**.
- When a learner has to think multiplicatively they think **simultaneously about units of ones and about units of more than one.**
- In order to carry out any operation there is a need for algorithms: **step-by-step procedures for computation** (Kilpatrick et al, 2001).
- Multiplicative reasoning is the capacity to **work flexibly with concepts, strategies and representation of multiplication.**
- Multiplication differs from addition even if it is considered **as a process of equal grouping it is the union of discrete equivalent sets 7 + 7 + 7 = 21.**

1.4 STRATEGIES ON MULTIPLICATION

What are the multiplication strategies?

Strategies on Multiplication

- Unitising requires the learner to count groups simultaneously.
- Use of one finger to tally each group that have been counted, whereas some showed unitary counting which develops to rhythmic counting in groups and later to number patterns.
- Rhythmic counting occurs when interim numbers are progressively internalized which can be detected by whispering, a silent mouthed acknowledgement which indicates that counting has happened.
- Skip counting,
- Double strategy,
- Distributive property which involves understanding about the structure of the part/ whole relationship realizing that 9 x5 can be solved by adding 5 x5 and 4 x 5. .
- Associative property (2 x 3) x 5 = 2 x (3 x 5).
- Array comes from their understanding of spatial structures, array of squares in a rectangle.

Five levels in Early Multiplication and Division (Wright et al: 2003)

Level 1: Initial Group

A learner uses perceptual thinking to establish numerosity of collections of equal groups when items are visible, and counts by ones not in multiples; in that, the child uses perceptual counting to make groups of specified size from collection of items.

Level 2: Perceptual Counting in Multiples

A learner uses multiplicative counting strategies to count visible items in equal groups that involves in counting in multiples. For counting strategies that include rhythmic, double, skip counting, the child relies on visible items.

Level 3: Figurative Composite Grouping

A learner uses multiplicative counting strategies to count items in equal groups in cases where the individual items are not visible.

Level 4: Repeated Abstract Composite Grouping

A learner counts composite units in repeated addition or subtraction, that is, uses the composite units a specified number of times. The learner is simultaneously aware of both the composite and unitary aspects.

Level 5: Multiplication as Operations

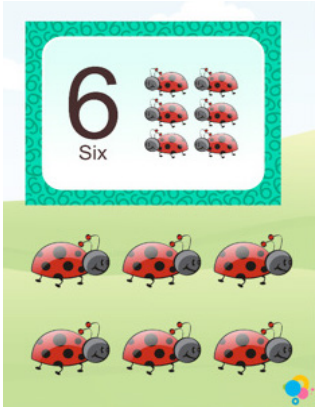
A learner can regard both the number in each group as a composite unit, and can immediately recall many basic facts for multiplication and division. A learner is able to use a known fact to work out an unknown fact, the learner use 3 x 6 = 18 to work out 18 ÷ 3.

SECTION 2

LAYING THE FOUNDATION FOR MULTIPLICATION AND DIVISION

Learners obtain knowledge and understanding of mathematical concepts in different ways and we need to have an understanding of pre-required knowledge in order for them to successfully understand the concepts of multiplication and division. Children obtain knowledge through:

1.1 Different types of Mathematical knowledge



Social Knowledge

- We refer to knowledge that needs to be **told** to learners and remembered by them as **social knowledge**. The only way in which learners can acquire social knowledge is to be informed about it and having been informed about it, they need to remember it. It deals with language of mathematics and how the society uses the language.
- The implication of social knowledge for the numeracy classroom is that teachers have to tell (teach) children this knowledge. For example teachers need to teach learners vocabulary such as:
  - Number names;
  - The names of shapes and objects etc

Physical Knowledge

- Physical knowledge is derived through **touching; playing with and using concrete/ physical material**.
- Children need a lot of **concrete experiences** in the classroom to develop their physical knowledge.
- Children need ample time to explore the physical world. No one can do this for them.
- It is through **counting physical objects** that children:
  - experience that two objects are different to three objects; it takes longer and
  - more actions to count 10 than it does 3 ; and 20 takes a lot more action than counting to 10..
- Five counters can be held in one hand; 10 in two hands; but if you want more groups you need to pack it out - there are too many for our hands. Physical knowledge forms the basis for logical-mathematical thinking.

- Learners obtain this knowledge by learning through free play

Conceptual knowledge - (also referred to as logico mathematical knowledge)

- Conceptual knowledge is **internal knowledge**. This is developed or constructed within the mind of the learner for themselves.
- It cannot be told
- Physical knowledge, work with objects, forms the basis for logical-mathematical knowledge. e.g. "different", "similar", "the same in weight", and 'the numerosity of 'two'
- The teacher's role in the development of children's conceptual knowledge is two-fold.
    - Learners reflect on what he or she has learnt
    - Learner use the conceptual knowledge to solve problems

The teacher:

- Is responsible for creating activities for children that will reveal the underlying structures of numbers, operations, and mathematical relationships
- Needs to actively encourage children to reflect on what they are doing and what they are thinking.
- Must help children to verbalise their observations so that they can explain these to the other children as well as learn to interpret the explanations of the other children

Before multiplication and division are thus taught formally, children should be familiar with the following experiences and skills. These skills are transferred from a very early age.

Children should:

1. Be able to count securely;
2. Understand basic addition and subtraction;
3. Be able to form groupings of the same size
    - without 'remainders'
    - with 'remainders'.

The key challenge of the Foundation Phase classroom is to assist children to develop a strong sense of number.

A child's sense of number develops through three levels with the levels following one after the other:

Level 1: Counting all

- Children at Level 1 are **counting all**.
- When we ask a child who is operating at Level 1 to add two numbers then they will first recreate each number using fingers or other representations.
- As the numbers get larger and the child can no longer rely on the fingers of their two hands to create the numbers, they will use either objects such as counters or bottle tops or they will reconstruct the number on paper by drawing stripes or circles.

Level 2: Counting on

- Children at Level 2 are counting on. When we ask a child who is operating at Level 2 to add/subtract two numbers then the child is able to conceptualise at least one of the numbers without having to recreate it, and recreates only the other number.
- As the numbers get larger and the child can no longer rely on the fingers of their hand to create the number(s) they will also resort to using objects or drawings.

Level 3: Breaking down and building up of numbers

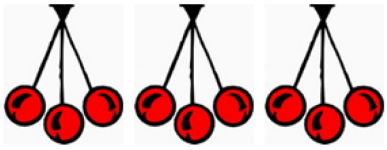
- Children at Level 3 are able to work with numbers in flexible ways often breaking numbers down (decomposing), reorganising.
- Children at Level 3 are said to have a "numerosity" of the numbers with which they are working, that is, they have a sense of the "muchness" of the numbers (pointing): and can think of those numbers in a large range of different ways.

Skip counting

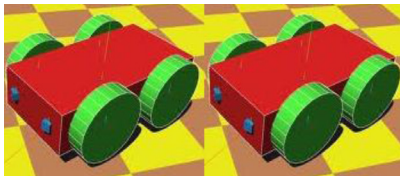
Skip counting is counting by numbers other than one. Counting by 2s, 3s, 4s, 5s, 10s etc is skip counting. Skip counting not only helps children see patterns in numbers, but also plays a big role in setting the stage for learning multiplication facts. In Grade R learners begin to count in:

Counting in 2s. How many socks?

Counting in 3s .How many fruits?



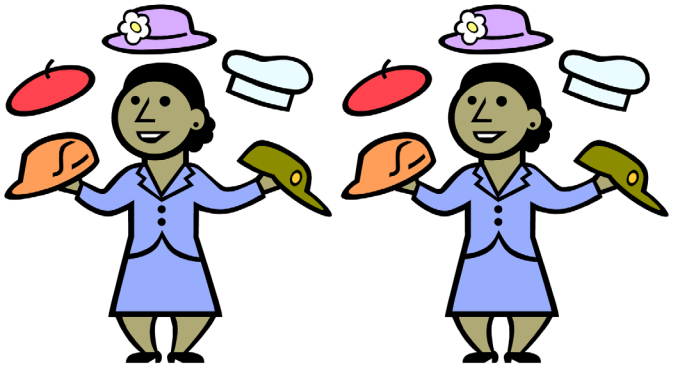
Counting in 4s. How many wheels?



Counting in 5s. How many fingers?

Once children are confident about counting in multiples of small numbers, **repeated addition** can be introduced as an extension of standard addition using the multiple-type counting as a repeated addition strategy.

How many hats are there?



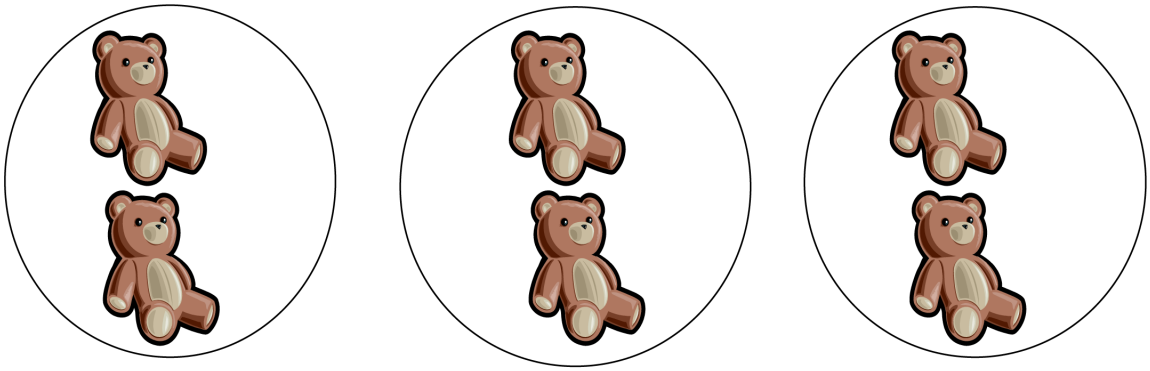
5 + 5 = 10 or 2 x 5 = 10

Notice how the level of abstraction gradually increases from using real objects (all of which can be counted) to using abstract numbers (with intervening numbers absent). Recording at these early stages is initially largely either practical or pictorial and then a mixture of pictorial/practical and repeated addition equations (which may be scribed by the teacher to begin with). Most teachers delay the introduction of the standard x symbol for multiplication until repeated addition has been firmly established.

1. Repeated Addition

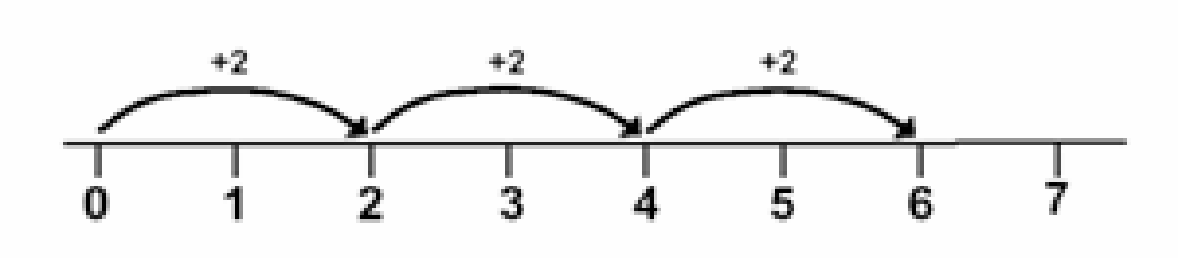
This is the first multiplication structure to which children should be introduced. It builds upon the already established understanding children have about addition but extends this from adding the contents of a grouping to adding the contents of one group and then using this to add the contents of several equally-sized groups. For understanding this multiplication structure, prior experience of equal groupings and of addition is important.

With objects: 2 plus 2 plus 2 gives me 6



2 +2 + 2 = 3 x 2 (2 multiplied 3 times)

On a number line



2 + 2 + 2 = 3 x 2 (2 multiplied 3 times)

SECTION 3

DEVELOPING MULTIPLICATION FLUENCY

The selection of the activities by the teacher will be determined by the developmental needs and ability of the learners in the class. These activities accommodate learners who experience barriers and those who learn at a faster pace.

SUGGESTED RESOURCES

- *Manipulatives*
- *Arrays*
- *Containers*
- *Paper plates*
- *Worksheets*

COUNTING ACTIVITIES

ACTIVITY 1 [10 minutes]

The teacher counts aloud with the learner from

1 to 30

[You may start from any number]

- The teacher will say 1 and the learner will say 2 and follow the pattern 3, 4.
- The teacher count very softly and the learner loudly.
- The teacher nod and the learner say the number.

[1_____ 3, _____5, _____9.....]

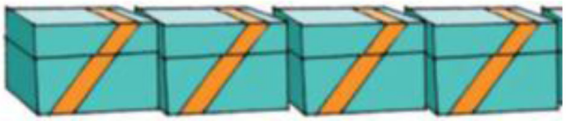
Use different number ranges to reinforce the patterns

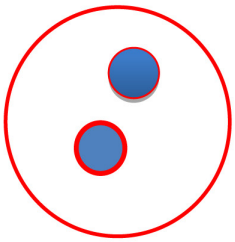
ACTIVITY 2 [20 minutes]

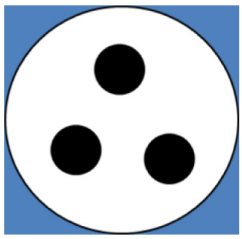
Resources: Dot cards/ Counters

Counting in groups of 2/ 3/ 4/ 5

The teacher will use 2- dot cards/ manipulative to show the learners.







- The teacher will show the learners 2- dot cards. They both count on in two's from 2 to 30 and keep on removing the cards one at a time
- The learners count in two's without the cards forward and backwards

  NB: The dot cards show each group as a unit

The teacher can use three's, four's, five's, etc. increasing the number range

ACTIVITY 3 [10 minutes]

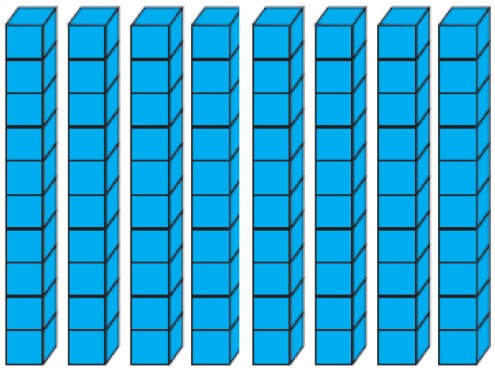
Resources: Counters

Counting in groups of 10 using counters

The teacher uses a group of 10 counters one at a time.

The learners then understand that each group of 10 is a unit

- The learners count forward in 10 as the teacher puts the bundles (10, 20, …,100)
- The learners count backwards in tens as the teacher remove the bundles
- They count without the bundles



SECTION 4

4.1 DEVELOPING MULTIPLICATION AND DIVISION

NB:The resources for dot cards and arrays are attached to this document (ANNEXURE 2) for the teacher to use them when engaging the learners with these activities.

1. Equal Groups

1.1 **ACTIVITY**: Making equal groups [30 minutes]

Individual or group work

> **Focus: Assist children in forming equal groups and to focus on the number of groups, the number of items in each group**

Learners are given 15 counters and are asked to make three groups with four in each group.

How many groups? / How many in each group?

1.2 ACTIVITY: Describing equal groups [30 minutes]

Counters/ unifix cubes/ containers

Individuals

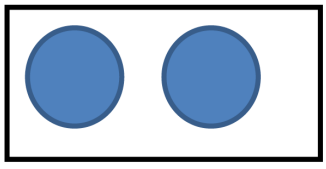
Each learner is given 10 counters and 2 containers, ask them to share it equally into the containers, how many counters in each containers?

> **Encourage children to use systematic sharing strategy and encourage the learners not to count by ones.**

Use the same activity but increase the number range

1.3 Combining and counting equal groups

Place out ten 2 – dot cards, put each 2- dot card and let the learner count putting each card after the other.



Similar with 3 dots, 4 dots, 5 dots

1.4 Determining the number in an equal share

Resources: Using dots cards/ unifix cubes

Here are 6 counters. Share them amongst three people.

How many will each get?

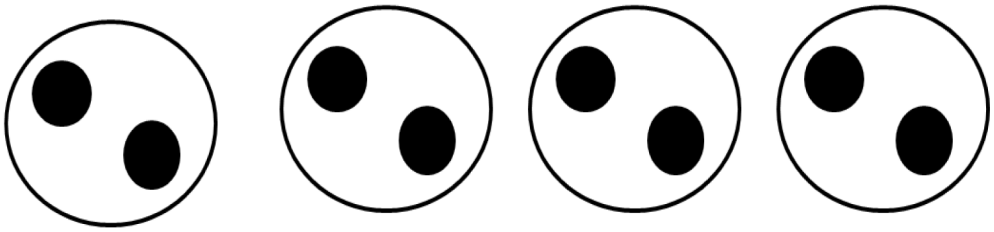
Use 10 and 2, 12 and 6, 18 and 3

1.5 Determining the number of equal groups

Resources: Using dots cards/ unifix cubes

The key focus is developing learner's initial ideas, and it is important to observe carefully learner's action, language and ways of reasoning. The learners have to realize that each group has the same number of items.

Place out four 2-counter cards:



How many counters are there on each card?

How many cards are there?

How many counters are there altogether?

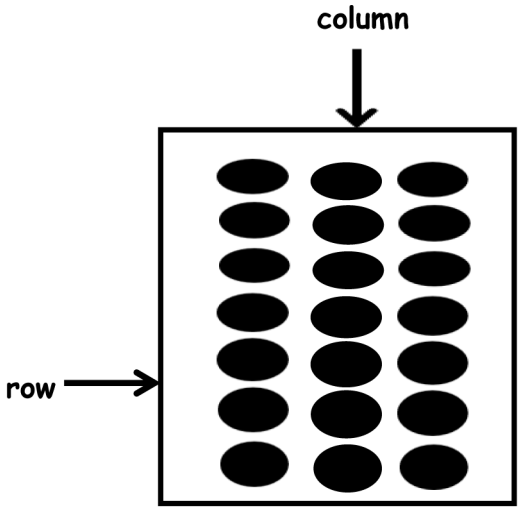
1.6 Describing visible arrays

Resources: Arrays

Arrays

Arrays provide children with a visual image of multiplication, depending on the arrangement of the array. The calculation can be read differently.

Explain the array to the learners that it has rows and column. Show the learners the row and columns.



How many rows?

How many dots are altogether?

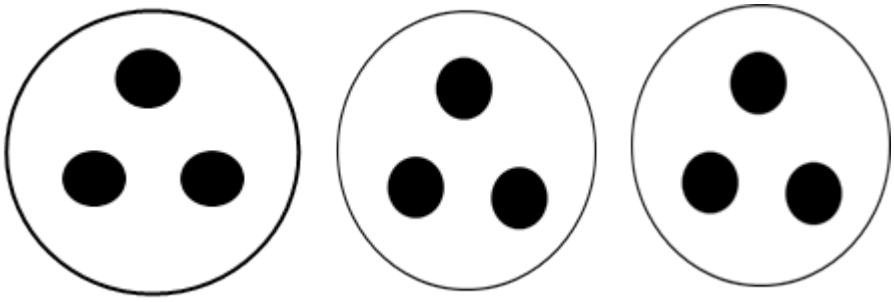
Use the same activity with different arrays

Check what strategies the learners are using. Some learners will count in ones, but in multiplication the learners must count in groups. Ensure that the learners count in groups

1.7 Developing counting in groups of 3's/ 4's/ 5's using screen items

ACTIVITY : [15 minutes] Resources: *2 dots / 4 dots/ 5 dots cards*

- Place out a plate containing three dots in it. Tell the learners that one plate has three dots, place another plate then and ask the learners how many dots are there altogether in two plates. Put more plates under a screen and ask how many dots are altogether.

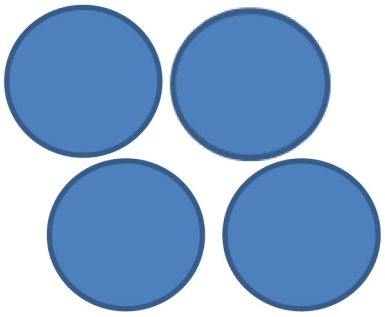


Ensure that the learners don't count in ones.

1.8 USE OF DOT CARDS

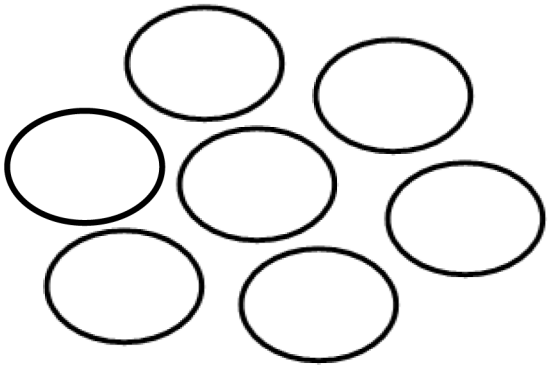
ACTIVITY [20 minutes] Resources: dot cards

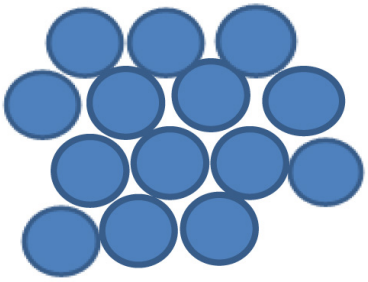
There are 5 dot cards, under each card there are 4 dots how many dots are there altogether?



Use the same activity but increase the number of dots under each card.

The teacher tells the learners that there are 7 cards with 14 dots altogether. Ask how many dots in each card? Learners will not be allowed to touch the dots.

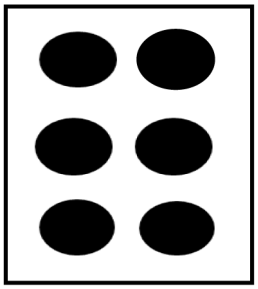




Use the same activity but increase the number range on the dot card

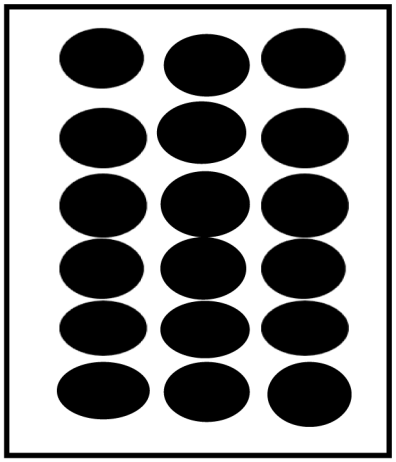
1.9 Resources: Arrays

Display the first row for a second while other rows are covered. Let the learners look at the first row and then show the others for another second or two, ask learners as to how many dots are altogether.

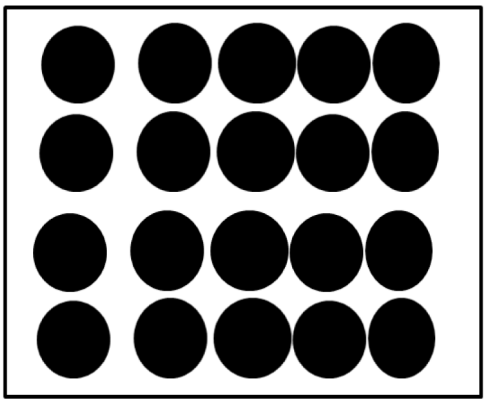


Use the same activity with different arrays

• The teacher uncovers the first row and covers the rest and tells the learners that there are 6 rows altogether, how many dots are there altogether.



Use the same activity with different arrays

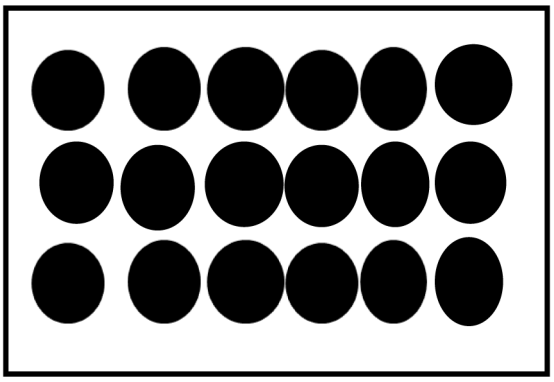


Place a 4 x 5 array and cover one row,

- How many 20 dots altogether?
- How many rows are there?
- How many columns?

The teacher turns the array at 90°

- How many 20 dots altogether?
- How many rows are there?
- How many columns?

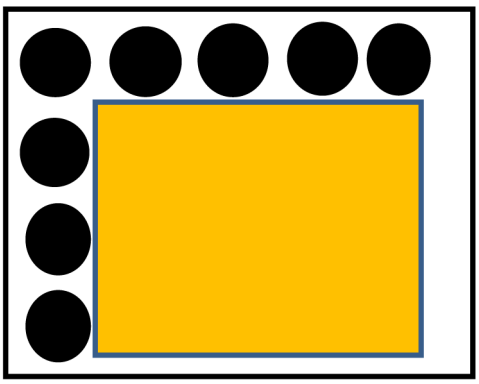


Use the same activity with different arrays

There are 18 dots altogether and there are 3 rows, how many dots in a row.

Use the same activity but increase the number range on the dot cards.

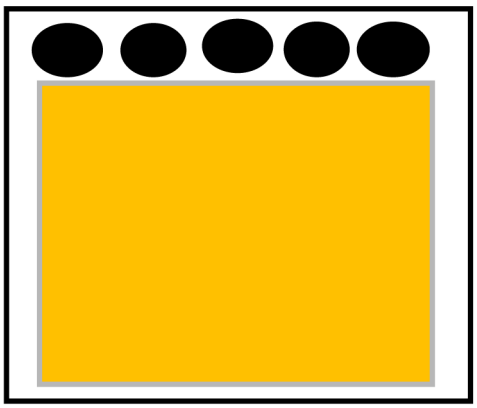
1.10 USE OF A COVERED ARRAY



- How many dots are there altogether?
- Explain how did you get the answer?

Use the same activity but increase the number range.

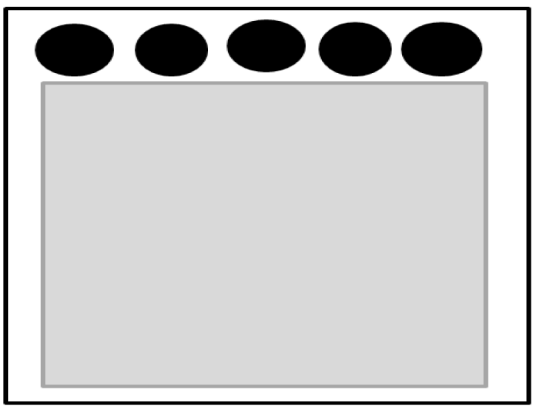
Use the array below to answer the following question



If there are six rows with five dots in each, how many dots are altogether?

There are…….dots altogether.

Now do the same with the following:

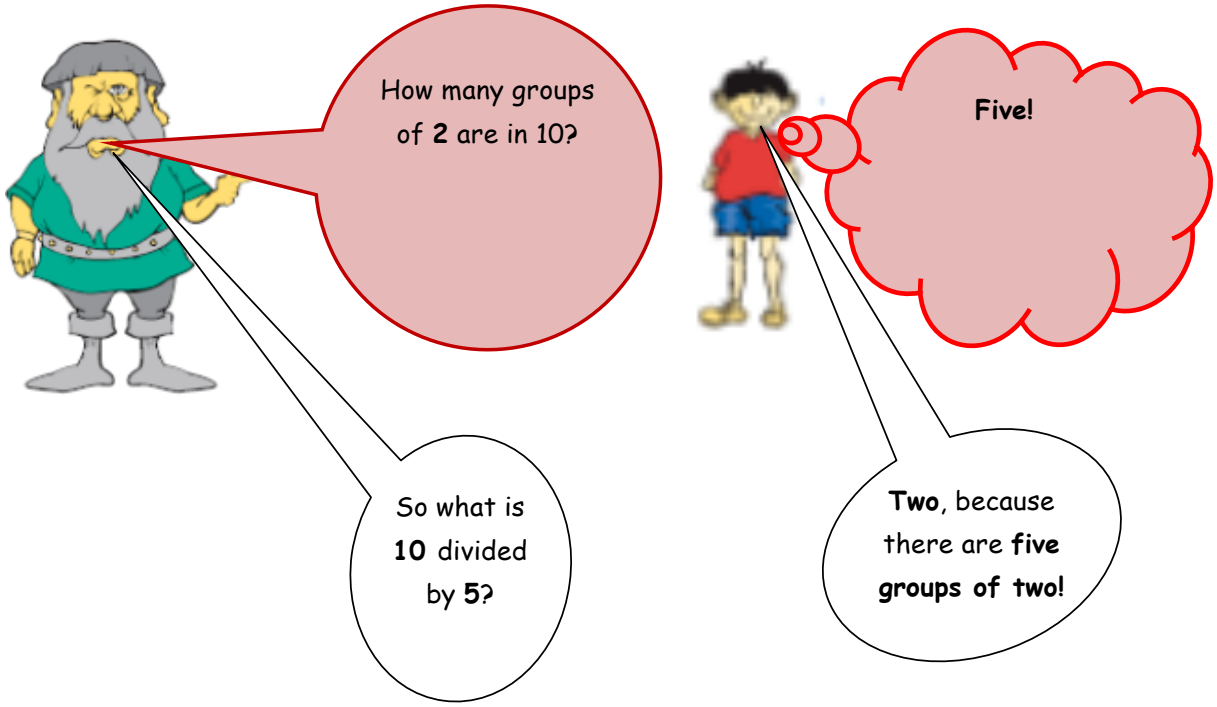


There are 15 dots altogether. Each row has 5 dots. How many rows are there?

There are…….. rows

4.2 GROUPING AND DIVIDING

When we divide we separate into group. **[5 minutes]**



1. How many groups of 3 are in 12? ..... 12 ÷ 3 =.....
2. How many groups of 4 are in 16? ..... 16 ÷ 4 = ...
3. How many groups of 1 are in 15? ..... 15 ÷ 1 =.....
4. How many groups of 4 are in 12? ..... 12 ÷ 4 =.....

Give learners the counters to check the solutions.

SECTION 5

5.1 SHARING AND GROUPING PROBLEMS

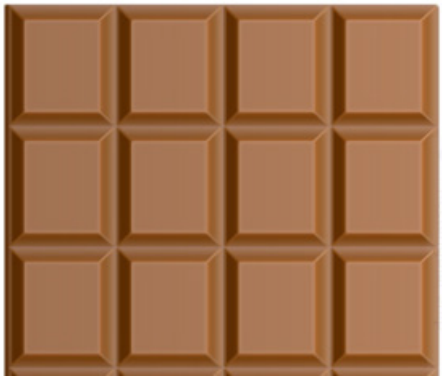
Children in the Foundation Phase find sharing and grouping problems, a lot more accessible than problems that can be solved through addition and/or subtraction – this is in part because sharing is a natural part of their day to day experiences and also because sharing and grouping problems give a clearer more obvious sense of what the child needs to do in order to solve the problem than addition and subtraction problems do. In other words children find it easier to "make a plan" for solving sharing and grouping problems.

When solving the problems learners should be encouraged to draw the diagrams and explain how they get the answer.

- All references to sharing and grouping refer to remainders right from Grade R.

This is, once again, deliberate. Unless children experience sharing/grouping problems with remainders from an early stage they will make an incorrect assumption that **"if there is a remainder then it is not a division problem"**.

- In Grades 2 and 3, the remainder is in turn also shared among the people doing the sharing and this leads to the introduction of fractions in the Foundation Phase.



Grade 2 examples of problems that deal with remainders:

3 people are sharing 4 rows of a slab of chocolate between themselves. Each person gets one row of chocolate and there is one row left over.

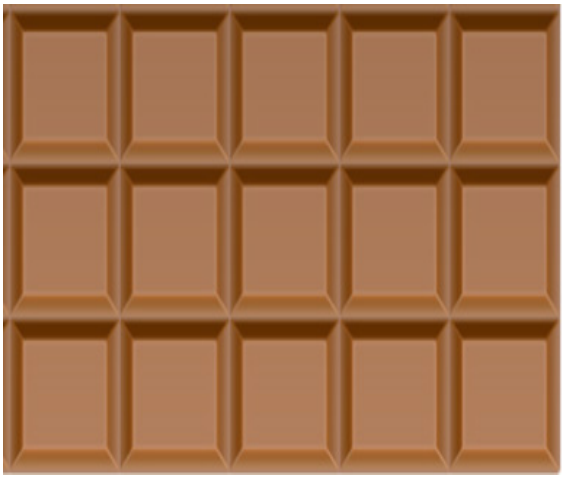
To share this row among the three people, it must be cut into three equal pieces: each piece is called one-third (a unitary fraction).

Grade 3 examples of problems that deal with remainders.

3 people are sharing a slab of chocolate with 5 rows between themselves. Each person again gets one row of chocolate and there are two rows left over.

To share these remaining rows among the three people, each row must be cut into three equal pieces. The remaining 6 pieces are shared amongst the 3 people and each person gets two pieces – they get two one-thirds (a non-unitary fraction).

Other than the addition of the words multiplication in Grade 2 and division in Grade 3 there is not a lot that is different between the Problem Types cluster from one Grade to the next. This is largely because the progression from one Grade to the next lies in both the increasing number range and the expectation that children will be working at more sophisticated levels of number development and using more efficient solution strategies.



5.2 PROBLEM SOLVING ACTIVITIES

ACTIVITY 1: [5 minutes]

Mazuki shares 12 cookies amongst 2 children, one will child gets 6 cookies.

Is Mazuki correct? Yes, because 12 ÷ 2 = 6

1. Share 10 sweets among 5 children.

   How many will each get? ……., 10 ÷ 5 = ……

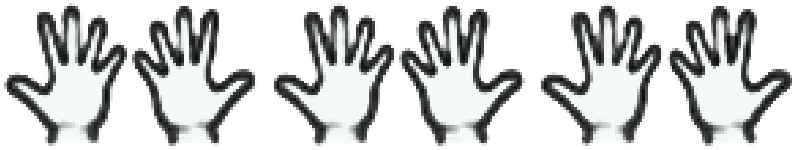
2. Share 20 apples among 4 children.

   How many will each get? ……., 20 ÷ 4 = …….

3. If you give 8 sweets to 1 child.

   How many for 1 child? …… ….. So, 8 ÷ 1 = …….

ACTIVITY 2:



How many hands? _____ How many fingers? _____
Write your answer in 2 ways.

ACTIVITY 3: [10 minutes]

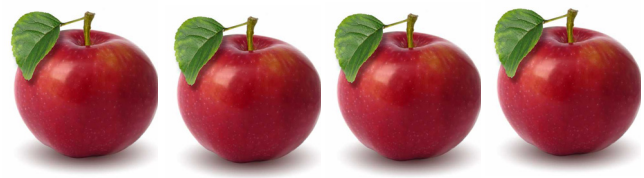
a) How many pots in a row? _____
b) How many legs in a row? _____
c) How many rows of pots? _____
d) How many legs are altogether? Show how you work it out.

ACTIVITY 4: Grouping [20 minutes]

Grouping, discarding the remainder

Stella sells apples in bags with three apples in each. She has 12 apples. How many bags with three apples each can she make up?

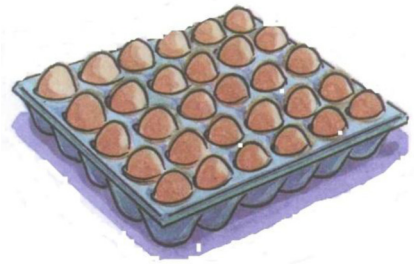
Stella sells apples in bags with 10 apples each. She has 80 apples. How many bags with 10 apples each can she make up?



Grouping, incorporating the remainder in the answer

Ben wants to take 15 eggs to his grandmother. He puts the eggs in an egg box that can take 6eggs. How many boxes will he need to take all the eggs?

A farmer has 47 eggs. How many egg boxes that can take six eggs each does he need to pack all the eggs?



Sharing

Sharing, discarding the remainder

Share 14 oranges among three friends so that they all get the same number of oranges.

Repeated addition

How many wheels do four bicycles have?

How many wheels do 20 bicycles have?

DIFFERENT STRATEGIES FOR PROBLEM SOLVING

Repeated subtraction

40 ÷ 8 = □

40 – 8 ➔32 – 8 ➔24 – 8➔ 16 – 8 ➔ 8 – 8➔ 0

Learners count the number of times they subtracted 8 to get to 0, which is 5

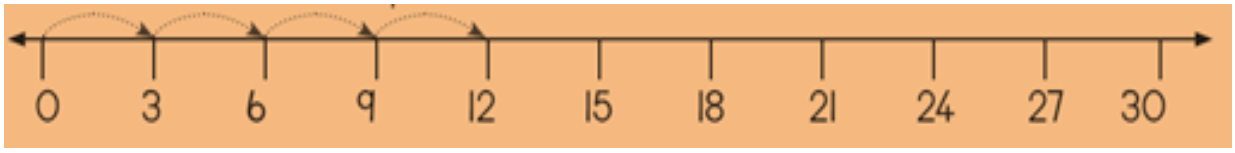
Repeated addition:

40÷8=□

8 + 8 ➔16 + 8 ➔ 24 + 8 ➔ 32 + 8 ➔ 40

Learners count the number of times they added 8 to get to 40, which is 5

Use of a number line in:

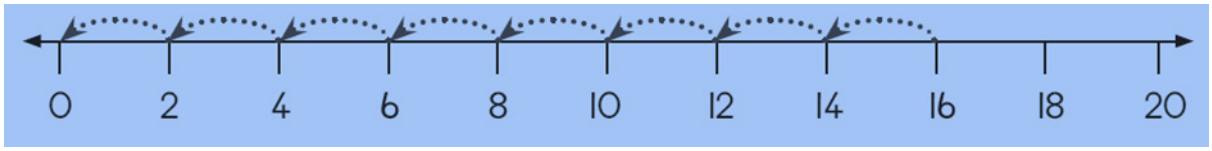


a) Addition and multiplication using a number line.

3 + 3 + 3 + 3 = 12,

4 x 3 = 12

b) Subtraction and division number line.



16 – 2 – 2 – 2 – 2 – 2 – 2 – 2 – 2 = 0

16 ÷ 2 =

Distributive property:

This means that the number can be broken up into parts that are easier to calculate.

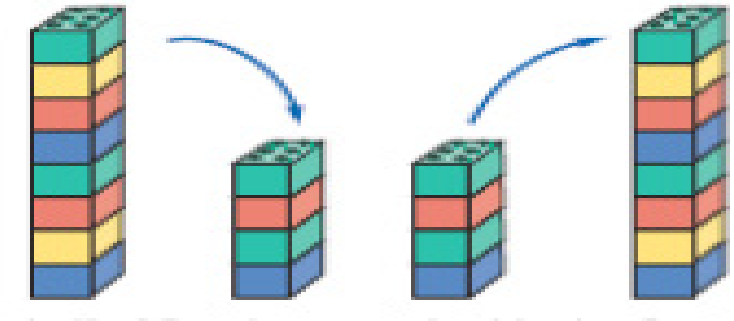
39 ÷ ÷ 3 = □

30 + 9 = 39

(30 ÷ ÷ 3) + (9 ÷ ÷ 3)

10 + 3 = 13

Breaking up numbers into halves.



Half of 8 is 4

8 ÷ 2 = 4

Double 4 is 8

4 x 2 = 8

60 ÷ 5 =□

Half of 60 is 30

30 ÷ 6 = 5

6 + 6 = 12

Understand the rules of dividing by 1 and 0

6 ÷ 1= 6

6 ÷ 0= 0

Left over or remainder

12 ÷ 5 ÷ 5 = 2 remainder 2

The Grid Method

X	3	3	
5	15	15	30
4	12	12	24
			54

Study the following Grid and explain step by step how you derived your response for 9 x 6

9 x 6 = 54

Use of a table and fill in the missing numbers

x	1	2	3	4	5	6	7	8	9	10
4	4									

ANNEXURE 1

The following are alternative strategies for multiplication and division of 3-digit numbers. It is important that the learners are exposed to a number of different strategies, particularly those where whole 10's and 100's are involved. Do not force the learners to use these strategies. These are simply alternatives.

59 x 3	248 x 4
59 = 50 + 9 **59 = 10 + 10 + 10 + 10 + 10 + 9** **50 x 3 = 150** **9 x 3 = 27** **150 + 27 = 177**	**248 = 200 + 40 + 8** **248 = 200 + 10 + 10 + 10 + 10 + 8** **200 x 4 = 800** **40 x 4 = 160** **8 x 4 = 32** **800 + 160 = 960** **960 + 32 = 992**
96 ÷3	**96 ÷3**
96 = 30 + 30 + 30 + 6 **30 ÷ 3 = 10** **30 ÷ 3 = 10** **30 ÷ 3 = 10** **6 ÷ 2 = 3** **10 + 10 + 10 + 2 = 32**	**96 = 90 + 6** **90 ÷ 3 = 30** **6 ÷ 3 = 2** **30 + 2 = 32**
76 ÷ 4	**135 ÷5**
76 = 40 + 20 + 10 + 6 **40 ÷ 4 = 10** **20 ÷ 4 = 5** **16 ÷ 4 = 4** **10 + 5 + 4 = 19**	**135 = 50 + 50 = 35** **50 ÷ 5 = 10** **50 ÷ 5 = 10** **35 ÷ 5 = 7** **10 + 10 + 7 = 2**

MULTIPLICATION AND DIVISION PROBLEM TYPES

REPEATED ADDITION

1. Mother buys 3 bags of oranges. Each bag contains 7 oranges. How many oranges did she buy?
2. Father buys 24 apples that are packed in 6 bags. If each bag contains the same number of apples, how many apples are in each bag?
3. Mother buys 32 pears. She wants to pack them into plastic bags, with 8 pears in each bag. How many bags does she need?
4. I fill 5 cups with 200ml milk in each. How much milk did I have before filling the cups?
5. 2 litres of cool-drink is poured into 10 cups so that each cup holds the same amount. How many millilitres of cool-drink is in each cup?
6. How many cups each holding 100 ml can be filled from a litre bottle of cool-drink?

RATE

1. Sipho walks at 6km per hour. How far does he walk in 4 hours?
2. Potatoes are sold at R12 per kilogram. If I buy 4 kilograms of potatoes , how much will I have to pay?

COMPARISON (TIMES AS MANY AS)

1. Zanele has 4 crayons . Pinky has 3 times as many crayons as Zanele . How many marbles does Pinky have?
2. The length of a car in a photograph is 4cm. If the photograph is enlarged 3 times, what will the length of the car be on the enlargement?
3. Radha has 9 marbles, which is 3 times as many marbles as Krishna has. How many marbles does Krishna have?
4. If a photograph is enlarged 3 times, the length of a car on the enlargement is 12cm. How long is the car in the original photograph?

ARRAYS (ARRANGEMENTS)

1. A slab of chocolate has 4 pieces along the shorter side and 6 pieces along the longer side. How many pieces does the slab contain?

2. A vegetable patch has 12 rows of cabbage plants, with 6 plants in each row. How many cabbage plants are there in the patch?

COMBINATIONS

1. Werna has 3 skirts of different colours and 4 tops of different colours. All the colours match. In how many different ways can she dress?

SHARING – WITH AND WITHOUT REMAINDERS, LEADING TO FRACTIONS

1. My sister and I found 5 sweets. We each took the same number. How many did we each take?

2. Mom bought 8 sausages and her 4 children shared them equally. How many sausages did each child eat?

3. Granny bought 10 sausages and her 4 children shared them equally. How many sausages did each child eat?

GROUPING – WITH AND WITHOUT REMAINDERS

1. Nsiki has 12 bananas and she put them into 3 baskets. How many bananas are in each basket?

2. Pieter bought 14 oranges. He has 4 packets. How many can he pack in each packet equally?

REPEATED ADDITION AND SUBTRACTION

1. How many wheels do 4 bicycles have?

2. Father has R20. He gives R5 to each of his three children. How much money will he have left?

ANNEXURE 2

SAMPLES OF GAMES

1. **PLACE VALUE WITH DICE** activities may vary according to the number range in grades as specified by the policy document.

   Instruction

   - Throw two or three dices and ask learners what is the biggest number they can make
   - Let the learners :
     - add the numbers together
     - subtract the smallest number from the biggest number

2. **USE OF NUMBER CARDS AND A DICE**

   Throw a dice, use the top number in a dice then ask the questions in a number card

How many more do I need to make 12?

What comes before the answer?

Double the answer

Halve the answer

Multiply by 3

Add 7 onto the answer

Subtract 8

What is 5 more than the answer?

What is 3 less than the answer?

1. TO REINFORCE BASIC OPERATIONS

Use of playing cards/ dice

Group Activity/ Paired activity

Play "SNAP"

- The leader will pick 2 cards / 2 dices and ask the learners to add/ subtract/ multiply/ divide/ double/ halve the numbers or say which number comes after/ before/is more than /less than the number. If they see the matching number they shout "snap".
- The cards can also be used for mixed operations

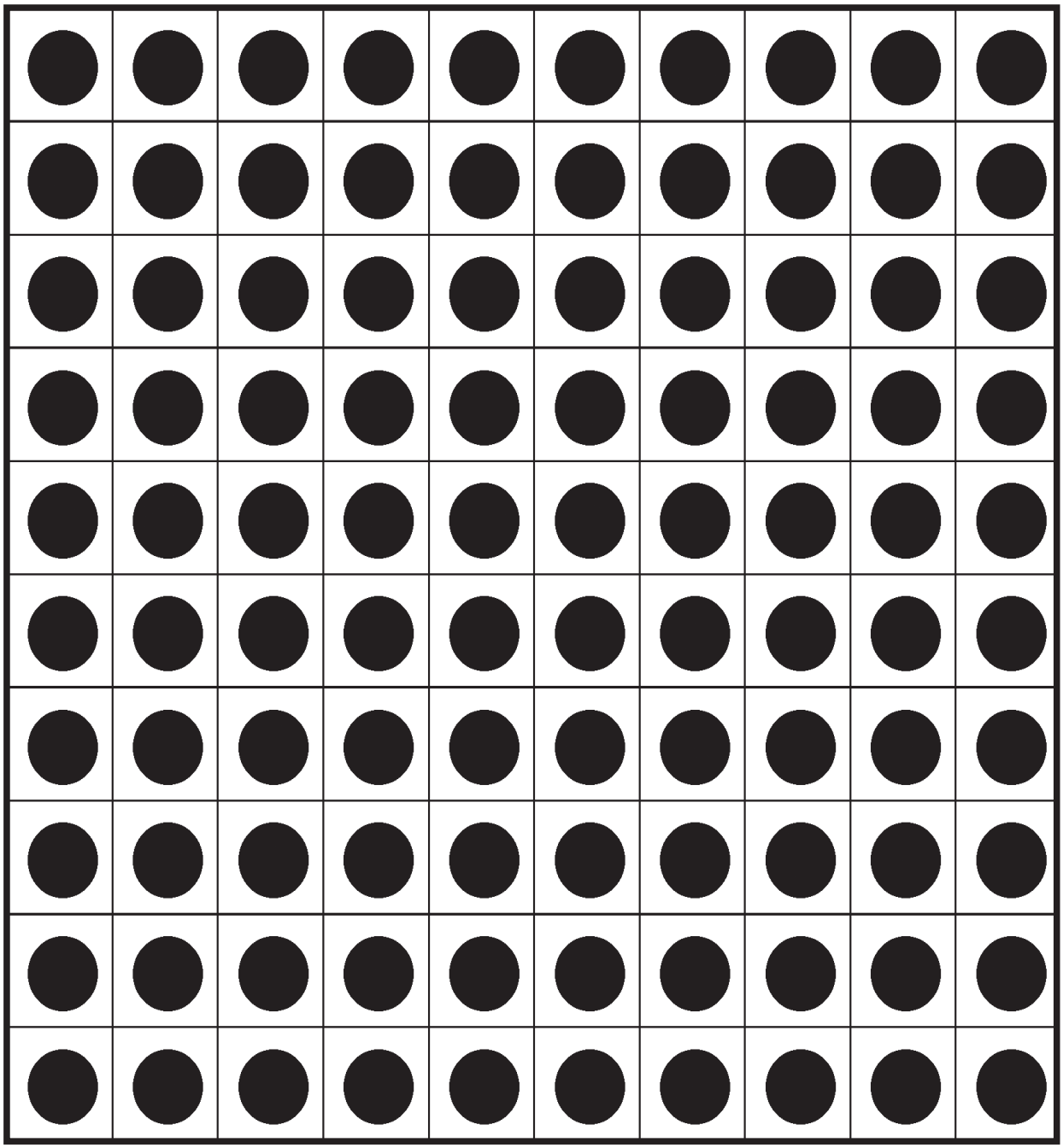
4. USE OF A NUMBER GRID for re-enforcing basic operations (+, - , x, ÷)

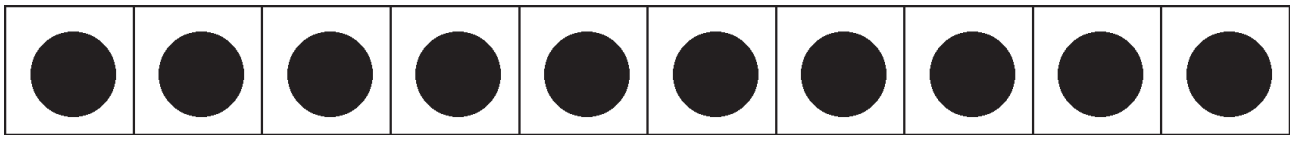
This game is suitable for grade 3

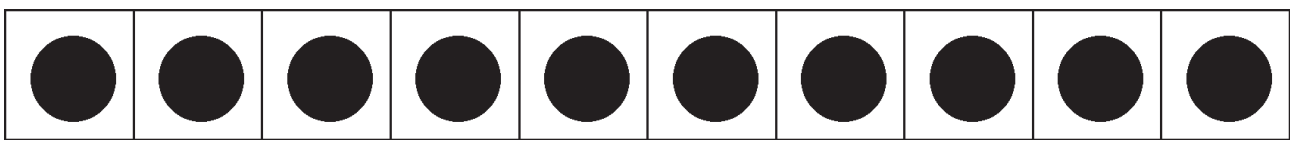
Circle many number combinations by looking horizontally, vertically and diagonally.

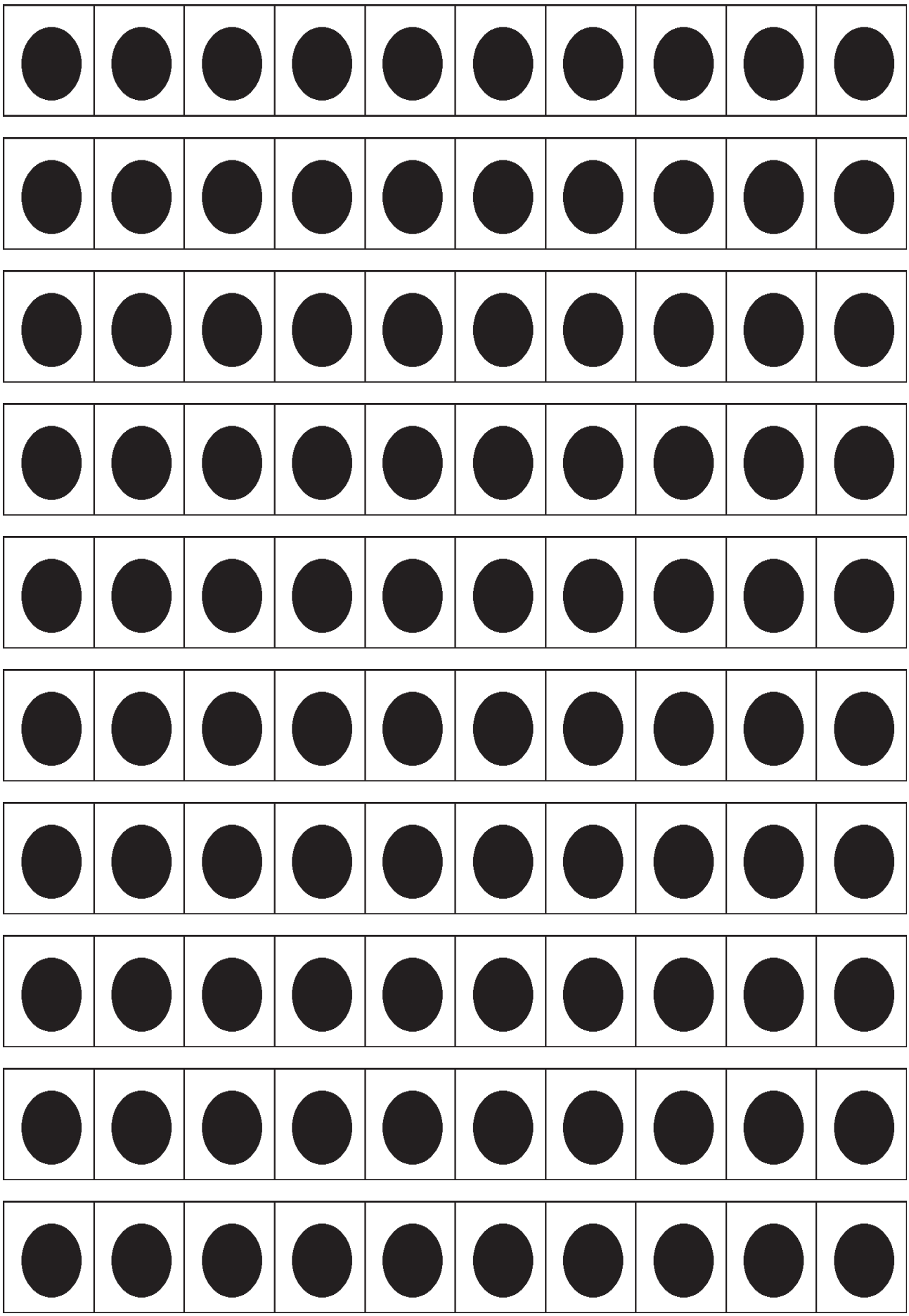
9	3	6 ×	81	60	7	12	19	57	76	23
45	2	3 =	12	48	6	8	14	25	33	58
3	35	18	4	72	42	30	56	3	44	66
15	5	10	9	12	5	28	11	17	35	16
76	38	8	36 ÷	6 =	6	4	27	31 +	5	1
48	8	80	4	20	11	7	4	9	36	2
64	4	24	8	3	5	15	45	40	3	10
7	13	12	32	35	19	8	3	6	18	3
32	32	28	4	7	6	9	4	19	21	5
58	18	40	36	3	7	21	28	36	4	9

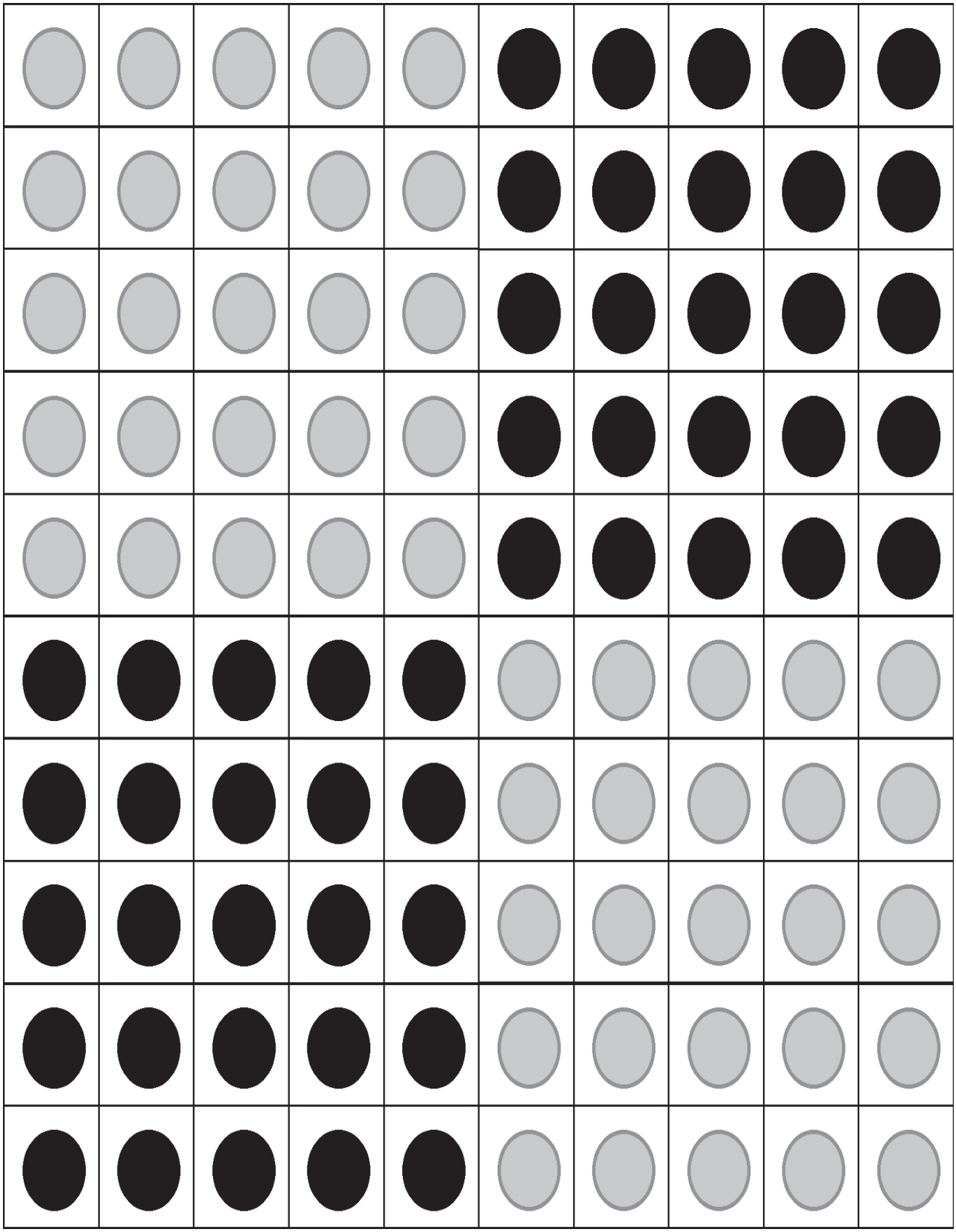
DIFFERENT ARRAYS

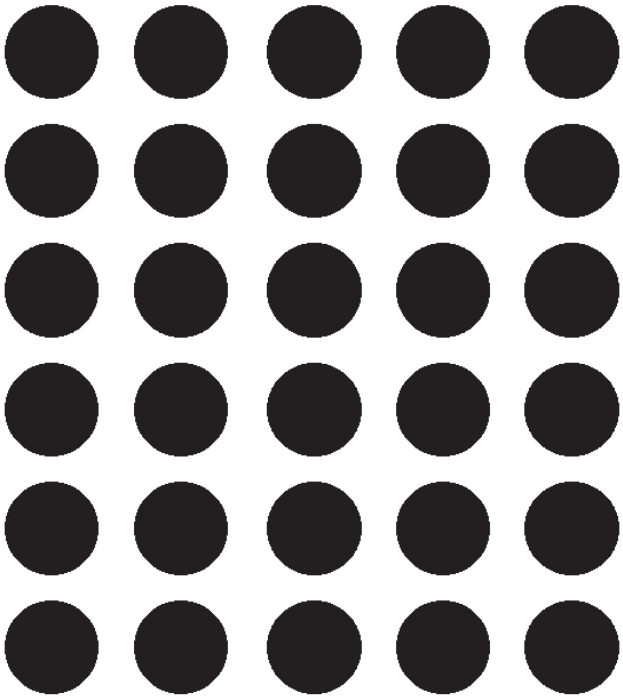


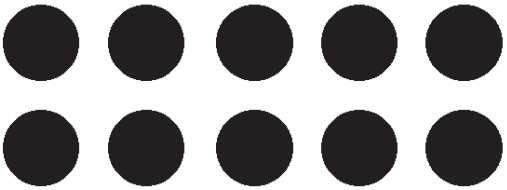


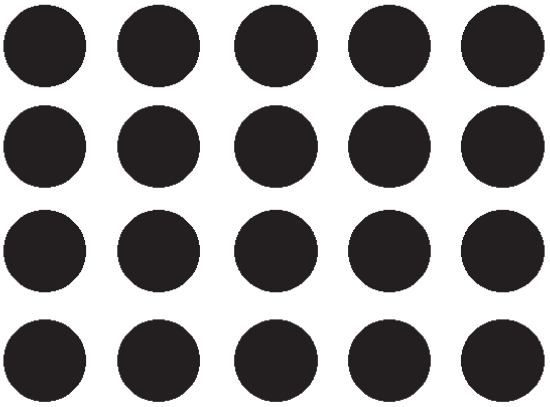


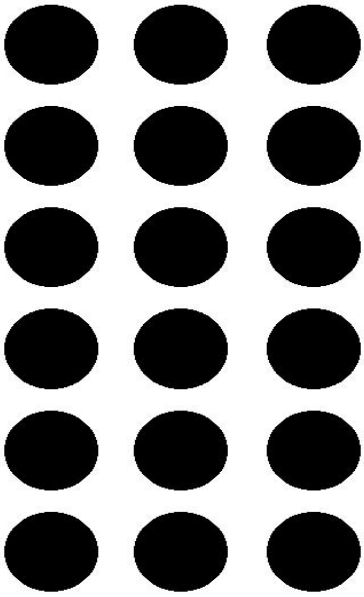


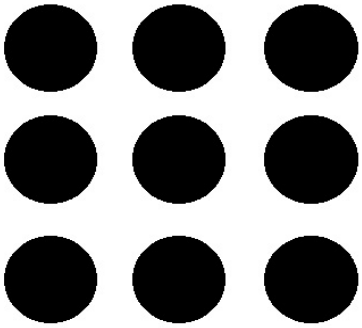


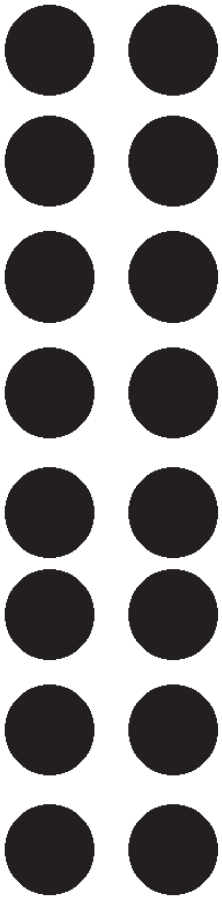


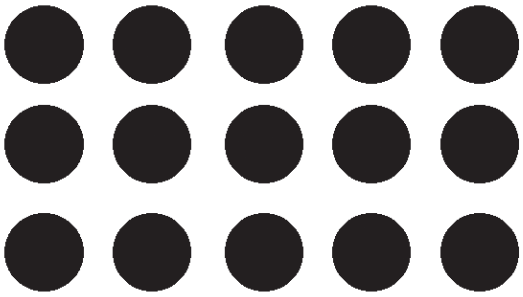


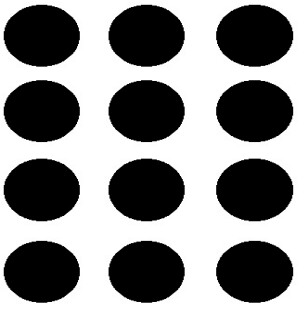


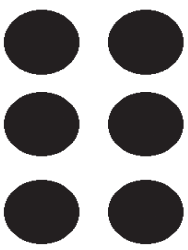


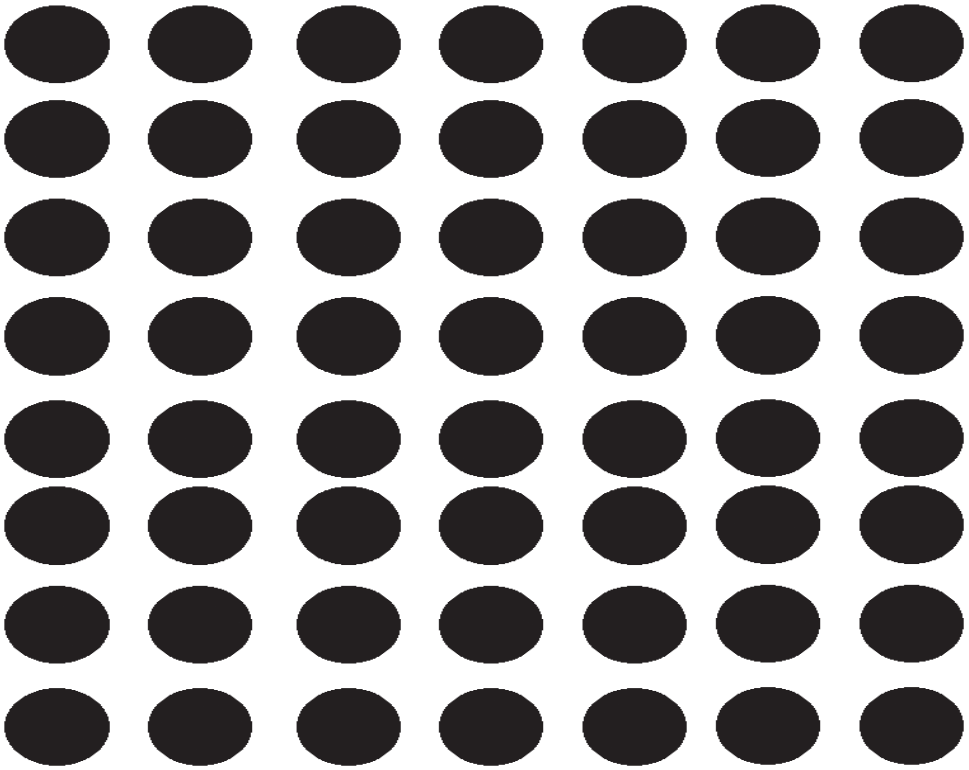




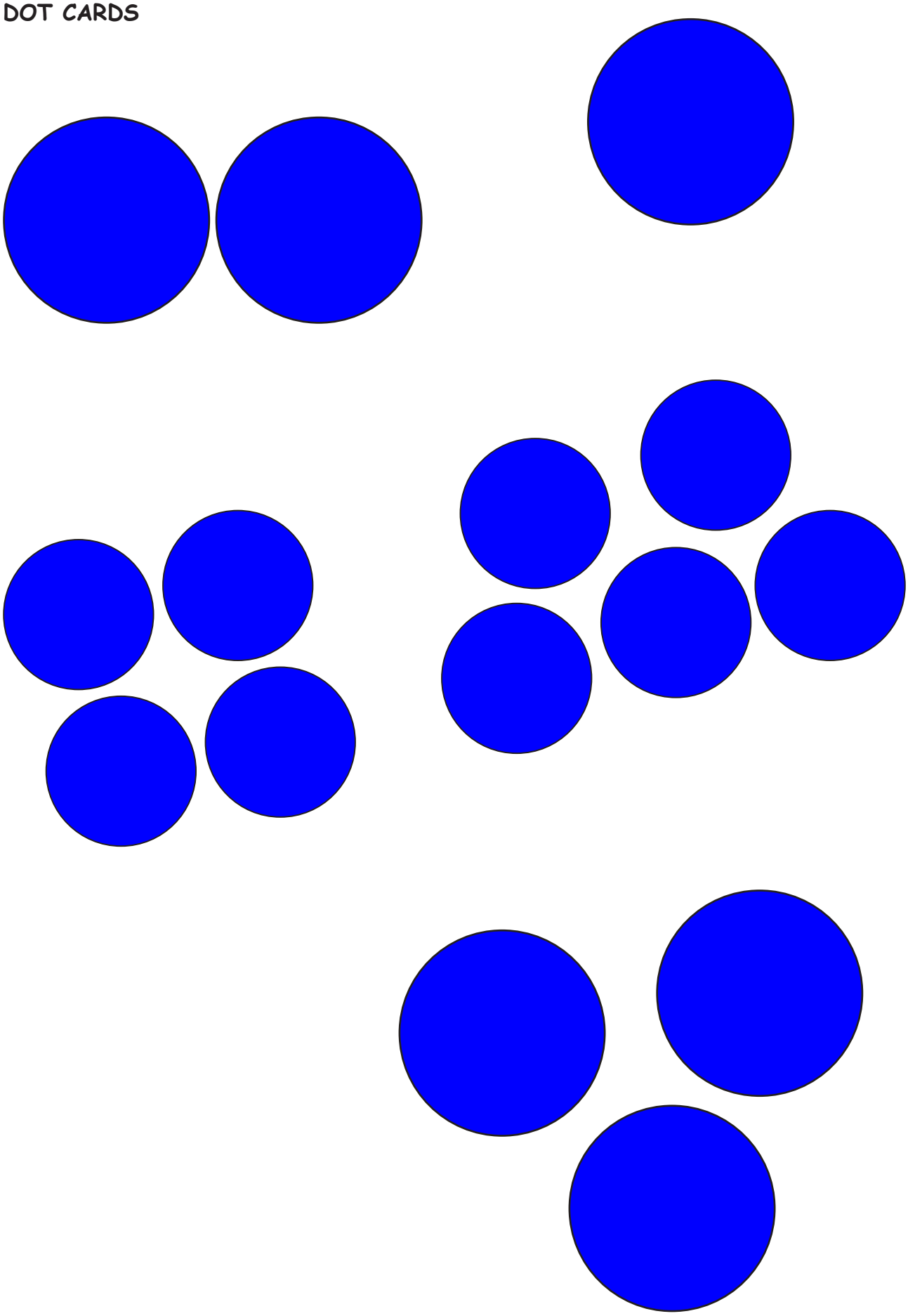








DOT CARDS



ACKNOWLEDGEMENTS:

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1. Department of Education (2011), Curriculum Assessment Policy (CAPS) Foundation Phase Mathematics Grades R-12 Government Printers

2. Department of Education (2002), Revised national Curriculum Statement Grades R-9 (schools) Policy, Mathematics, Government Printer

3. Department of Basic Education (DBE) Workbooks

4. Kilpatrick, J. Swafford, J. & Findell, B.(Eds.) (2001). **Adding it up: Helping children to learn mathematics**. Washington: National Academy Press.

5. Wright, R.J. (2003). A Mathematics Recovery: Programme of intervention in early number learning. Australian Journal of Learning Disabilities, 8(4).

6. Wright, R. J. Martland, J., & Stafford, A. K. (2006). Early numeracy: assessment for teaching and intervention. London: Paul Chapman Publishing Ltd.

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