



education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

EXEMPLAR 2008

MEMORANDUM

This memorandum consists of 12 pages.

QUESTION 1

1.1.1	$x^2 - 10x = 24$ $x^2 - 10x - 24 = 0$ $(x-12)(x+2) = 0$ $x = 12 \text{ or } x = -2$ OR $(x-5)^2 = 49 = 7^2$ $\therefore x-5 = 7 \text{ or } -7$ $\therefore x = 12 \text{ or } -2$	✓ standard form ✓ factors ✓ answers (3)
1.1.2	$x^2 - 6x = 10(1-3x)$ $x^2 - 6x = 10 - 30x$ $x^2 + 24x - 10 = 0$ $x = \frac{-24 \pm \sqrt{(24)^2 - 4(1)(-10)}}{2(1)}$ $x = \frac{-24 \pm \sqrt{616}}{2}$ $x = 0, 41 \text{ or } x = -24, 41$	✓ standard form ✓ substitution ✓ 616 ✓✓ answers (5)
1.1.3	$(x-1)(x-2) \leq 6$ $x^2 - 3x - 4 \leq 0$ $(x-4)(x+1) \leq 0$ $\begin{array}{r} + 0 \\ - 1 \end{array} \quad \begin{array}{r} - 0 \\ 4 \end{array} \quad +$ $-1 \leq x \leq 4$	✓ standard form ✓ factors ✓✓ answer (4)
1.2	$x+3y=5$ $x=5-3y$ $(5-3y)y+y^2=3$ $5y-3y^2+y^2=3$ $0=3-5y+2y^2$ $0=(3-2y)(1-y)$ $y=\frac{3}{2} \text{ or } y=1$ $x=\frac{1}{2} \text{ or } x=2$	✓ solve for x ✓ substitution ✓ standard form ✓ factors ✓✓ y -answers ✓ x -answers (7) [19]

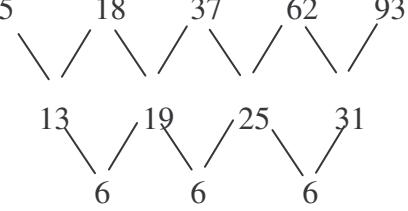
QUESTION 2

2.1	$F_V = P_V(1-i)^n$ $0,25P = P(1-0,21)^n$ $n \log 0,79 = \log 0,25$ $n = \frac{\log 0,25}{\log 0,79}$ $n = 5,88 \text{ years}$	✓ formula ✓ substitution ✓ simplification ✓ n = ✓ answer (5)
2.2.1	$Fv = Pv(1-i)^n$ $491520 = 1200000(1-i)^4$ $(1-i)^4 = 0,4096$ $i = 0,2$ $r = 20,00\%$	✓✓ substitution ✓ simplification ✓ answer (4)
2.2.2	$Fv_{(\text{sinkingfund})} = 1,2000000(1,15)^4 - 491520$ $= R1607287,50$	✓ substitution ✓ answer (2)
2.2.3	$(1,15)^4 = 1,74900625$ <p style="margin-left: 40px;">∴ an increase of 74,90 %</p>	✓ substitution ✓ increase (2)
2.2.4	Let x be the monthly repayment $i = \frac{9}{1200} = 0,0075$ $1607287,50 = x \left[\frac{(1,0075)^{48} - 1}{0,0075} \right]$ $1607287,50 = x[57,5207111]$ $x = R27 942,76$	✓ i ✓ 1607287,50 ✓ 1,0075 ✓ substitution ✓ 57,5207111 ✓ answer (6) [19]

QUESTION 3

3.1	2 ; 21	$\checkmark 2$ $\checkmark 21$ (2)
3.2	$(2 + 2 + \dots + 2) + (5 + 9 + 13 + \dots)$ $\quad \quad \quad \text{for 50 terms} \quad \quad \quad \text{for 50 terms}$ $= \sum_{i=1}^{50} 2 + \sum_{i=1}^{50} (4i + 1)$ $= 2(50) + \left[\frac{50}{2} (2(5) + 49(4)) \right]$ $= 100 + 25(10 + 196)$ $= 100 + 5150$ $= 5250$	$\checkmark 2 + 2 + \dots + 2$ $\checkmark 5 + 9 + 13 \dots$ $\checkmark 100$ $\checkmark \text{substitution}$ $\checkmark \text{answer}$ (5) [7]

QUESTION 4

4.1	130 ; 173	$\checkmark \checkmark$ (2)
4.2	 <p>The second difference is constant $\therefore T_n$ is quadratic $\therefore an^2 + bn + c = T_n$ $2a = 6$ $a = 3$</p> $T_n = 3n^2 + bn + c$ $5 = 3(1)^2 + b(1) + c$ $b + c = 2 \dots (\text{i})$ $18 = 3(2)^2 + b(2) + c$ $2b + c = 6 \dots (\text{ii})$ $(\text{ii}) - (\text{i}): \quad b = 4$ $c = -2$ $T_n = 3n^2 + 4n - 2$	$\checkmark a = 3$ $\checkmark \text{substitution}$ $\checkmark \text{equation (ii)}$ $\checkmark b = 4$ $\checkmark c = -2$ (5)

	<p>OR</p> $a(1)^2 + b(1) + c = 5$ <p>(i) $a + b + c = 5$</p> $a(2)^2 + b(2) + c = 18$ <p>(ii) $4a + 2b + c = 18$</p> $a(3)^2 + b(3) + c = 37$ <p>(iii) $9a + 3b + c = 37$</p> <p>(ii) - (i): $3a + b = 13$ $b = 13 - 3a$</p> <p>Substitute $b = 13 - 3a$ into (iii)</p> $9a + 3(13 - 3a) + c = 37$ $9a + 39 - 9a + c = 37$ $c = -2$ <p>Substitute $b = 13 - 3a$ and $c = -2$ into (ii)</p> $4a + 2(13 - 3a) + (-2) = 18$ $-2a = -6$ $a = 3$ $\therefore b = 4$	<p>✓ subst. $x = 1, 2, 3$</p> <p>✓ Equations (ii) - (i):</p> <p>✓ Substitution into (iii)</p> <p>✓ $c = -2$</p> <p>✓ $b = 4$</p>
	<p>OR</p> <p>Let T_n be the n^{th} term of the sequence</p> <p>Then</p> $\left. \begin{array}{l} T_2 - T_1 = 13 \\ T_3 - T_2 = 19 \\ T_4 - T_3 = 25 \\ T_5 - T_4 = 31 \\ T_n - T_{n-1} = \dots \end{array} \right\} \text{add both sides}$ <p>$T_n - T_1 = 13 + 19 + 25 + \dots \text{(to } n-1 \text{ terms)}$</p> $T_n - 5 = \left(\frac{n-1}{2} \right) [2(13) + (n-2)6]$ $T_n = (n-1)(3n+7) + 5$ $T_n = 3n^2 + 4n - 2$	<p>✓ Let T_n</p> <p>✓ subtracting terms</p> <p>✓ $T_n - T$</p> <p>✓ $T_n - 5$</p> <p>✓ $T_n = 3n^2 + 4n - 2$</p>
4.3	$3n^2 + 4n - 2 = 1278$ $3n^2 + 4n - 1280 = 0$ $(3n + 64)(n - 20) = 0$ $n = \frac{-64}{3} \text{ or } n = 20$ $n = \frac{-64}{3} \text{ is not valid}$ $\therefore n = 20$	<p>✓ substitution</p> <p>✓ factors</p> <p>✓ $n \neq \frac{-64}{3}$</p> <p>✓ $n = 20$</p> <p style="text-align: right;">(4) [11]</p>

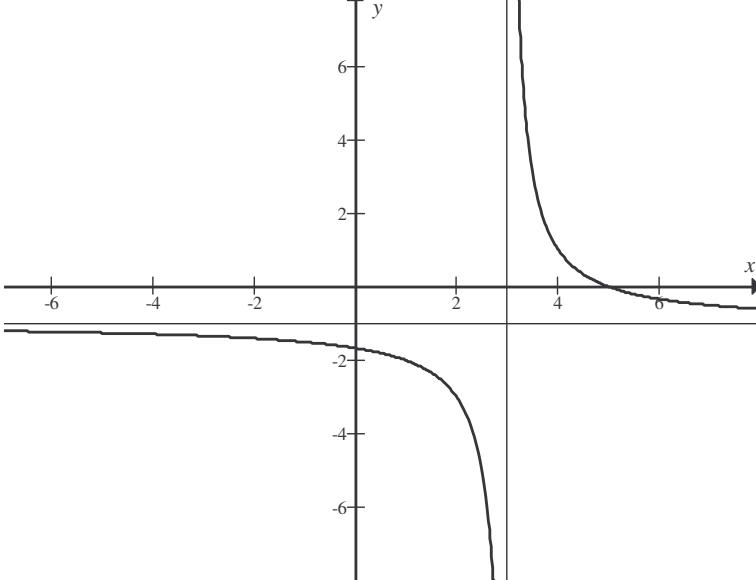
QUESTION 5

5.1	<p>Pattern 3</p> $\frac{1}{4} + \frac{3}{16} + \frac{9}{64}$ <p>Pattern 4</p> $\frac{1}{4} + \frac{3}{16} + \frac{9}{64} + \frac{27}{256}$	✓ sum ✓ $\frac{9}{64}$ ✓ sum ✓ $\frac{27}{256}$ (4)
5.2	$\frac{1}{4} + \frac{3}{16} + \frac{9}{64} + \dots + \frac{3^{n-1}}{4^n}$ $\sum_{k=1}^n \frac{3^{k-1}}{4^k}$	✓✓ nth term ✓ sigma notation (3)
5.3	$a = \frac{1}{4} \quad r = \frac{3}{4}$ $S = \frac{a}{1-r} = \frac{\frac{1}{4}}{1-\frac{3}{4}} = 1$	✓ r ✓ answer (2) [9]

QUESTION 6

6.1	$b(1)^2 = \frac{1}{2}$ $b = \frac{1}{2}$ $a^1 = \frac{1}{2}$ $a = \frac{1}{2}$	✓ answer ✓ answer (2)
6.2	$y = 2^{-x}$ $y = \left(\frac{1}{2}\right)^x$ $f^{-1} : x = \left(\frac{1}{2}\right)^y$ $y = \log_{\frac{1}{2}} x$	✓ substitution ✓ $f^{-1} : x = \left(\frac{1}{2}\right)^y$ ✓ answer (3)
6.3	The inverse is not a function because for example, $g^{-1}\left(\frac{1}{2}\right) = 1 \text{ or } -1$	✓✓ answer (2)
6.4	$x \in [0 ; \infty)$ or $x \in (-\infty ; 0]$	✓ $x \in [0 ; \infty)$ ✓ $x \in (-\infty ; 0]$ (2)
6.5.1	$0 < x < 1$	✓✓ answer (2)
6.5.2	$f(x) - 1 = g(x)$ $f(x) - g(x) = 1$ $x = 0$	✓ statement ✓ answer (2) [13]

QUESTION 7

7.1	$x = 3$ $y = -1$	✓ answer ✓ answer (2)
7.2	x -intercept: $0 = -1 + \frac{2}{x-3}$ $x-3 = 2$ $x = 5$ x -intercept $(5 ; 0)$ y -intercept $\left(0 ; -\frac{5}{3}\right)$	✓ substitution ✓ x -intercept ✓ y -intercept (3)
7.3		✓ shape ✓ intercepts ✓ asymptotes (3) [8]

QUESTION 8

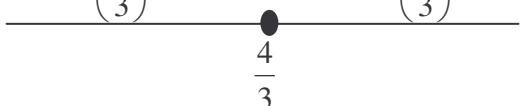
8.1	Period = 360°	✓ answer (1)
8.2	The shift changes the range of g and will become now $[-1 ; 3]$	✓ - 1 ✓ 3 (2)
8.3	$h(x) = \cos(x + 30^\circ - 30^\circ)$ $h(x) = \cos x$	✓ answer (1) [4]

QUESTION 9

9.1	$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{x - (x+h)}{x(x+h)} \\ &= \lim_{h \rightarrow 0} \frac{x - (x+h)}{x(x+h)} \div h \\ &= \lim_{h \rightarrow 0} \frac{-h}{x(x+h)} \times \frac{1}{h} \\ &= \lim_{h \rightarrow 0} \frac{-1}{x(x+h)} \\ &= \frac{-1}{x^2} \end{aligned}$	✓ substitution ✓ common denominator ✓ simplification ✓ simplification ✓ answer (5)
9.2.1	$\begin{aligned} D_x[-5x^2 + 2x] \\ = -10x + 2 \end{aligned}$	✓✓ answer (2)
9.2.2	$\begin{aligned} y &= \sqrt{x^3} + \frac{1}{3x^3} \\ y &= x^{\frac{3}{2}} + \frac{1}{3}x^{-3} \\ \frac{dy}{dx} &= \frac{3}{2}x^{\frac{1}{2}} - x^{-4} \end{aligned}$	✓ $x^{\frac{3}{2}}$ ✓ $\frac{1}{3}x^{-3}$ ✓✓ answer (4) [11]

QUESTION 10

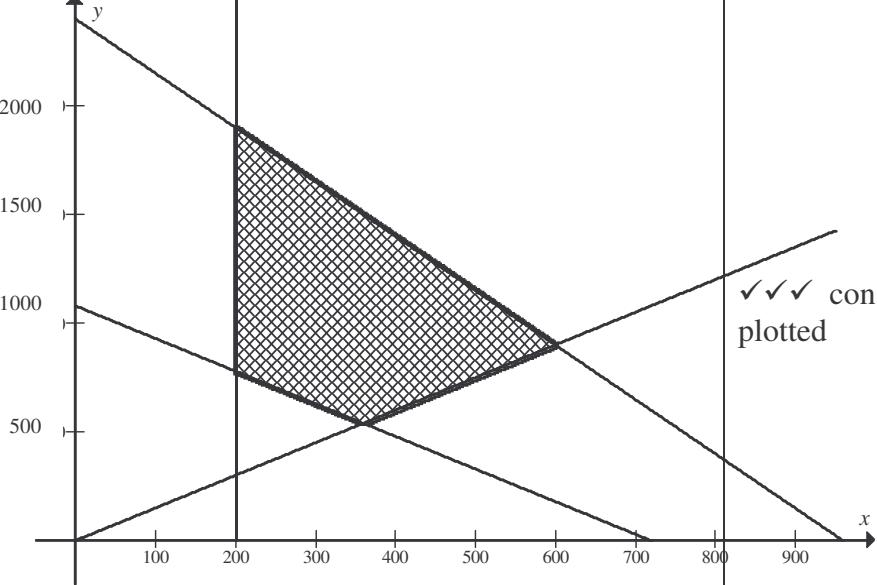
10.1	$f'(x) = 3x^2 - 8x - 11$ $0 = (3x - 11)(x + 1)$ $x = \frac{11}{3}$ or $x = -1$ $A(-1; 36)$ and $B\left(\frac{11}{3}; -14,81\right)$ Or Some candidates may know Horners' method $f\left(\frac{11}{3}\right) = \left(\left(\left(\frac{11}{3} - 4 \right) \times \frac{11}{3} \right) - 11 \right) \times \frac{11}{3} + 30$ <p>(this makes calculator work much easier) i.e. $f(x) = ((x - 4)x - 11)x + 30$</p>	✓ derivative ✓ derivative = 0 ✓ factors ✓ x-values ✓ points (5)
10.2	$(1; 36)$ $\left(\frac{17}{3}; -14,81\right)$	✓ $x = 1$ ✓ y-values remain the same (2)
10.3	Average rate of change $= \frac{36 - (-14,8)}{-1 - \frac{11}{3}}$ $= \frac{50,8}{-4,6}$ $= -10,89$	✓ formula ✓ substitution ✓ answer (3)
10.4	$f'(1) = 3x^2 - 8x - 11$ $= 3(1)^2 - 8(1) - 11$ $= -16$ $f(1) = 1^3 - 4(1)^2 - 11(1) + 30$ $f(1) = 16$ $y - 16 = -16(x - 1)$ $y = -16x + 32$	✓ $m = -16$ ✓ point (1 ; 16) ✓ substitution ✓ answer (4)
10.5	$-16x + 32 = x^3 - 4x^2 - 11x + 30$ $0 = x^3 - 4x^2 + 6x - 2$ $0 = (x - 1)(x^2 - 3x + 2)$ $0 = (x - 1)(x - 1)(x - 2)$ $x = 1$ or $x = 2$ the tangent cuts the graph again at $x = 2$	✓ substitution ✓ standard form ✓ factors ✓ answer (4)
10.6	$k > 36$ or $k < -14,81$	✓✓ answers (2)

10.7	$f'(x) = 3x^2 - 8x - 11$ $f''(x) = 6x - 8$ $0 = 6x - 8$ $x = \frac{4}{3}$ $f''\left(\frac{4}{3}\right) < 0$ $f''\left(\frac{4}{3}\right) > 0$  Point of inflection $\left(\frac{4}{3}; \frac{286}{27}\right)$ or $(1,33; 10,59)$	✓ $f''(x) = 6x - 8$ ✓ $f''(x) = 0$ ✓✓ argument ✓ x -value ✓ y -value (6) [26]

QUESTION 11

11.1	area = $\frac{\text{volume}}{\text{height}} = \frac{x^3 - 8x^2 + 5x + 50}{5-x} = -x^2 + 3x + 10$	✓ division by 5 - x ✓✓ answer (3)
11.2	$f(x) = x^3 - 8x^2 + 5x + 50$ $f'(x) = 3x^2 - 16x + 5$ $0 = 3x^2 - 16x + 5$ $0 = (3x-1)(x-5)$ $x = \frac{1}{3}$ or $x \neq 5$ But $x = 5$ is not valid $\therefore x = \frac{1}{3}$	✓ $f'(x)$ ✓ = 0 ✓ factors ✓ answers ✓ reject $x = 5$ ✓ dimensions (6) [9]

QUESTION 12

12.1	$\frac{1}{2}x + \frac{1}{5}y \leq 480$ $\therefore 5x + 2y \leq 4800$ $\frac{y}{x} \geq \frac{3}{2}$ $\therefore y \geq \frac{3x}{2}$	✓✓ inequality ✓✓ inequality (4)
12.2 and 12.3		✓✓✓ constraints correctly plotted (3) ✓ feasible region (1)
12.4	$P = 12\ 000x + 4\ 000y$	✓✓ equation (2)
12.5	$x = 600$ and $y = 900$	✓✓ answer (2)
12.6	$P = 12\ 000(600) + 4\ 000(900)$ $= R\ 10\ 800\ 000$	✓ substitution ✓ answer (2) [14]

Total: 150