Grade 12 Mathematics: Question Paper 2

MARKS: 150 marks **TIME: 3 hours**

QUESTION 1

1.1 A(0;1) ; B(-2;-3); C(8;2); D(d;6) are the vertices of the parallelogram ABCD.

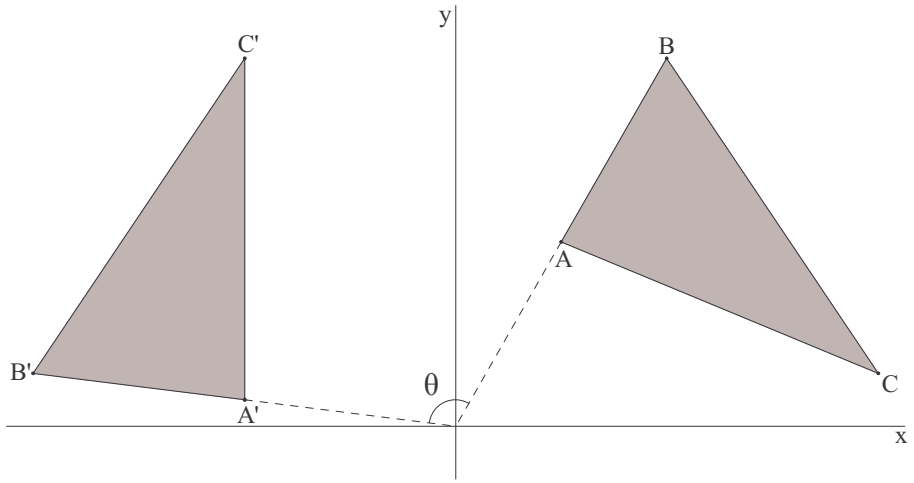
1.1.1 Calculate the gradient of BC (2)

1.1.2 Hence, determine the equation of AD and use it to calculate the value of d. (4)

1.1.3 Calculate the equation of the altitude AE of ΔABC with E on BC. (2)

1.1.4 Determine the coordinates of F, the point of intersection of the diagonals. (2)

1.2 In the diagram below triangle A'B'C' is the image of triangle ABC after a rotation of $\theta°$ about the origin.



1.2.1 If the coordinates of A and A' are (4;7) and (-8;1) respectively, show that $\theta = 112{,}62°$ (6)

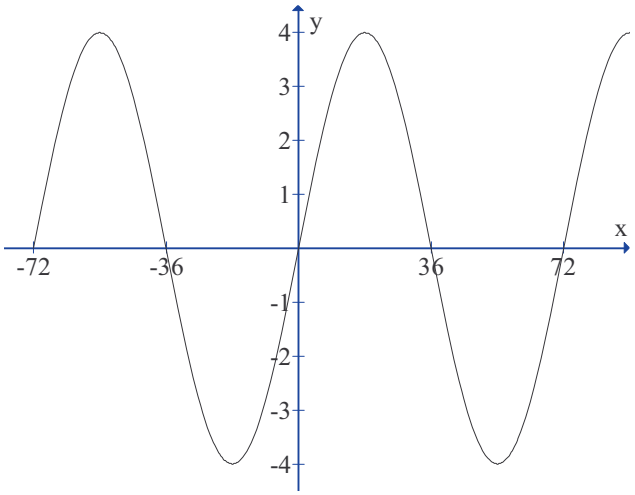
1.2.2 Hence, or otherwise, determine the coordinates of B' if B is the point (8;14). (4)

1.3 Solve the following equations for: $0° \leq x \leq 90°$

1.3.1 $2\tan x = -0{,}6842$ (2)

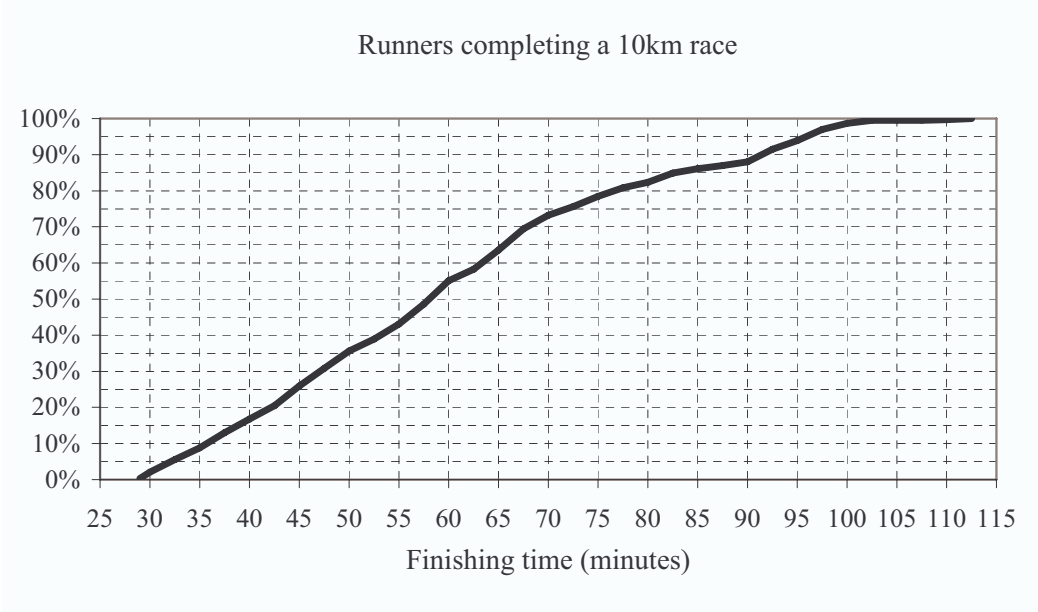
1.3.2 $\sin 2x.\cos x - \sin x.\cos 2x = 0{,}500$ (2)

1.4 What function is represented by the following graph:



(2)

1.5



The (cumulative frequency) ogive curve above represents the finishing times of the 590 runners who completed a 10km race.

1.5.1 Estimate in how many minutes a runner would have had to complete the race in order to place at the 15th percentile or better (2)

1.5.2 If a silver medal is awarded to all runners completing the race in under 40 minutes, estimate the number of runners who would have received a silver medal. (3)

1.5.3 Draw a box and whisker plot to summarise the data represented on the graph (5)

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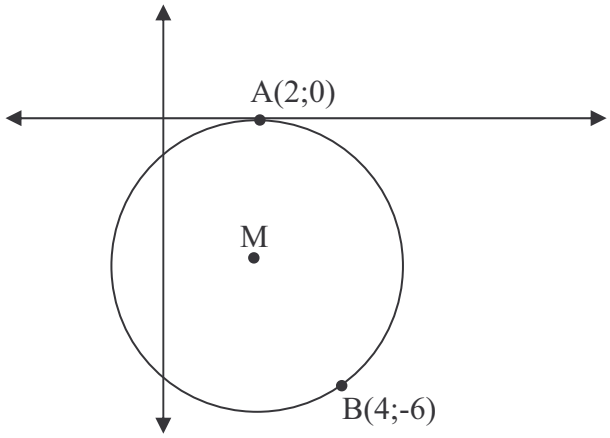
QUESTION 2

2.1



Refer to the diagram above and determine:

2.1.1 The equation of a circle, centre M, which touches the x- axis at A(2;0) and passes through B(4;-6). (7)

2.1.2 The equation of the tangent to the circle at B. . (4)

2.2 Given the equation of the circle $x^2 + y^2 - 12x - 6y + 20 = 0$ and the line $2x + y - 5 = 0$

2.2.1 Show that the line and the circle cut each other at A(1;3) and B(3;-1). (6)

2.2.2 Determine the length of AB (leave your answer in simplified surd form). (3)

2.2.3 Determine the equation of the perpendicular bisector of AB. (5)

2.2.4 Hence or otherwise, determine the centre of the circle. (6)

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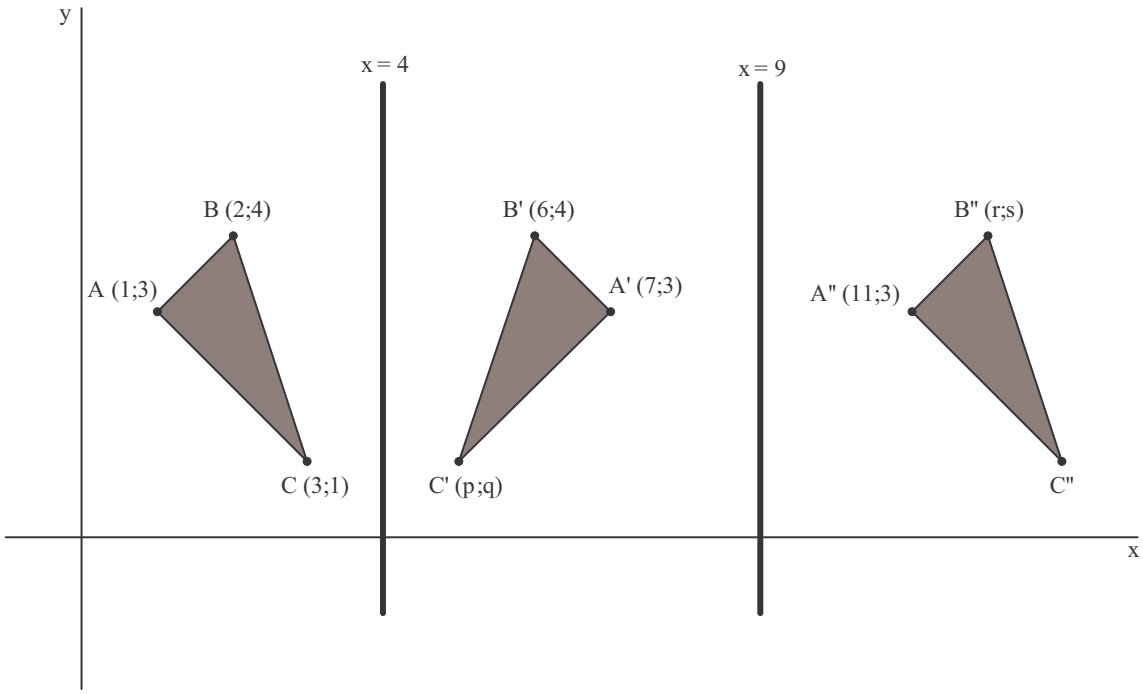
QUESTION 3

3.1



In the diagram above:

- Triangle A'B'C is the reflection of triangle ABC about the line $x = 4$; and
- Triangle A"B"C" is the reflection of triangle A'B'C' about the line $x = 9$

This combination of transformations is called a composition of transformations

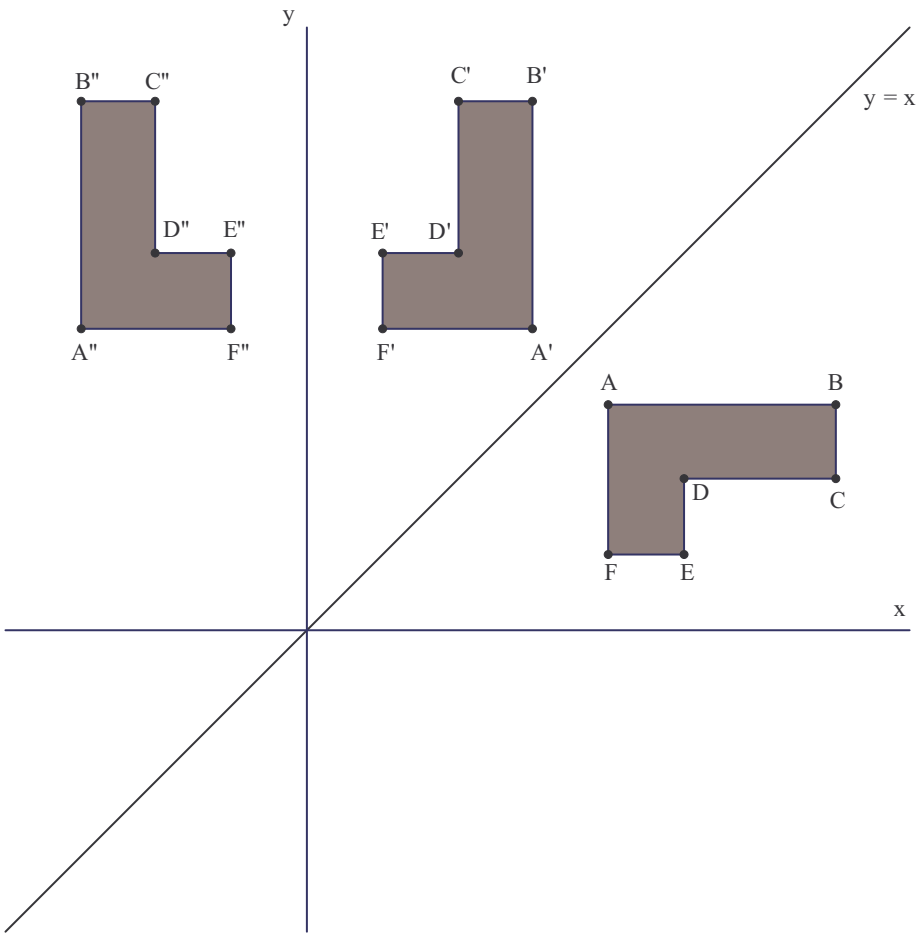
3.1.1 Determine (with justification) the values of p and q: the co-ordinates of C' (3)

3.1.2 Determine (with justification) the values of r and s: the co-ordinates of B" (3)

3.1.3 Describe (with justification) a single transformation that would have the same result as the composition of transformations: *reflection about the line $x = 4$ followed by reflection about the line $x = 9$*, i.e. a single transformation that could be used to transform triangle ABC into triangle A"B"C" (2)

3.1.4 By considering the point A(1;3) discuss whether or not the composition of transformations: reflection about the line $x = 9$ followed by reflection about the line $x = 4$ will give the same result as the composition above. (3)

3.2



In the diagram above hexagon A"B"C"D"E"F"C is the image of hexagon ABCDEF after the following composition of transformations:

- Reflection about the line $y = x$ followed by:
- Reflection about the y-axis

3.2.1 Show that the co-ordinates of A" are (-3;4) if A is the point (4;3) (3)

3.2.2 Hence, or otherwise, show that the composition of transformations above is equivalent to a rotation of 90° about the origin. (4)

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QUESTION 4

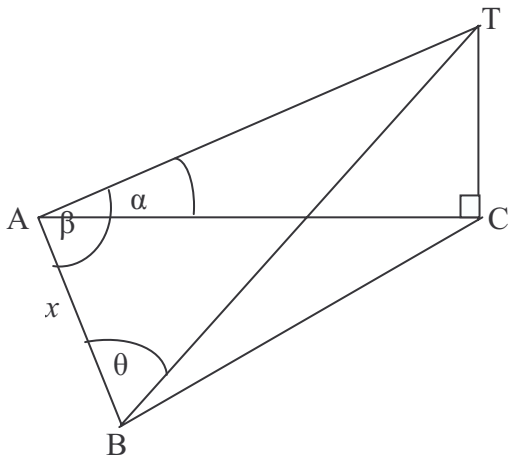
4.1 $$\frac{\sin 3\theta}{1+2\cos 2\theta} = \sin\theta$$

4.1.1 Prove the above identity. (6)

4.1.2 Why is the identity not valid for $\theta = 60°$ (2)

4.1.3 Without any further calculations, determine another value of θ for which the identity will not be valid. (1)

4.2 In the figure, TC is a vertical tower. Two wires leading from the top of the tower are staked at position A and position B so that A; B and C are all in the same horizontal plane. The angles TAB and TBA are measured and found to be β and θ respectively. The distance between the stakes is x metres. If the angle of elevation from stake A to the top of the tower is α, calculate the height of the tower in terms of x; β; θ and α .



(5)

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QUESTION 5

The table below shows the time of sunrise and sunset in Cape Town for every 15th day of the year 2007 (source: http://www.saao.ac.za) and the graph shows the models referred to in the questions below.

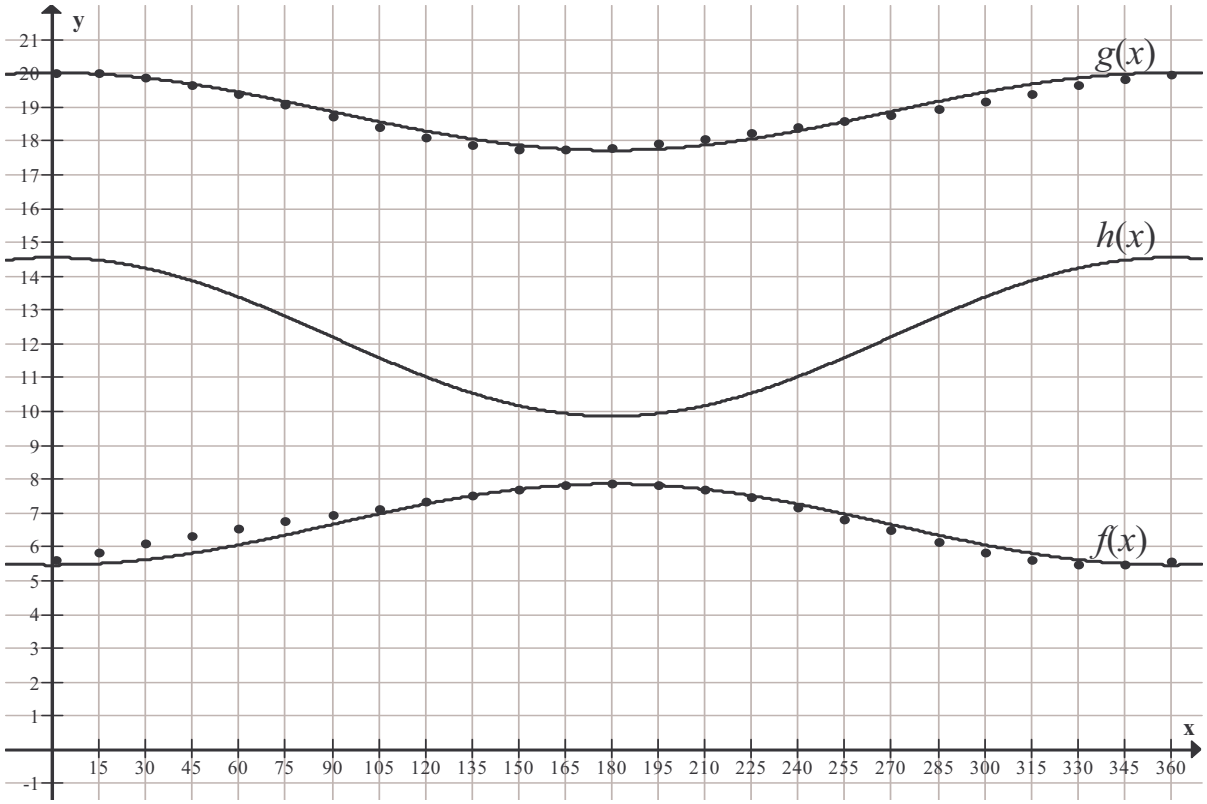
		sunrise	sunset
day of year	date	hours:min	hours:min
1	01-Jan-07	05:38	20:01
15	15-Jan-07	05:50	20:00
30	30-Jan-07	06:05	19:53
45	14-Feb-07	06:20	19:40
60	01-Mar-07	06:33	19:23
75	16-Mar-07	06:46	19:04
90	31-Mar-07	06:57	18:43
105	15-Apr-07	07:08	18:24
120	30-Apr-07	07:20	18:07
135	15-May-07	07:31	17:54
150	30-May-07	07:41	17:46
165	14-Jun-07	07:49	17:44
180	29-Jun-07	07:52	17:47
195	14-Jul-07	07:49	17:55
210	29-Jul-07	07:41	18:04
225	13-Aug-07	07:28	18:15
240	28-Aug-07	07:10	18:25
255	12-Sep-07	06:50	18:35
270	27-Sep-07	06:29	18:46
285	12-Oct-07	06:09	18:57
300	27-Oct-07	05:51	19:10
315	11-Nov-07	05:37	19:23
330	26-Nov-07	05:29	19:38
345	11-Dec-07	05:28	19:50
360	26-Dec-07	05:34	19:59

After plotting the times of sunrise and sunset on the graph, Frank has determined that he will use the cosine function to model the data with each day of the year corresponding to a degree, i.e. the 15th day of the year is represented by $x = 15°$.

Frank's model for sunrise is given by:

$f(x) = -1,2\cos(x \quad 180^\circ) \quad 6,66$

The graphs are shown on the next page.



5.1 Show that this model gives the correct time for sunrise on day 180. (3)

5.2 By how many minutes does the time of sunrise predicted by Frank's model differ from the actual time of sunrise on the 60th day of the year? (3)

Developing a model for the time of sunset:

5.3 Show that the time difference, in decimals, between the earliest and the latest time of sunset ≈ 2,283. (3)

5.4 Hence, or otherwise, explain how the time of sunset can be modeled with the function: $g(x) = a\cos(x \quad p) + q$; where $a = 1{,}142$; $p = 0$; and $q = 18{,}875$ (4)

5.5 By how many minutes does the time of sunset predicted by this model differ from the actual time of sunset on the 285th day of the year? (3)

$h(x)$, shown on the graph, is a model that predicts the number of hours of sunlight for each day of the year.

5.6 Express $h(x)$ in terms of $f(x)$ and $g(x)$ (2)

5.7 Hence of otherwise determine the day of the year and the number of hours of sunlight on the day with:

(a) the most hours of sunlight (2)

(b) the least hours of sunlight (2)

5.8 By how many minutes does the actual number of hours of sunlight and the predicted number of hours of sunlight differ on the 75th day of the year? (4)

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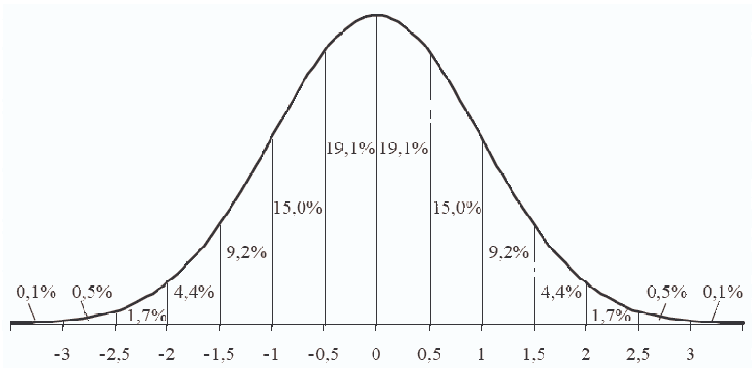
QUESTION 6

Refer to the Standard Normal distribution curve supplied alongside to answer questions 6.1 and 6.2:



6.1 Mrs. Hlope is a real estate agent. Last month, the sale prices of homes in her area approximated a normal distribution with a mean of R150 000 and a standard deviation of R25 000.

6.1.1 A house had a sale price of R175 000. What is the percentile rank of its sale price, to the *nearest whole number*? Explain what that percentile means. (4)

6.1.2 Mrs. Hlope told a customer that most of the houses sold last month had selling prices between R125 000 and R175 000. Explain why she is correct. (5)

6.2 The test results and various calculations for 20 students appear in the table alongside.

Test results	$(x-\bar{x})$	$(x-\bar{x})^2$
59	-3,7	13,69
58	-4,7	22,09
57	-5,7	32,49
55	-7,7	59,29
68	5,3	28,09
54	-8,7	75,69
89	26,3	691,69
73	10,3	106,09
60	-2,7	7,29
98	35,3	1246,09
65	2,3	5,29
18	-44,7	1998,09
59	-3,7	13,69
48	-14,7	216,09
58	-4,7	22,09
56	-6,7	44,89
72	9,3	86,49
60	-2,7	7,29
74	11,3	127,69
73	10,3	106,09

$\bar{x} = 62,7$

$\sum(x-\bar{x})^2 \quad 4910,2$

6.2.1 Determine the standard deviation of the test results (2)

6.2.2 Determine what percent of the students scored within one standard deviation of the mean. (3)

6.2.3 Hence, or otherwise, determine whether the test results approximate a normal distribution. Justify your answer. (3)

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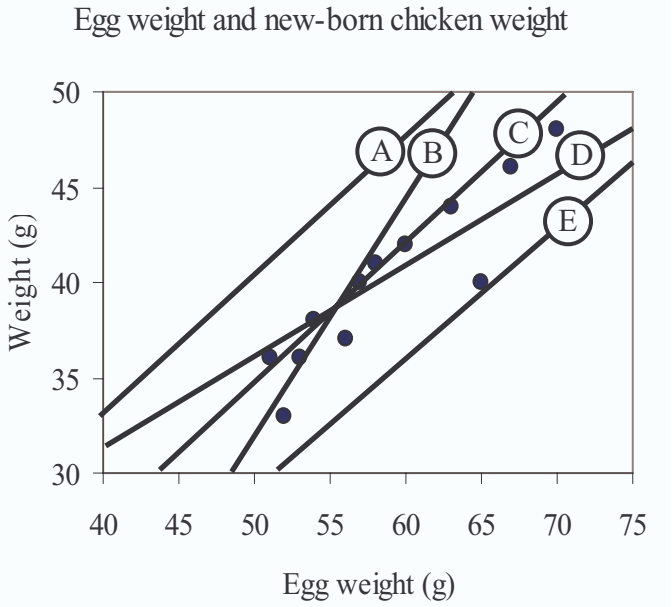
QUESTION 7

The scatter plot on the right compares egg weight and new-born chicken weight for a number of chicken eggs.



Each of the lines can be used to predict new-born chicken weight for given egg weights. Which line:

7.1 Predicts new-born chicken weights that are too high? How can you tell this from the plot? (2)

7.2 Predicts new-born chicken weights that are too low? (1)

7.3 Overestimates new-born chicken weight for lighter eggs? How can you tell this from the plot? (2)

7.4 Overestimates new-born chicken weight for heavier eggs? (1)

7.5 Tends to be the best predictor of new-born chicken weight? How can you tell this from the plot? (2)

[8]

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