Province of the EASTERN CAPE EDUCATION

Mathematical Proficiency

Development of a strong sense of number and the role of problems in teaching Mathematics in Grade 3

JULY 2011

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INTRODUCTION

In order to be able to participate effectively in our world in which we live in, it is essential that individuals know "basic mathematics".

The Numeracy Programme in the Foundation Phase is critical to developing both a sense of what mathematics is and what it means to engage with mathematics.

The results of both national and international studies reveal quite clearly that South African schools are failing our children in helping them to develop the necessary skills to be able to "do mathematics".

The average achievement for the **Grade 3 Numeracy Common Test** in 2010 was 45.3 %. This is of great concern in the light of the **Foundations for Learning Campaign** goal that by 2011 no learner should achieve less than 50 %.

Analyses of the results of the 2010 Common Tests showed that the learners had difficulty with **number skills** and the **basic operations.** This manual seeks to address the identified weaknesses and problems and provides guidance for Foundation Phase teachers on how to support children to develop the required knowledge and skills in numbers and operations.

For students to be successful in later mathematics activities and to use mathematics effectively in life, they must have a sound understanding of elementary mathematics concepts, a positive attitude towards learning mathematics, and the belief that an understanding of mathematics is attainable (Kilpatrick & Swafford, 2003).

The Foundation Phase is a crucial period to foster the basic skills and love for the subject – the role of the teacher cannot be underestimated.

It is hoped that the training sessions will provide the necessary guidance and will add value to the teaching and learning process in the classroom.

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The Eastern Cape Department of Education acknowledges the authors of *Numeracy Handbook for Foundation Phase Teachers Grades R-3, DBE* from which valuable extracts have been used to compile this guideline document.

Section 1:	Mathematical Proficiency

OUTCOMES

At the end of Section 1 participants should be:

- Aware and understand the strands of mathematical proficiency to improve their teaching practice;
- Aware and understand that teaching mathematics is much more than the teaching of methods or procedures only.

The challenge for all Numeracy teachers is to answer the question:

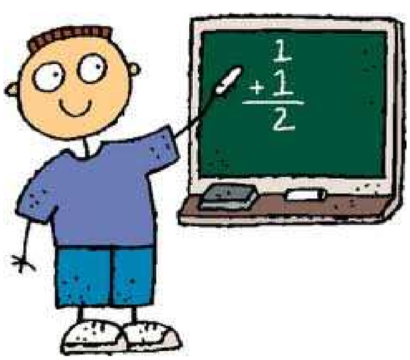
How do I teach in a way that will help children to think mathematically?

In order to develop a response to this question we will introduce a number of ideas. In particular we will consider the following:

The notion of Mathematical Proficiency incorporating the **five strands**:

- Conceptual understanding (**Understanding**)
- Procedural fluency (**Computing**)
- Strategic competence (**Applying**)
- Adaptive reasoning (**Reasoning**)
- Productive disposition (**Engaging**)

U.C.A.R.E about Mathematics



The five strands of Mathematical Proficiency

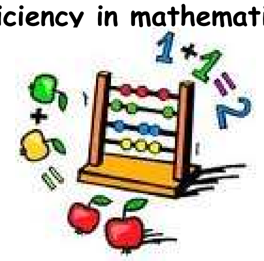
Mathematical Proficiency is a term used by the authors of: *Adding it up: helping children learn mathematics* (NRC, 2001) to describe:

"what it is to be successful in mathematics"

Being mathematically proficient (numerate) means:

1. **Understanding** what you are doing (conceptual understanding)
   - Understand problems or task;
   - Make connections to similar problems;
   - Use models and multiple representations.
2. Being able to **calculate/compute** (procedural fluency) with confidence.
   - Accurate computation;
   - Proper use of algorithm (Intermediate Phase);
3. Being able to **apply** what you have learnt (strategic competence);
   - Formulate and carry out a plan;
   - Can create similar problems;
   - Can solve using appropriate strategies.
4. Being able to **reason** about what you have done (adaptive reasoning);
   - Justify responses logically;
   - Reflect on and explain procedures;
   - Explain concepts clearly.
5. Recognising that you need to **engage** (productive disposition) with a problem in order to solve it;
   - Tackle difficult tasks;
   - Persevere;
   - Show confidence in own ability;
   - Collaborate/Share ideas.

These five strands are interwoven and interdependent in the development of proficiency in mathematics (see diagram below).

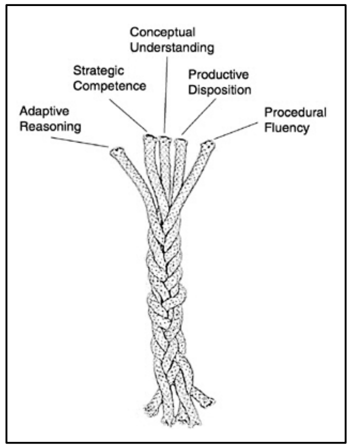


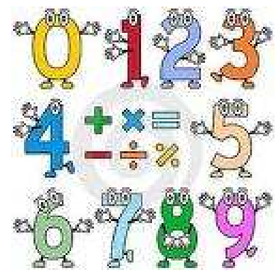
Five Strands

Understand | Compute | Apply | Reason | Engage

U.C.A.R.E about Mathematics

Teachers should not teach in a manner based on the extreme positions that learners learn solely by internalizing what a teacher or book says, or solely by inventing mathematics on their own. The **integrated** and **balanced development of all five strands** of mathematical proficiency should guide the teaching and learning of school mathematics.





1. Understanding

When children learn with understanding they have fewer things to remember. Children who learn with understanding know that 3 × 5 is the same as 5 × 3 and that the four times table is simply double the two times table.

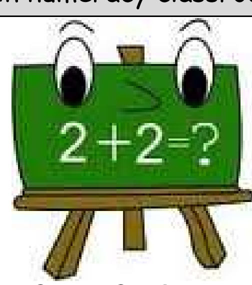
EXAMPLE: When children understand concepts and procedures such as **place value** and **operations** with single digit-numbers, they can **extend these concepts** and procedures to new areas e.g.

- 9 + 4 = 13; 19 + 4 = 23; 69 + 4 = 73 and
- 4 + 3 = 7; 40 + 30 = 70; 400 + 300 = 700.

Thus, learning how to add and subtract multi-digit numbers does not have to involve entirely new and unrelated ideas.

Note:

- Any concepts that learners learn **with understanding** will support them to remember and to use mathematical facts in unfamiliar situations.
- Learning **without understanding** contributes to so many of the problems experienced by children in numeracy classrooms.



Task 1: 5 minutes

Ask the participants to solve the following problem individually using the concept of doubling:

1. 6 + 7	**2. 8 + 9**	**3. 15 + 16**
4. 35 + 36	**5. 65 + 66**	**6. 45 + 47**

1.
2.
3.
4.
5.
6.

Ask a few participants to explain the strategies they used with the rest of the group.

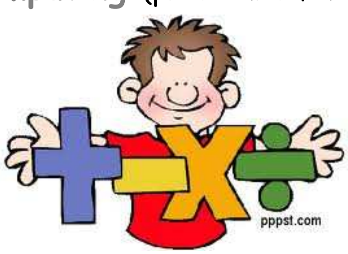
Note:

- Because young children tend to learn the doubles fairly early, they can use them to find the answer to sums that are closely related.
- They may see that 6 + 7 is just 1 more than 6 + 6 or double 6.
- These relations make it easier for students to learn new addition combinations e.g. 8 + 9; 5 + 6; 7 + 8 etc.

Conceptual understanding is an investment that learners can apply in many ways.

The implication for teaching: We need to provide children with the opportunity to make sense of and reflect on procedures and practices so that they can develop deep conceptual understanding.

2. Computing (procedural fluency)



Computing involves being able to perform the procedures for the basic operations:

- addition;
- subtraction;
- multiplication; and
- division

Children should be efficient and accurate in performing basic computations (7 + 5; 13 – 4; 8 x 3; etc.) **mentally**.

To be fluent a learner should:

- be able to perform the procedure **quickly**, **accurately**, and **flexibly** both mentally and with pencil and paper or even a calculator.
- have a good conceptual understanding of **place value** which will support the development of **fluency in multi-digit computation**.
- be able to **use a variety of mental strategies** to perform operations such as 199 + 67 or 4 x 26 rather than relying on pencil and paper.

U**nderstanding and computing are interwoven**. For example, it is difficult for learners to understand multi-digit calculations if they have not attained some reasonable level of skill in single-digit calculations.

Both **accuracy** and **efficiency** can be improved with practice, which can also help students maintain **fluency**.

Without procedural fluency, students have trouble deepening their understanding of mathematical ideas or solving mathematics problems. When learners practice procedures they do not understand, there is a danger they will practice incorrect procedures (see the example below).

Task 2: 5 minutes

- Ask the participants to identify the common errors in the 3 examples below that often occur in multi-digit subtraction.
- Let them discuss in their groups why they think some learners will make this error.

62	95	346
– 48	– 47	– 189
26	52	243

Note: If children learn to subtract with understanding, they do not often make this mistake. When they learn skills without understanding, the skills are learned as **isolated bits of knowledge**.

Task 3: 5 minutes

Ask the participants to use quick mental strategies to perform the following operations and to only write down the answers in the space provided:

1. 299 + 68
2. 341 – 25
3. 69 x 5

1.

2.

3.

Ask a few participants to explain their strategies to the rest of their group.

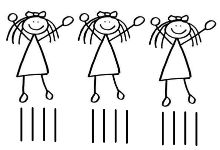
The implication for teaching: Teachers need to support their children in developing a strong sense of number (see Section 2) and they need to help children develop their computational strategies in ways that makes sense to them.

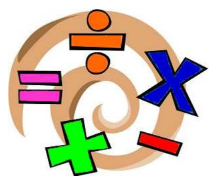
3. Applying (strategic competence)

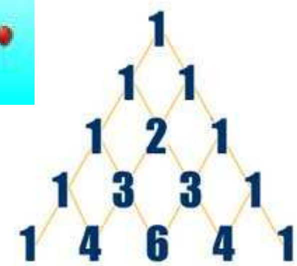
Strategic competence refers to the ability to formulate mathematical problems, represent them, and solve them. This strand is similar to what is called **problem solving**.

The learner's first step in solving a given problem is to represent it mathematically in some fashion, whether:

- numerically,
- symbolically,
- verbally or
- graphically.





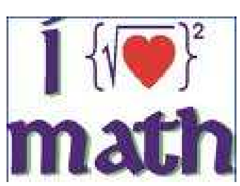


To solve a problem successfully, learners must:

- **understand** the situation and then
- be able to **express problems mathematically** and
- be able to **devise strategies** for solving those using appropriate concepts and procedures. This step may be facilitated by making a **drawing, writing** an **equation** or creating some other representation.

Applying is **using understanding** and **computing skills** to solve **non-routine problems**. These are problems for which the **learner does not immediately have a method** to solve the problem.
In contrast, a **routine problem** is when a learner applies a **known solution procedure**. For example the sum of 9 and 6 is a routine problem for most learners because they know what to do and how to do it.

A student should learn not only to solve routine problems but also apply the same problem solving techniques to invent a solution to a non-routine problem.
(see Task 4 below).



Task 4: 10 minutes

Step 1: Ask the participants to solve the following problem individually using any method they like. Read the problem through twice, while participants follow the written text.

Do not interpret the problem in any way! Neither should you discuss it at all with the participants. Simply present it to them and leave them to work it out in the space provided.

Step 2: While the groups are busy, walk around and listen to their discussions. Do not engage in any discussions.

Step 3: At the end of 5 minutes stop the discussions and check if anyone has reached a solution. The solution is not important at this stage. You are looking at the **process** rather than the answer.

- Ask a few participants to explain their methods with the rest of the group. They can go to the front and explain it using a flip chart.
- Discuss the importance of children explaining their own method and thinking.
- Discuss how the teacher will facilitate the process of problem solving with children who appear unable to even start.

Step 4: Define "Problem solving"

*A problem is a task that requires the person solving the problem to use knowledge, **understanding** and skills that he/she has acquired from other activities and to **apply** these to the new and unfamiliar situation and come up with a solution.*

At Calta petrol sells for R8 per liter.
This is 20c less per liter than petrol at Shellow.
How much does 5 liters of petrol cost at Shellow?

Note: The quantities R8 and 20c are followed by the key wor*d "less",* which suggests that the learners should subtract 20c from R8 to get R7,80. Then the keywords how much and liters suggest that 5 should be multiplied by the rest giving an answer of R39,50. **This is not correct**.

Less successful problem solvers tend to focus on specific numbers and keywords such as R8, 20c, less and 5 liters rather than the **relationships** among the quantities.

A more skillful approach would be to construct a problem model i.e. any form of **mental representation** about the relations among the variables in the problem. For example a learner can envision a number line and locate each cost per liter on it to solve the problem.

Task 5: 10 minutes

Step 1: Ask the participants to solve the following problem individually using any method they like. Read the problem through twice, while participants follow the written text.

Steps 2 & 3: Follow the same steps as in Task 3

A bicycle shop has a total of 11 bicycles and tricycles in stock.
Altogether there are 28 wheels.
How many bicycles and how many tricycles are there?

Note: A learner with strategic competence could **flexibly apply the knowledge and skills** that they have developed to solve the problem.

The implication for teaching: Children need to be exposed to non-routine problems in which they have to apply the knowledge and skills that they have developed

4. Reasoning (adaptive reasoning)

Reasoning is central to the development of mathematics and mathematical proficiency. Children must be able to **justify** their work i.e. "provide sufficient reason". They must therefore reflect on and think about how they solved a problem or completed a task. **Reflection on their actions contributes to the development of conceptual knowledge.**

It is through reasoning that children develop their understanding of the task in hand and come to see mathematics as sensible and **doable**.

Task 6: 5 minutes

Step 1: Ask the participants to solve the following problem individually using any method they like. Read the problem through twice, while participants follow the written text.

Do not interpret the problem in any way! Neither should you discuss it at all with the participants. Simply present it to them and leave them to work it out in the space provided.

Step 2: While the groups are busy, walk around and listen to their discussions. Do not engage in any discussions.

Step 3: At the end of 5 minutes stop the discussions and check if anyone has reached a solution. The solution is not important at this stage. You are looking at the **process** rather than the answer.

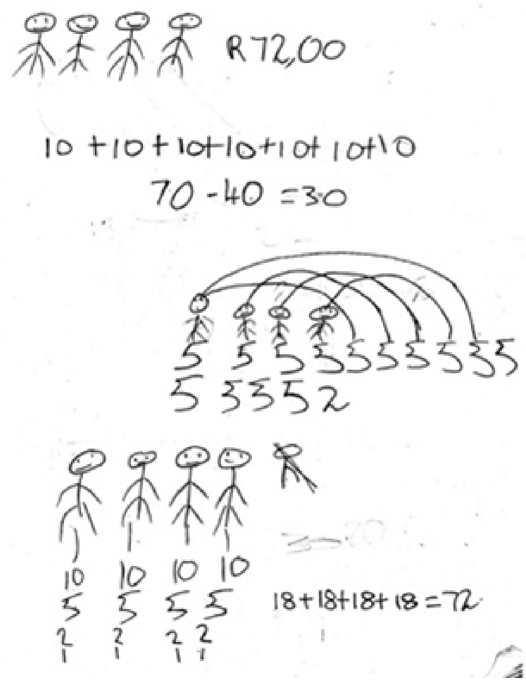
- Ask 2 or 3 participants to explain their methods with the rest of the group and to show their working on a flip chart in the front of the room.

Step 4: Now refer the participants to the work of Zintle, a Grade 2 learner, below and have a discussion around the strategies she used to solve the same problem.

Four children are paid R72 altogether.
If they share the money equally between themselves, how much will each person get?

Zintle's (Grade 2) work below illustrates what can happen when a child can **reason**.

Zintle



Zintle drew 4 children and started by giving each child R10.
She then established that there was still R30 left over. She reasoned that there is not enough money to give each child another R10, so she started again and tried to share out the money using amounts of R5.
She wrote 5 beneath each child and then connected each child with another 5.
When she started to give each child a third R5, she realised that she had actually given each child R10 already and that there would be enough 5's for another round.

She then started all over again, giving each 10, then 5 and then 2 and lastly 1 each.
So every child got R18.
Finally she checked that the 18 + 18 + 18 + 18 is in fact R72.
Through careful reasoning Zintle was able to establish that 72 ÷ 4 = 18
Numeracy Handbook for Foundation Phase Teachers Grades R-3, DBE;

Task 7: 5 minutes

Ask the participants to tick the correct answers to the following:

If 38 + 95 = 133 is true, which of the following is true?

38 = 95 + 133

38 + 133 = 95

133- 38 = 95

95 – 133 = 38

38 – 133 = 95

Have a group discussion after 5 minutes.

The implication for teaching is that teachers need to encourage reflection/ reasoning through classroom discussion. To be clear, discussion will often focus on children simply explaining their thinking.

5. Engaging (productive disposition)

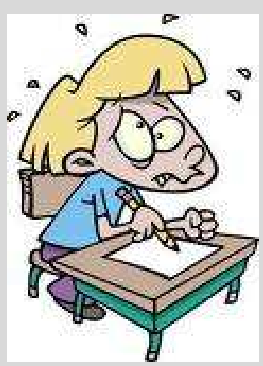
Productive disposition refers to the tendency to see sense in mathematics, to see it as worthwhile and useful\ i.e. to believe that **steady effort in learning mathematics pays off**. To engage is to see mathematics as sensible, useful, and doable – if you work at it – and are willing to do the work.

Children who are engaged in mathematics (solving a problem) do so knowing that:

- they may have to struggle and put in some effort;
- they may have to try a few different approaches before the problem is solved;
- Mathematical proficiency is an important part of their future.

Children who by contrast experience mathematics as a set of rules and instructions to be remembered and reproduced without understanding, become:

- Become indifferent (not interested)
- Become frustrated and
- Regard mathematics as meaningless and difficult.



Summary:

1. **Productive disposition develops through problem solving** (strategic competence).
2. **The more mathematical concepts children understand, the more sensible mathematics becomes**.

In contrast, when children are seldom given challenging mathematical problems to solve, **they come to expect that memorizing rather than sense making paves the road to learning mathematics** (*Adding it up: helping children learn mathematics* (NRC, 2001).

The implication for teaching is that teachers need to demonstrate their faith in the ability of the children in their class (allow children to be comfortable in doing mathematics and sharing their ideas) to solve problems – providing the necessary support without taking away the independence of the child.

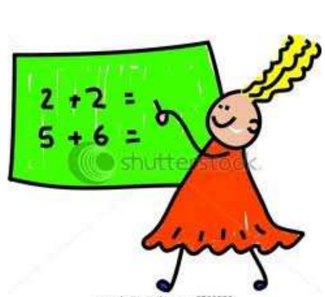
Awareness of the strands of mathematical proficiency helps teachers in the following ways:

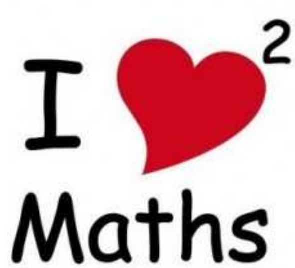
- They remind teachers that teaching mathematics is much more than the teaching of methods or procedures only. Teaching mathematics involves helping children to **understand**, to **apply** and to **reason** about and with the mathematics that they learn.

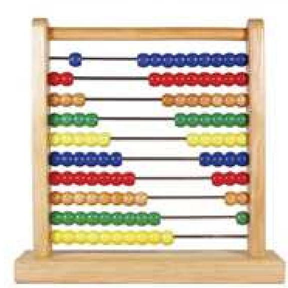
- They remind teachers that proficiency is acquired over time. Children need to **spend enough time to engage in activities** around a specific topic if they are to become proficient with it. They need to spend a lot of time doing mathematics – solving problems, reasoning, developing understanding, practising skills – and **building connections between their previous knowledge and new knowledge**.

- They allow teachers to answer the question: "Am I doing the right thing?" If the children in your class are **willing** and **able** (i.e. engaging) to **apply** their mathematical knowledge with **understanding** to solve non-routine problems and **justify** their solution method(s) they are becoming numerate/mathematically proficient.

(Adapted from: *Numeracy Handbook for Foundation Phase Teachers*, DBE)







SECTION 2:	Criteria for developing a strong sense of number

OUTCOMES

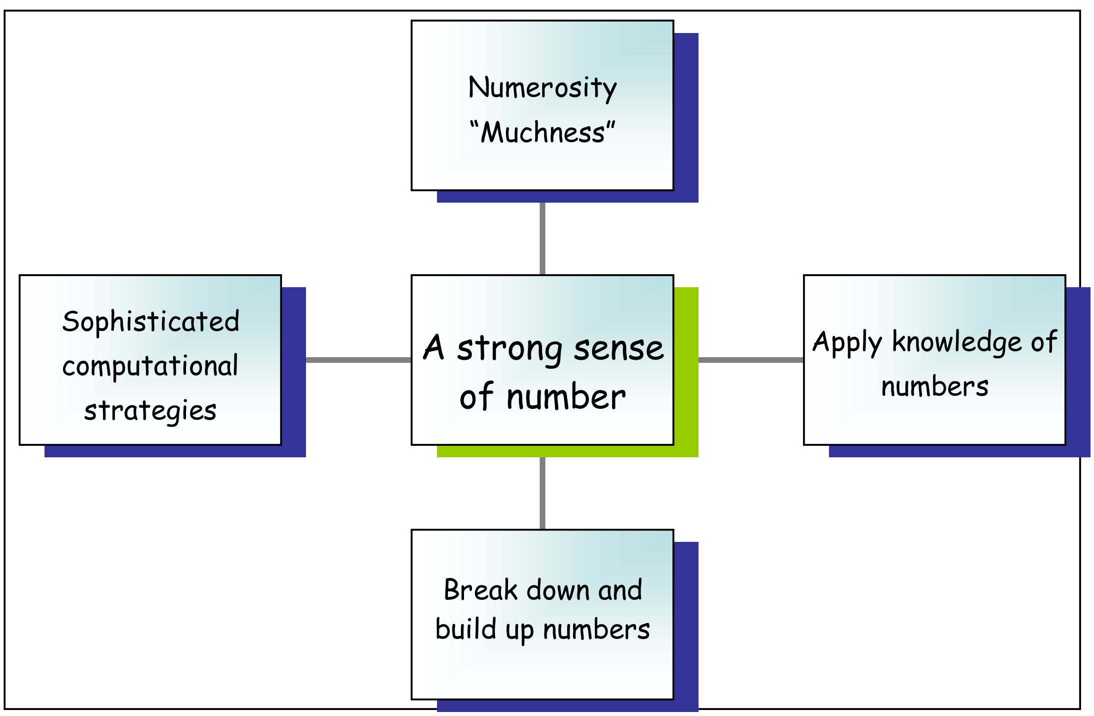
At the end of Section 2 participants should:

- Understand the criteria that is necessary for a child to develop a strong sense of number
- Have a solid knowledge of the three levels of number sense development.

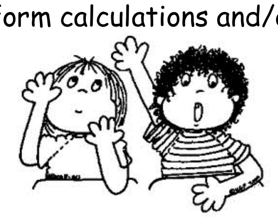
What is a "strong sense of number"?

One of the most critical things to be achieved in the Foundation Phase is the **development of a strong sense of number.**

The flow diagram below illustrates the criteria necessary for a child to develop a strong sense of number:



Children with a poorly developed sense of number will not advance in a significant way in their mathematical development. There are many children at fairly advanced ages still using stripes to perform calculations and/or solve problems.



Levels of counting

The key challenge of the Foundation Phase classroom is to assist children to develop a strong sense of number.

A child's sense of number develops through **three levels** with the levels following one after the other:

3 levels of counting

1. Counting All
2. Counting on
3. Break down and build up

Level 1: Counting all

Children at Level 1 are counting all. When we ask a child who is operating at Level 1 to add two numbers then they will first recreate each number using fingers or other representations. As the numbers get larger and the child can no longer rely on the fingers of their two hands to create the numbers, they will use either objects such as counters or bottle tops or they will reconstruct the number on paper by drawing stripes or circles. See Xola's solution to the "cookie problem" on p 17.

Level 2: Counting on

Children at Level 2 are counting on. When we ask a child who is operating at Level 2 to add/subtract two numbers then the child is able to conceptualise at least one of the numbers without having to recreate it, and recreates only the other number.
As the numbers get larger and the child can no longer rely on the fingers of their hand to create the number(s) they will also resort to using objects or drawings.
See Sandla's solution to the "cookie problem" on p 17.

Level 3: Breaking down and building up numbers

Children at Level 3 are able to work with numbers in flexible ways often:

- Breaking numbers down (decomposing);
- Reorganising them; and
- Building them up again to perform a calculation and/or to solve a problem.

Children at Level 3 are said to have a **"numerosity"** of the numbers with which they are working – that is, they have a sense of the **"muchness"** of the numbers and can think of those numbers in a large range of different ways.
See Lwazi's solution to the "cookie problem" in the illustration below.

Below are three childrens' solutions to the same problem. They each demonstrate a different level of number development.		
Pat and Lee made 52 cookies altogether. If Pat made 31 of them, how many did Lee make?		
OOOOOOOOOO OOOOOOOOOO OOOOOOOOOOO OOOOOOOOOO OOOOOOOOOO o 21	31 ||||||||||||||||||||| 21	10 10 10 10 10 2 10 10 10 1
Xola draws 31 biscuits and then continued to draw cookies until she reaches 52. Next she counted the number of extra cookies and wrote down her answer of 21. Xola demonstrates a **Level 1** number sense – she had to **count all**.	Sandla wrote down 31 and counted on up to 52. He then counts how many extra cookies were needed – 21. Sandla demonstrates a **Level 2** number sense – he did not have to construct the 31 again – he simply **counted on**.	Lwazi broke the 52 into five tens and two and the 31 into three tens and one. He still has to determine the number of cookies baked by Lee. Lwazi demonstrates a **Level 3** number sense – he did not construct either number – instead he simply **broke the numbers down in order to rearrange them and build up again.**

Adapted from: *Numeracy Handbook for Foundation Phase Teachers Grades R-3,* DBE;

Note: The above examples are a clear illustration of how a weak number sense can limit a child's ability to develop increasing levels of sophistication.

SECTION 3:	Supporting the development of Level 2 and Level 3 number sense

OUTCOMES

Participants will be:

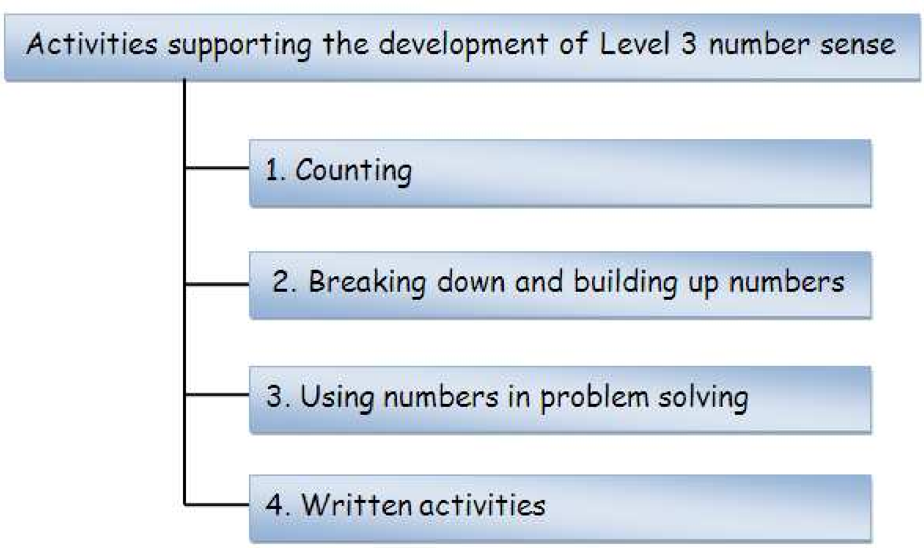
- aware of the activities to support the development of Level 3 number sense;
- able to teach in a manner to support learners to develop a strong sense of number.

Icebreaker (Mental Maths) **(5 min)**

Tell the participants you are going to ask them a sequence of questions and they must close their eyes and work out the answers in their heads. When you are sure all eyes are closed you ask the following questions with a pause after every question:

- I have 27, what is 3 more? (30)
- Now what will I have left if I give away 8? (22)
- What is 7 less? (15)
- What will I get by adding another 20? (35)
- What will I get when I divide by 5? (7)
- What will I get if I multiply by 10? (70)

- Ask a few participants to share their answers with the rest of the group.
- Tell them that this is a typical example of a **mental maths activity** that they can use when they work with small groups of children on the mat.
- They will have to change the numbers according to the level of the children.



1. Counting

Two kinds of counting

Most children are introduced to number through counting. It is important to make the distinction between **two different kinds of counting**:

1. rote counting; and
2. rational counting.

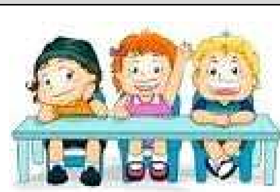
1. Rote counting

One, two, buckle my shoe;
Three, four, open the door;
Five, six, pick-up sticks;
Seven, eight, lay them straight;
Nine, ten, a big fat hen;

Rote counting involves:

- what children do when they chant/sing numbers in sequence;
- teachers and parents perceiving that children are able to count when they can recite the number names in sequence from one to twenty and beyond – this is not the case;
- when children can recite the numbers, these children do not (yet) see any relationship between the number words and a quantity of items. It is as if they are reciting the lines of a poem or the words of a song without having any sense of what the words mean;
- the importance of children counting in **small groups and individually** on a regular basis.

It is important that children are able to rote count – through rote counting they develop their **knowledge of the number names.**



2. Rational counting

Note: There is no need to spend time on rote counting with children who have reached Level 1 and beyond.

As children develop from Level 1 to **Levels 2 and 3** they **still need to engage in a lot of rational counting activities.**

Rational counting involves:

- the counting of **physical objects** touching objects one by one
- matching the **number names** to objects.
- developing a **sense of the muchness** of a number – that is; children develop the sense that 25 is more than 5 and 500 is much more than 25.

The nature of the activities focuses on:

- **counting larger quantities** in efficient ways including the **use of grouping**; and
- **counting on** from a given point e.g. count in 10s from 300 to 450 (using physical objects).

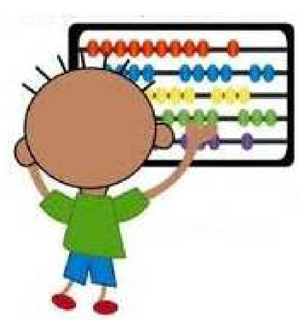
Examples of practical activities to develop counting

Note: The following are examples of activities that should take place in **small groups** working with the teacher while sitting on the mat.

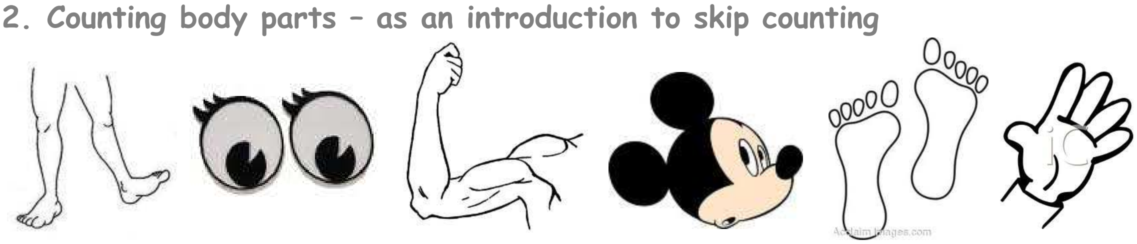
1. Counting using counting frames

Counting using **counting frames** can help children both with their ability to count in 10's and also to learn how to complete tens and count on.

For example if asked to count out 73 on a counting frame we would expect children to develop the confidence to count: 10, 20, 30 , 40, 50, 60, 70, 71, 72, 73 and when asked how many more beads are needed to reach 80, the child will count on and see that 7 more are needed.



2. Counting body parts – as an introduction to skip counting



Task 1: 10 minutes

Step 1: Let the group appoint a group leader to ask the following questions to the rest of the members in the group:

1. "How many members are there in the group?" and he/she appoints one of the members to walk around the group touching each of the other members and counting aloud: 1, 2, 3, ...
2. "How many eyes are there in the group?" and the leader appoints someone else to walk around the group touching each member's eyes, counting aloud: 1, 2, 3, ...
3. "Could we have counted the eyes more quickly?" and let another member walk around the group touching each member's pair of eyes, counting aloud: 2, 4, 6, ...

Repeat the above activities for:

1. Ears;
2. Hands and fingers (counting first in fives and then in tens); and
3. Feet and toes (counting first in fives and then in tens)

3. Counting as far as possible

- Provide a very large pile of counters and let the children on the mat count the counters in the pile by taking turns.
- Let one child start the counting and ask the next child to take over when the first child reaches a number inside a decade (e.g. 37 or 83 etc).

- The pile should contain several hundreds of counters so that each child, in their turn, counts through at least two or three decade transitions.

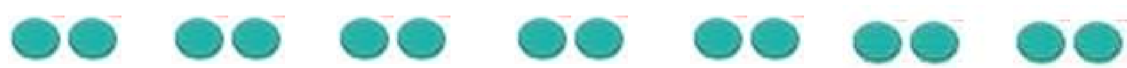
- **It is important to count well beyond one hundred in this way** as many children will only develop their sense of two-digit numbers when working with three-digit numbers.
- As the numbers get larger the children should be helped to group the objects in sensible groups (twos, fives, tens etc.).

4. Counting on

- A variation of the counting as far as possible activity is to have the children count the counters into a container (say a jar) and to write the number reached on any one day on the side of the jar.
- On the next day the group continues to add counters to the jar and the new total is written on the side.
- Continue in this way and see how far each group on the mat can get each week.

5. Grouping and counting

- Give each child 100 counters and ask each child to arrange his/her counters in groups – (two's or fives or tens or whatever they want)



- The children count all the counters (as a group) in the groups as they are arranged.
- The teacher decides about the order in which the counters are going to be counted.
- It is unrealistic to expect them to count large numbers in e.g. 4's or 6's.)

6. Throwing dice and adding the numbers

- The children take turns to **throw** two or three dice
- They add the numbers together as quickly as possible.
- Ask questions such as:
  - What is double/half the answer; multiply the answer by 3/5/10; what is 20 more etc.

7. Estimating the number of counters in a pile and counting

As children are gaining confidence with the basic counting skills described above they will be starting to develop a sense of the "muchness" of numbers – they are ready to start estimating and checking their estimates.

An example of an estimation activity with a group of learners in the classroom:

- Place a **large** pile of counters on the mat.

- Ask the children to estimate the number of counters in the pile – they should either write down their estimate or tell it to the group.
- Let the children work together in the group to arrange the counters in sensible piles (5s, 10s, 50s and 100s)
- One of the children is asked to count the pile of counters while the other children listen to see if it is done correctly.
- Compare the estimates that the children made with the actual number.

Task 2: 10 minutes

Step 1: Pass a bottle of counters around and ask the participants to **estimate** how many counters there are in the bottle.
Step 2: They write their estimation down in the space provided.
Step 3: Call 4 participants to the front to arrange the counters in groups of 10.
Step 4: Call another 2 participants to the front to count the number of counters in 10's aloud while the other participants listen.
Step 5: The participants write down the accurate answer in the space provided.
Step 6: They calculate the difference between their estimation and the accurate answer.
Step 7: Ask the participants to put their hands up if their estimate was more and whose was less than the actual number. You can ask whose was 10 more/less etc. – but never "Whose estimate is correct?" as **we should not have an expectation that estimates are correct – only better or worse.**

Estimation: ___________

Accurate number: ________

Difference: ________

Note: At first we should expect that children's estimates will vary widely from the actual amount, but with time and as they develop a sense of the "muchness" of numbers so they will become more accurate. In comparing their estimates with the actual numbers, the teacher should encourage children to get a sense of whether their estimate was more or less than the actual number and even whose estimate was closest to the actual number – but never "Whose estimate is correct?" as **we should not have an expectation that estimates are correct – only better or worse.**

2. Breaking down and building up numbers

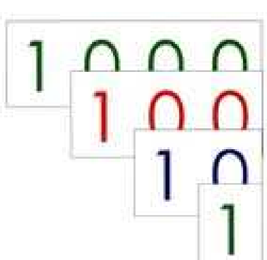
Level 3 number sense involves children being able to do three related actions:

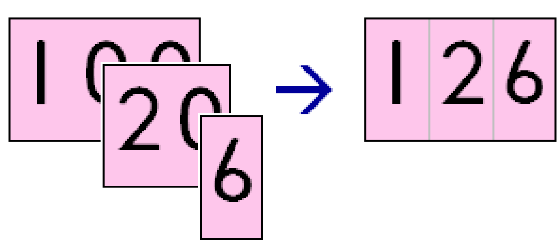
- Breaking down (decomposing) numbers;
- Reorganising the parts; and
- Recombining them to perform a calculation and/or to solve a problem.

When children are able to **break down, rearrange** and **build up numbers with confidence** they have developed:

- a **strong sense of number**; and
- the **confidence** to use numbers **to solve problems.**

Breaking down, rearranging and building up numbers should be regarded as the aim/goal of the Foundation Phase: having children work with confidence at Level 3.





Tens and Units

Many teachers think that Tens and Units are place value. However, that is the **final** part of developing an understanding of place value, **not the first**. Teachers are only just beginning to develop the concept of place value in Grade 2. Learners need to still develop an understanding of numbers and their relationship e.g. 25 is 5 more than 20. The social knowledge they need is that the symbol for twenty is 20. The 2 and the 0 are both part of the number. The 0 is not a place holder – it is part of how 20 is written. The logico-mathematical knowledge learners need to develop is that 25 is 20+5 as well as 24+1 or 30-5 and so on.

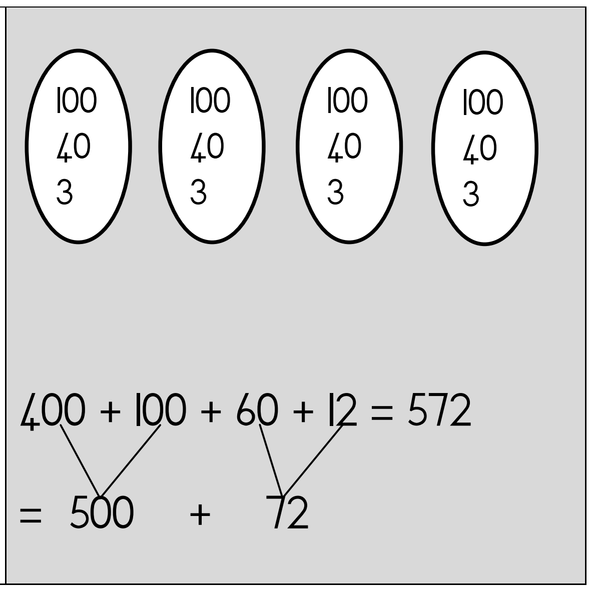
This is why "Numerosity" is done every day as it helps learners build up an understanding of the relationship between numbers. It will be a long time before learners reach the point of being able to understand column arithmetic and it is for this reason that working vertically in columns is found for **the first time in the Grade 5 Assessment Standards NOT Grade 3!**

Numeracy Handbook for Foundation Phase Teachers Grades R-3, DBE;

Children who can confidently break down, rearrange and build up numbers are ready to work with numbers in a more abstract sense – the way that is expected when we introduce them to the more formal (abstract) algorithms of the late Intermediate and early Senior Phases.

The following is an example of a problem that involves repeated addition. The child solved the problem by **breaking down, reorganising** and **combining** numbers.
This child has a **Level 3** sense of number.

This Grade 3 child is solving a problem that involves the repeated addition of four lots of 134.
First the child decomposes 143 into 100 and 40 and 3.
She then reorganises the parts to combine the four 100s to get 400, the four 40s to get 160 and the four 3s to get 12.
Then she decomposes the 160 into 100 and 60, reorganises and combines the 400 and the 100; and the 60 and the 12. Finally she combines 500 and 72 to give an answer of 572.



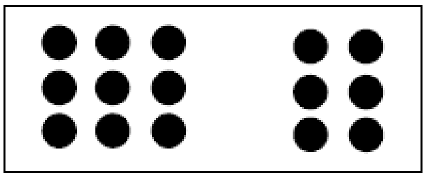
Note: In order to assist children in developing a sense of all the ways in which they can break down, reorganise and build up numbers teachers need to provide opportunities that help them to think about various ways of doing so.

Examples of activities that involve breaking down, rearranging and building up numbers

1. Cards with drawings/pictures on them

The following examples are extracts from: *Numeracy Handbook for Foundation Phase Teachers Grades R-3*, DBE

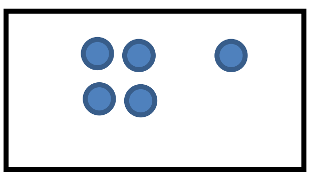
Given cards such as those on the right, children are asked first to count each group of objects and then to count the total.



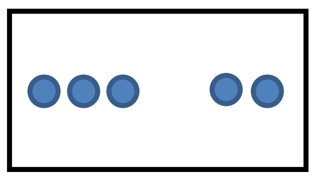
9 + 6 = 15

Variations include having children make their own arrangements of pairs of numbers for example:

- Make as many pairs of numbers that together make 5, 10, 15, 20 and so on.

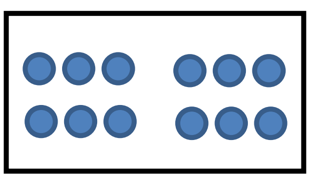


4 + 1 = 5

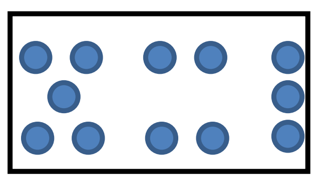


3 + 2 = 5

- Make the number 12 using two numbers, using three numbers, using four numbers and so on.



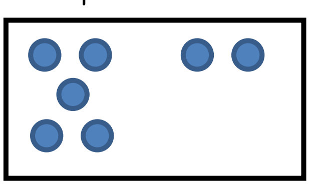
6 + 6 = 12



5 + 4 + 3 = 12

- For a given set of cards determine how many more dots are needed to make 10, 20, 30 and so on.

  Example: **Make 10**

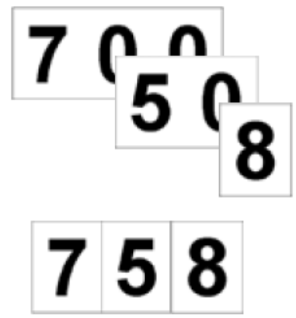


These questions and more like them will help children to develop a sense of all of the different ways in which different numbers can be made.

2. The use of Flard Cards

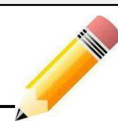
Flard cards are cards that allow children to build up numbers from their constituent hundred, ten and unit elements.

Foundation Phase children who are working with a number sense at or below Level 3 are not able to think of 758 as 7H + 5T + 8U because this is too abstract for them. By contrast these same children can work with the idea of "seven hundred plus fifty plus eight" (700 + 50 + 8) much more easily because this way of thinking matches the way in which they read and say the number names more closely. *(Numeracy Handbook for Foundation Phase Teachers*, DBE)



Task 3: 15 minutes

Step 1: Give each participant a set of Flard cards.
Step 2: Ask them to pack out the cards in sequence.
Step 3: Let the participants count in 1's from 220 to 268
Step 4: Ask them to point to find 268 on the number grid.
Step 5: Now ask them to make 268 using the flard cards.
Step 6: Ask one of the participants to share with the groups which cards he/she used– Tell the participants that by sliding the cards apart the learners will come to realise that: 268 = 200 + 60 + 8.
Step 7: Give the participants a few more numbers to build up and break down.
Step 8: Ask them to make 385 in at least three different ways using the Flard cards.
Step 9: Let them write the combinations in the space provided.



1. 385 = ______________________________
2. 385 = ______________________________
3. 385 = ______________________________

NOTE to teachers: You cannot expect in Step 8 for the learners to place the cards on top of each other to reveal the number being constructed. What these latter activities do is to force the child to explore number relationships/bonds and in particular the combinations of numbers that can be used to make other numbers.

Examples of combinations of 157:

100 50 7

70 80 7

60 40 50 5 2

TASK 4: 10 minutes

Step 1: Ask the participants to solve the following problem using their flard cards.
Step 2: While the groups are busy, walk around and listen to their discussions. Do not engage in any discussions.
Step 3: Simply present it to them and leave them to work it out.
Step 4: Let them write down their steps in the space provided.
Step 5: At the end of 5 minutes stop the discussions and check if anyone has reached a solution. The solution is not important at this stage. You are looking at the **process** rather than the answer.
Step 6: Ask 2 or 3 participants to explain their strategies with the rest of the group and to show their working on a flip chart in the front of the room.

Mr Dondolo had 367 cows on his farm. Last Friday he bought 498 more cows at the stock fare. How many cows does he now have on his farm?

Reflection: 3 minutes
Participants reflect on what they have learnt in this session.

Discuss the following problem solving strategy used by a Grade 2 child to solve Mr Dondolo's problem:

300 + 400 = 700
60 + 90 = 150
8 + 7 = 15
700 + 150 = 850
850 + 15 = 865

This Grade 3 child is solving a problem which involves determining the value of 367 + 498. She breaks the numbers into 300 + 60 + 7 and 400 + 90 + 8 and then uses her knowledge of combinations of single-digit numbers (8 + 7 = 15) and combinations of multiples of ten (60 + 90 = 150) and combinations of multiples of 100 (300 + 400 =700) as tools determining 865 as her answer.

3. "Up and Down the Number Line"

Being able to break down, reorganise and build up numbers requires that children have a well-developed mental number line. This mental image allows children to sequence numbers and to move flexibly between them.

To assist children to develop this number line and to move flexibly along it the teacher can, while working with children on the mat, have children participate in a sequence of questions and answers such as the one in the box below.

T: I have 27, what do I get when I add 3? (30)
T: Now what will I have left if I give away 8? (22)
T: What am I left with if I give away 7? (15)
T: What will I get by adding another 20? (35)
T: How many more do I need to get to 40? (5)

"Up and down the number line" is an activity that should be completed mentally – if children are doing the calculations on their fingers then they are probably still at Level 1 and in need of some of the less sophisticated activities mentioned below or the same activity but with smaller numbers at each stage.

With children who are in the early stages of achieving level 2 number sense, the numbers used in the activity will be very small (1s, 2s and 3s). As the children's confidence grows so the activity can be used to develop skills used by people with a strong sense of number.

Such skills include:

- **Completing tens**
  - I have 17, what do I get if I add 3?
  - What must I add to 27 to get 30?
- **Bridging tens:**
  - I have 17, what do I get if I add 8?
    The expectation is that the child will first complete the ten and then add on the remainder. Their thinking could be summarised as: "I have 17, I need 3 to make 20, 8 – 3 = 5 so the answer is 20 + 5 = 25."
  - What must I add to 27 to get 32?
- **Adding to multiples of ten**
  - I have 17, what will I get if I add 33?
  - What must I add to 27 to get 60?
- **Adding and subtracting multiples of ten**
  Children need to realise that if 2 + 3 = 5 then 20 + 30 = 50 and 200 + 300 = 500
  - I have 30, what will I get if I add 50?
  - I have 70, what will I get if I take away 30?
  - What must I add to 10 to get 70?
  - What must I take away from 90 to be left *with 30?*
- **Subtracting to multiples of ten**
  - I have 27, what do I get if I take away 7?
  - What must I take from 45 to get 40?
- **Subtracting from multiples of ten**
  - I have 70 what do I get when I take away 8?
  - What must I take away from 40 to get 36?

(*Numeracy Handbook for Foundation Phase Teachers*, DBE)

As children gain confidence in completing this activity within a two-digit number range so teachers can extend exactly the same activity to numbers within the three-digit range. It is not likely that this will happen before Grade 3 at the earliest.

Task 5: 5 minutes

Develop your own sequence of questions for the learners to participate in.

1.

2.

3.

4.

5.

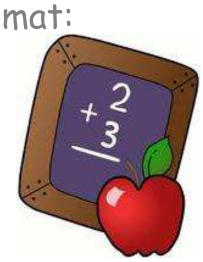
6.

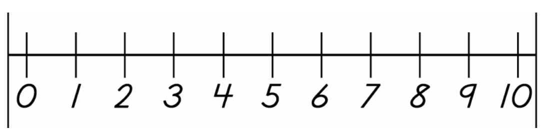
3. Using numbers in solving problems

The use of meaningful problems is one of the three crucial factors for developing

In terms of classroom management children should be exposed to problems on a frequent basis as they work with the teacher on the mat. They should be given the opportunity to solve the problems, to explain their solution methods and to listen to and learn from the solution methods of the other children in the group on the mat.

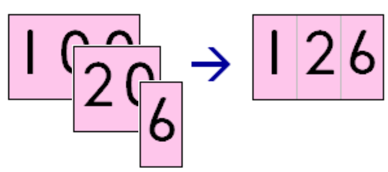
Children should have the following while working on problems on the mat:





- A book or slate in which to record their thinking and solutions strategies.

- Tools that may help them in solving problems such as:
  - Counters;
  - Counting frames;
  - Flard cards; and
  - Numbers lines.



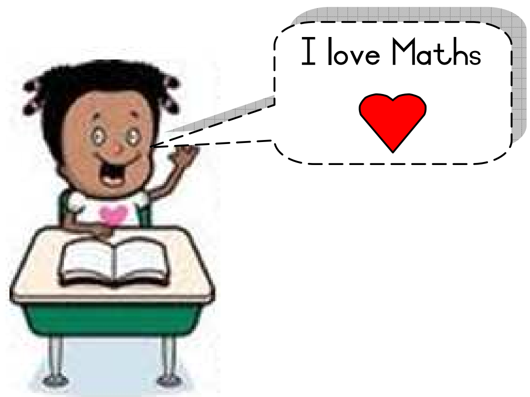
While these tools may help, they are no more than aids and with time as the child's number sense get stronger we would expect children to become less reliant on them.

Teachers should have the following while working on problems on the mat:

- A notebook in which to record their observations about the children in each group, in particular:
  - The number level at which each child is working; and
  - The types of problem solving strategies that they are using.

- A list of problems that they plan to work through – the success of using problems to develop both computational strategies and a strong number sense lies in the kinds of problems used by the teacher – random problems "invented" on the spur of the moment will not have the desired effect.

4. Written activities



As the teacher works with a small group on the mat, the rest of the class need to be working at their tables. It is important that the task set by the teacher reinforces the work being done on the mat.
The activities selected by the teacher should match the developmental stage of the child. In particular, the stage of number development at which the child is.

For children who are working at Levels 2 and 3 these written activities focus on the **breaking down, reorganising and building up** of numbers.

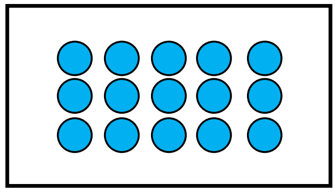
Activities include:

1. Counting

The kinds of counting activities used at this stage in a child's number level development (Level 2 and Level 3) include:

a. Counting grouped objects: grouped objects encourage children to learn to count in groups (to skip count).

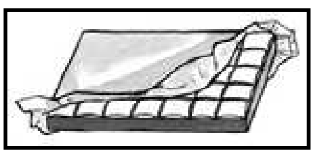
b. Counting objects arranged in grids: objects arranged in grids encourage children to skip count and also to construct their conceptual knowledge of the commutivity (3 x 2 = 2 x 3).

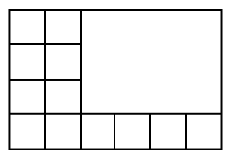


By counting the dots in the grid above children will come to realise that the number of dots can be counted as 5 + 5 + 5 or as 3 + 3 + 3 + 3 + 3
This observation leads to the realisation that 3 × 5 = 5 × 3 (commutivity)

When children realise that 3 × 5 = 5 × 3 they will have not only learnt an important pattern, but they will also have halved the number of number facts that they need to remember

c. Counting items where part of the collection of items is hidden.





An example of a collection to be counted where part of the collection is hidden
In order to count the number of small squares in this bar of chocolate, the child is left with no option but to count in multiples of 5 or 7 since they cannot actually count each and every square in the bar of chocolate.

Adapted from: *Numeracy Handbook for Foundation Phase Teachers*, DBE

2. Completing number patterns

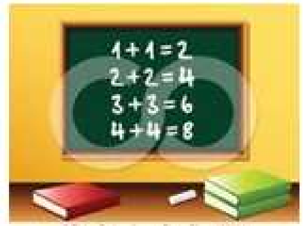
In order for children to develop a sense of the regularity and patterns within numbers they should be able to extend number patterns.

There are two key kinds of written activities involving patterns that can help in the development of number sense at this level.

a) The first is a sequence of numbers with some of the terms of the pattern given and the child must complete the pattern by either extending the pattern by a number of terms or filling in some missing terms – this kind of pattern is useful in developing skip counting and adding-on type skills.
Example: 5, 10, 15, ___, ___, 30, ___, ___, ___, 50

b) The second kind of pattern recognition task requires children to complete a series of simple arithmetic operations. In completing the calculations posed the child should also notice a pattern.

Example: 5 = 1 + 4   50 = 10 + 40
5 = 2 + ___   50 = 20 + ___
5 = 3 + ___   50 = 30 + ___
5 = 4 + ___   50 = 40 + ___



It is the recognition of the patterns within and between situations that help children in the development of conceptual knowledge – something that is crucial in the development of a robust number sense.
However, it should be noted the child is more likely to notice the pattern only if the teacher asks them to reflect on the activity – once again highlighting the importance of discussion.

Teachers can create many such examples using numbers that are appropriate to the number range within which her class is working.

Task 6: 10 minutes

1. Develop two number sequences that the learner must complete by either extending or filling in some missing terms. 2. Develop two series of simple arithmetic operations that the learner must complete (see the example in (b) above).

1.

2.

3. Tables

Tables provide a different way in which to represent patterns. The significance of tables lies in the relationship between two numbers that is evident – another pattern that we want children to realise.

Example:

Dogs	1	2	3	4	5		10	15	19	
Ears						18				50

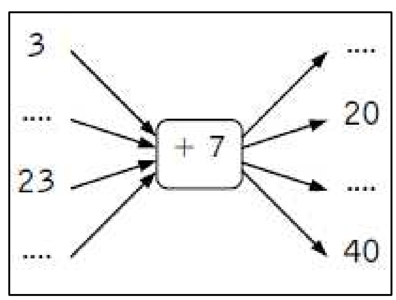
Task 7: 5 minutes

Develop a table in which a pattern is represented. Learners must fill in the missing numbers.

4. Flow diagrams

Flow diagrams provide another way of engaging children in productive thinking about numbers and the patterns within numbers.

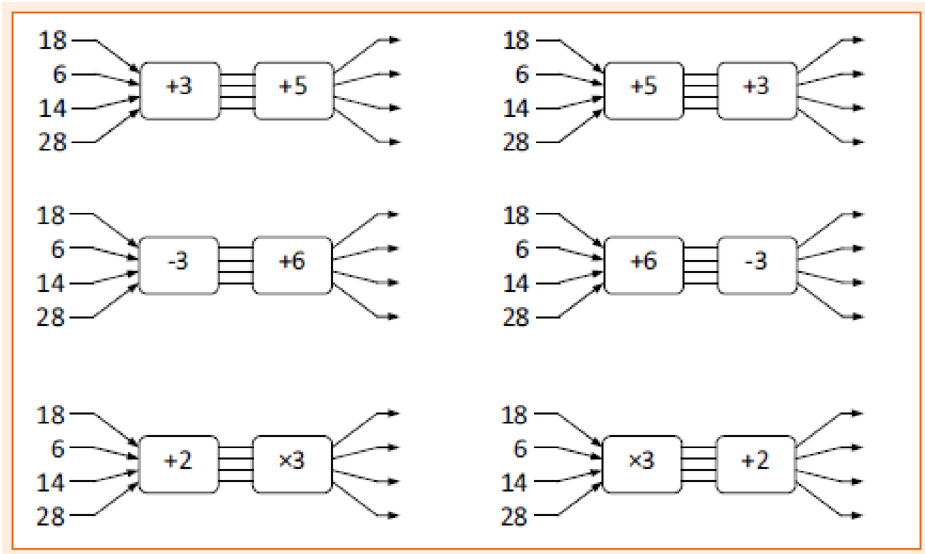
Children who are at Level 3 in terms of their number development are children who are able to break down and build up numbers in a range of different ways. "**Completing tens (or hundreds)**" is an important skill that children need to develop in order to break down and build up numbers with confidence. To support children in developing this skill a teacher might ask her class to complete a number of flow diagrams such as the one alongside. The teacher wants the children to observe the pattern: "adding a seven to a number ending in 3; completes the ten".



By asking children "what did you do to complete this task so quickly?" the teacher is encouraging reflection on the task and the children will observe patterns and come to realise their value.
(Numeracy Handbook for Foundation Phase Teachers, DBE)

Task 8: 10 minutes

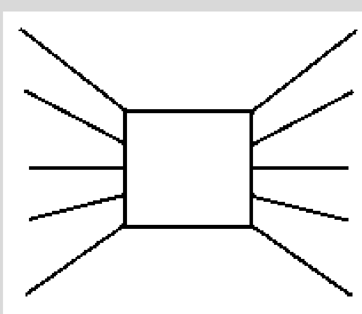
Complete the flow diagrams and discuss the patterns with your partner.

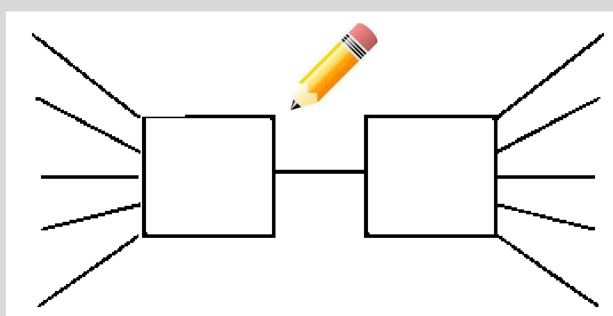


- Discuss in your group any addition and subtraction facts that you picked up in these flow diagrams.

Task 9: 10 minutes

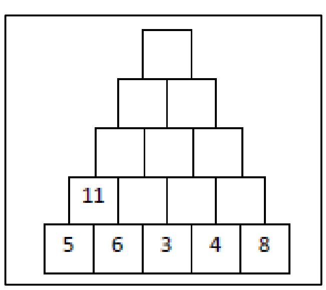
Design your own flow diagrams for the learners. The diagrams are drawn for you. Hint you can use any of the 4 basic operations.





5. Pyramids

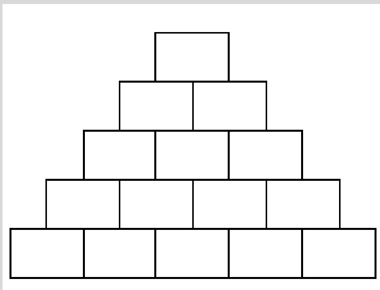
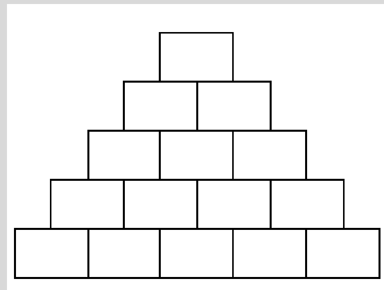
Pyramids provide another constructive activity for developing number sense



In a typical pyramid the number in each cell is determined by adding together the numbers in the two cells below the cell.
This pyramid provides an opportunity for children to work with numbers and in so doing to develop their number sense.

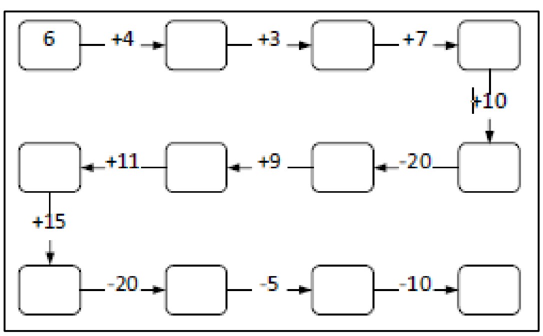
Task 10: 5 minutes

Design two pyramids for the learners. The diagrams are drawn for you.
Hint you can use any of the 4 basic operations.

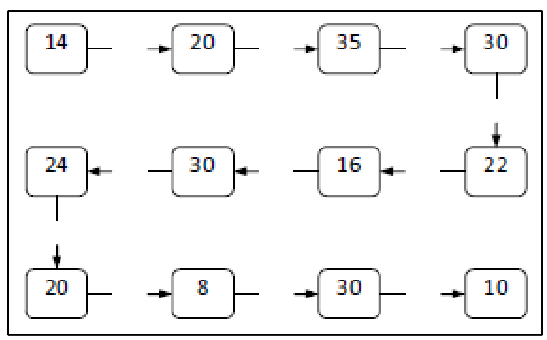




6. Number chains

Number chains are the written equivalent of the up and down the number line activities described earlier.



This number chain aims to develop the different skills listed under up and down the number line earlier.



This number chain forces the child to think about the operations required to achieve a particular "target".

7. Problems

In addition to children solving problems on the mat with the teacher they also need practice in doing so individually and as such should be challenged to **solve problems as part of their written work activities as well**.

The role of the teacher is to:

- Give the learners different types of problems so that they can develop an understanding of all four the operations (See Annexure 2).
- Observe which learners are more advanced in their approaches and which are still struggling. In so doing the teacher can select solutions from those developed by the children to be presented to the other children.
- Allow learners to use methods that suit them; otherwise they will become frustrated and copy from other learners.
- Introduce more sophisticated methods to learners who have reached a Level 3 sense of number to simplify the recording of what they did with numbers.

The following are examples of more "sophisticated" methods of recording:

What is the sum of 367 and 498?

367 + 400 →767 + 90 →770 + 87 →800 + 57 →857 + 8 = 865
or
300 + 400 = 700
60 + 90 = 150
7 + 8 = 15
700 + 150 + 15 →850 + 15 = 865

Note: Introduce the use of the arrow (→) notation to help the learners not to use the equal sign (=) incorrectly.

What is the difference between 531 and 247?

531 – 200 = 331
331 – 40 →331 – 31 – 9 →300 – 9 = 291
291 – 7 = 284

Multiply 28 by 3

20 x 3 = 60
8 x 3 = 24
60 + 24 = 84

Divide 75 by 5

75 = 50 + 25

20 x 3 = 60
8 x 3 = 24
60 + 24 = 84

Note to teachers when designing word problems:

- Make sure the problem has been posed at the correct level. If it is too easy, no thinking will take place. If it is too difficult the child will not even try. This can either be as a result of the number range or the language used.
- Therefore the **child's level of understanding** is important when designing problems.
- **Word problems come first.** They provide the context which builds understanding. They do not come last just to check technical workings.
- **Always read the problem to the children. The purpose of problem solving is to develop thinking skills and not to test reading**

8. Drill and practice

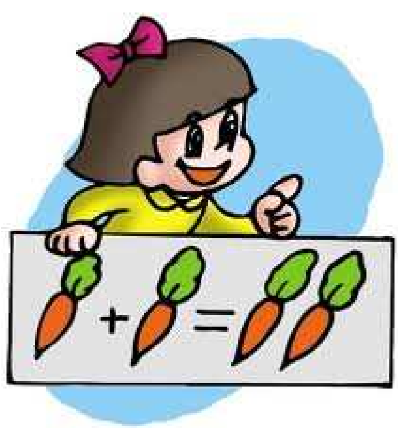
Being able to operate at Level 3 or higher in terms of number sense requires that each child simply knows a range of number facts.

For this reason there is much to be said for children also being set large collections of exercises involving the four basic operations and numbers appropriate to their stage of development.

Examples of activities:

3 + … = 10	18 – … = 10	3 × … = 12	16 ÷ … = 4
41 = 23 + …	7 = 18 – …	30 = 5 × …	9 = … ÷ 2
… + 16 = 43	… – 5 = 17	… × 7 = 21	… ÷ 4 = 11

Numbers sentences should be posed in the form illustrated above and children should be encouraged to complete these kinds of exercises against the clock.
(Numeracy Handbook for Foundation Phase Teachers: Grades R–3,DBE)



The following are alternative strategies for addition, subtraction, multiplication and division of 3-digit numbers. It is important that the learners are exposed to a number of different strategies, particularly those where whole 10's and 100's are involved. Do not force the learners to use these strategies. These are simply alternatives.

<table>
<tr><th colspan="4">352 + 137</th></tr>
<tr><td>1.</td><td>352 = 300 + 50 + 2
137 = 100 + 30 + 7

300 + 100 = 400
50 + 30 = 80
2 + 7 = 9
400 + 80 + 9 = 489</td><td>3.</td><td>352 = 300 + 50 + 2
137 = 100 + 30 + 7

300 + 100 →400 + 50→450 + 30 →480 + 2 →482 + 7 = 489</td></tr>
<tr><td>2.</td><td>137 = 100 + 30 + 7

352 + 100 → 452 + 30 →482 + 7 = 489</td><td></td><td></td></tr>
<tr><th colspan="4">532 - 264</th></tr>
<tr><td>1.</td><td>264 = 200 + 60 + 4

532 – 200 = 332
332 – 60 = 272
272 – 4 = 268</td><td>2.</td><td>264 = 200 + 60 + 4

532 – 200 → 332 – 60 → 272 – 4 = 268</td></tr>
<tr><td>3.</td><td>500 – 200 = 300
300 – 60 = 240
240 – 2 = 238
238 + 30 = 268</td><td>4.</td><td>264 + <u>200</u>→464 + <u>30</u> →494 + <u>10</u>→ 504 + <u>20</u>→524 + <u>8</u> →532

532 – 264 = 200 + 30 + 10 + 20 + 8 = 268</td></tr>
<tr><th colspan="2">59 x 3</th><th></th><th>248 x 4</th></tr>
<tr><td>1.</td><td>59 = 50 + 9

50 x 3 = 150
9 x 3 = 27
150 + 27 = 177</td><td>1.</td><td>248 = 200 + 40 + 8

200 x 4 = 800
40 x 4 = 160
8 x 4 = 32

800 + 160 = 960
960 + 32 = 992</td></tr>
<tr><th colspan="4">96 ÷ 3</th></tr>
<tr><td>1.</td><td>96 = 30 + 30 + 30 + 6

30 ÷ 3 = 10
30 ÷ 3 = 10
30 ÷ 3 = 10
6 ÷ 2 = 3
10 + 10 + 10 + 2 = 32</td><td>2.</td><td>96 = 90 + 6

90 ÷ 3 = 30
6 ÷ 3 = 2
30 + 2 = 32</td></tr>
<tr><th colspan="2">76 ÷ 4</th><th></th><th>135 ÷ 5</th></tr>
<tr><td>1.</td><td>76 = 40 + 20 + 10 + 6

40 ÷ 4 = 10
20 ÷ 4 = 5
16 ÷ 4 = 4
10 + 5 + 4 = 19</td><td>1.</td><td>135 = 50 + 50 = 35

50 ÷ 5 = 10
50 ÷ 5 = 10
35 ÷ 5 = 7
10 + 10 + 7 = 27</td></tr>
</table>

Addition and Subtraction Problem Types

Change Join Separate

1. Mary has 5 marbles. Jim gave her 8 more. How many marbles does Mary have now?
2. Mary has 5 marbles. How many more marbles does she need to have 13 marbles?

Change Separate

3. Mary had 13 marbles. She gave 5 marbles to Jim. How marbles does she have left?
4. Mary had 13 marbles. She gave some to Jim. Now she has 8 marbles left. How many marbles did Mary give to Jim?

Combine

5. Mary has 5 red marbles and 8 blue marbles. How many marbles does she have?
6. Mary has 13 marbles. Five are red and the rest are blue. How many blue marbles does Mary have?

Compare

7. Mary has 13 marbles. Jim has 5 marbles. How many more marbles does Mary have than Jim?
8. Mary has 13 marbles. Jim has 8 marbles. How many fewer marbles does Jim have than Mary?
9. Jim has 5 marbles. Mary has 8 more than Jim. How many marbles does Mary have?

Equalize

10. Mary has 13 marbles. Jim has 5 marbles. How many marbles does Jim have to get to have as many marbles as Mary?
11. Mary has 13 marbles. Jim has 5 marbles. How many marbles does Mary have to lose to have the same number of marbles as Jim?
12. Jim has 5 marbles. If he gets 8 marbles he will have the same number of marbles as Mary. How many marbles does Mary have?

Multiplication and Division Problem Types

Repeated Addition

13. Mother buys 4 bags of apples. Each bag contains 8 apples. How many apples did she buy?
14. Mother buys 32 apples that are packed in 4 bags. If each bag contains the same number of apples, how many apples are in each bag?
15. Mother buys 32 apples. She wants to pack them into plastic bags, with 8 apples in each bag. How many bags does she need?
16. I fill 10 cups with 200ml cool-drink each. How much cool-drink did I have before filling the cups?
17. 2l of cool-drink is poured into 10 cups so that each cup holds the same amount. How many millilitres of cool-drink is in each cup?
18. How many cups each holding 200ml can be filled from a 2l bottle of cool-drink?

Rate
19. A man walks at 6km per hour. How far does he walk in 3 hours?
20. Tomatoes are sold at R12 per kilogram. If I but 3 kilograms of tomatoes, how much will I have to pay?

Comparison (Times as many as)
21. Mary has 4 marbles. Jim has 3 times as many marbles as Mary. How many marbles does Jim have?
22. The length of a car in a photograph is 4cm. If the photograph is enlarged 3 times, what will the length of the car be on the enlargement?
23. Jim has 12 marbles, which is 3 times as many marbles as Mary has. How many marbles does Mary have?
24. If a photograph is enlarged 3 times, the length of a car on the enlargement is 12cm. How long is the car in the original photograph?

Arrays (Arrangements)
25. A slab of chocolate has 4 pieces along the shorter side and 6 pieces along the longer side. How many pieces does the slab contain?
26. A vegetable patch has 12 rows of onion plants, with 6 plants in each row. How many onion plants are there in the patch?

Combinations
27. Mary has 3 skirts of different colours and 4 tops of different colours. All the colours match. In how many different ways can she dress?

Sharing – with and without remainders, leading to fractions
28. My brother and I found 5 marbles. We each took the same number. How many did we each take?
29. Mom bought 8 sausages and her 4 children shared them equally. How many sausages did each child eat?
30. Mom bought 10 sausages and her 4 children shared them equally. How many sausages did each child eat?

Grouping – with and without remainders
31. I have 12 apples and put them into 3 baskets. How many apples are in each basket?
32. Mom bought 14 apples. How many packets of 4 apples can she make?

Repeated addition and subtraction
33. How many wheels do 4 bicycles have?
34. Father has R20. He gives R5 to each of his three children. How much money will he have left?

The key problems of the development of Numeracy in the Foundation Phase

As we support the development of numeracy in the Foundation Phase, the key problems or challenges include the following:

- **Too many children leave the Foundation Phase operating at Level 1**
  By the end of the Foundation Phase children should be working at Level 3. With a poorly developed sense of numbers children are unable to face the challenges of more advanced mathematics.

- **Too few teachers know the level at which each of the children in their class is working.**
  It is crucial that a teacher has a very clear sense of the level at which each of her children is working so that she can provide differentiated activities (see below) that are appropriate to the needs of each child.

  In part this is the result of too much whole class teaching in the Foundation Phase. By teaching the whole class only, teachers do not get to know the individual children and their needs sufficiently well. This sense is best developed by working with small groups of children on the mat on a regular basis.

- **Too few teachers are making deliberate plans to support the progression of children from one level to the next.**
  In the section that follows and in Unit 5 suggestions are made about what it takes to support children in their development.

- **Too many teachers believe that they are helping children by limiting the number range in which they are working.**
  It is critical that children are expected to work with as large a range of numbers as possible. It is, in part, through reasoning and engaging with and having to applying larger numbers that children develop a strong sense of number.

  If children are not exposed to larger numbers, i.e. the range of numbers with which they are working is limited in Grade 1 to say "less than 10 in the first term" and "less than 20 in the second term" etc. then there is no reason for them to develop a strong sense of number because they can survive using only Level 1 and at most Level 2 strategies.

References:

- The National Curriculum and Assessment Policy Statement **(CAPS);**
- Numeracy Handbook for Foundation Phase Teachers Grades R-3, DBE;
- Foundations for Learning Numeracy Lesson Plans, Grade 3;
- Foundation Phase Draft Training Toolkit Manual for CAPS (February-March 2011), DBE;
- Adding it up: helping children learn mathematics (NRC, 2001)