



education

Department:
Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MATHEMATICS P1

EXEMPLAR 2008

MEMORANDUM

This memorandum consists of 12 pages.

QUESTION 1

1.1.1	$x^2 - 10x = 24$ $x^2 - 10x - 24 = 0$ $(x - 12)(x + 2) = 0$ $x = 12$ or $x = -2$ OR $(x - 5)^2 = 49 = 7^2$ $\therefore x - 5 = 7 \text{ or } -7$ $\therefore x = 12 \text{ or } -2$	✓ standard form ✓ factors ✓ answers (3)
1.1.2	$x^2 - 6x = 10(1 - 3x)$ $x^2 - 6x = 10 - 30x$ $x^2 + 24x - 10 = 0$ $x = \frac{-24 \pm \sqrt{(24)^2 - 4(1)(-10)}}{2(1)}$ $x = \frac{-24 \pm \sqrt{616}}{2}$ $x = 0,41$ or $x = -24,41$	✓ standard form ✓ substitution ✓ 616 ✓✓ answers (5)
1.1.3	$(x - 1)(x - 2) \leq 6$ $x^2 - 3x - 4 \leq 0$ $(x - 4)(x + 1) \leq 0$ $\begin{array}{ccccccc} + & 0 & - & 0 & + & & \\ & -1 & & 4 & & & \end{array}$ $-1 \leq x \leq 4$	✓ standard form ✓ factors ✓✓ answer (4)
1.2	$x + 3y = 5$ $x = 5 - 3y$ $(5 - 3y)y + y^2 = 3$ $5y - 3y^2 + y^2 = 3$ $0 = 3 - 5y + 2y^2$ $0 = (3 - 2y)(1 - y)$ $y = \frac{3}{2}$ or $y = 1$ $x = \frac{1}{2}$ or $x = 2$	✓ solve for x ✓ substitution ✓ standard form ✓ factors ✓✓ y-answers ✓ x-answers (7) [19]

QUESTION 2

2.1	$F_v = P_v(1-i)^n$ $0,25P = P(1-0,21)^n$ $n \log 0,79 = \log 0,25$ $n = \frac{\log 0,25}{\log 0,79}$ $n = 5,88 \text{ years}$	✓ formula ✓ substitution ✓ simplification ✓ n = ✓ answer (5)
2.2.1	$F_v = P_v(1-i)^n$ $491520 = 1200000(1-i)^4$ $(1-i)^4 = 0,4096$ $i = 0,2$ $r = 20,00\%$	 ✓✓ substitution ✓ simplification ✓ answer (4)
2.2.2	$F_{v(\text{sinkingfund})} = 1,2000000(1,15)^4 - 491520$ $= R1607287,50$	✓ substitution ✓ answer (2)
2.2.3	$(1,15)^4 = 1,74900625$ $\therefore$ an increase of 74,90 %	✓ substitution ✓ increase (2)
2.2.4	Let x be the monthly repayment $i = \frac{9}{1200} = 0,0075$ $1607287,50 = x \left[\frac{(1,0075)^{48} - 1}{0,0075} \right]$ $1607287,50 = x[57,5207111]$ $x = R27\,942,76$	 ✓ i ✓ 1607287,50 ✓ 1,0075 ✓ substitution ✓ 57,5207111 ✓ answer (6)

[19]

QUESTION 3

3.1	2 ; 21	✓ 2 ✓ 21 (2)
3.2	$(2 + 2 + \dots + 2)_{\text{for } 50 \text{ terms}} + (5 + 9 + 13 + \dots)_{\text{for } 50 \text{ terms}}$ $= \sum_{i=1}^{50} 2 + \sum_{i=1}^{50} (4i + 1)$ $= 2(50) + \left[\frac{50}{2} (2(5) + 49(4)) \right]$ $= 100 + 25(10 + 196)$ $= 100 + 5150$ $= 5250$	✓ $2 + 2 + \dots + 2$ <i>for 50 terms</i> ✓ $5 + 9 + 13 \dots$ <i>for 50 terms</i> ✓ 100 ✓ substitution ✓ answer (5) [7]

QUESTION 4

4.1	130 ; 173	✓ ✓
4.2	$\begin{array}{ccccccccc} 5 & & 18 & & 37 & & 62 & & 93 \\ & \backslash & / & \backslash & / & \backslash & / & \backslash & / \\ & & 13 & & 19 & & 25 & & 31 \\ & & & \backslash & / & \backslash & / & \backslash & / \\ & & & & 6 & & 6 & & 6 \end{array}$ The second difference is constant $\therefore T_n$ is quadratic $\therefore an^2 + bn + c = T_n$ $2a = 6$ $a = 3$ $T_n = 3n^2 + bn + c$ $5 = 3(1)^2 + b(1) + c$ $b + c = 2 \quad \dots (i)$ $18 = 3(2)^2 + b(2) + c$ $2b + c = 6 \quad \dots (ii)$ (ii) – (i): $b = 4$ $c = -2$ $T_n = 3n^2 + 4n - 2$	(2) ✓ $a = 3$ ✓ substitution ✓ equation (ii) ✓ $b = 4$ ✓ $c = -2$ (5)

	OR $a(1)^2 + b(1) + c = 5$ (i) $a + b + c = 5$ $a(2)^2 + b(2) + c = 18$ (ii) $4a + 2b + c = 18$ $a(3)^2 + b(3) + c = 37$ (iii) $9a + 3b + c = 37$ (ii) – (i): $3a + b = 13$ $b = 13 - 3a$ Substitute $b = 13 - 3a$ into (iii) $9a + 3(13 - 3a) + c = 37$ $9a + 39 - 9a + c = 37$ $c = -2$ Substitute $b = 13 - 3a$ and $c = -2$ into (ii) $4a + 2(13 - 3a) + (-2) = 18$ $-2a = -6$ $a = 3$ $\therefore b = 4$ OR Let T_n be the n^{th} term of the sequence Then $\left. \begin{array}{l} T_2 - T_1 = 13 \\ T_3 - T_2 = 19 \\ T_4 - T_3 = 25 \\ T_5 - T_4 = 31 \\ T_n - T_{n-1} = \dots \end{array} \right\} \text{add both sides}$ $T_n - T_1 = 13 + 19 + 25 + \dots (\text{to } n-1 \text{ terms})$ $T_n - 5 = \left(\frac{n-1}{2} \right) [2(13) + (n-2)6]$ $T_n = (n-1)(3n+7) + 5$ $T_n = 3n^2 + 4n - 2$	✓ subst. $x = 1, 2, 3$ ✓ Equations (ii) – (i): ✓ Substitution into (iii) ✓ $c = -2$ ✓ $b = 4$ ✓ Let T_n ✓ subtracting terms ✓ $T_n - T$ ✓ $T_n - 5$ ✓ $T_n = 3n^2 + 4n - 2$
4.3	$3n^2 + 4n - 2 = 1278$ $3n^2 + 4n - 1280 = 0$ $(3n + 64)(n - 20) = 0$ $n = \frac{-64}{3} \text{ or } n = 20$ $n = \frac{-64}{3} \text{ is not valid}$ $\therefore n = 20$	✓ substitution ✓ factors ✓ $n \neq \frac{-64}{3}$ ✓ $n = 20$ (4) [11]

QUESTION 5

5.1	Pattern 3 $\frac{1}{4} + \frac{3}{16} + \frac{9}{64}$ Pattern 4 $\frac{1}{4} + \frac{3}{16} + \frac{9}{64} + \frac{27}{256}$	✓ sum ✓ $\frac{9}{64}$ ✓ sum ✓ $\frac{27}{256}$ (4)
5.2	$\frac{1}{4} + \frac{3}{16} + \frac{9}{64} + \dots + \frac{3^{n-1}}{4^n}$ $\sum_{k=1}^n \frac{3^{k-1}}{4^k}$	✓✓ nth term ✓ sigma notation (3)
5.3	$a = \frac{1}{4} \quad r = \frac{3}{4}$ $S = \frac{a}{1-r} = \frac{\frac{1}{4}}{1-\frac{3}{4}} = 1$	✓ r ✓ answer (2) [9]

QUESTION 6

6.1	$b(1)^2 = \frac{1}{2}$ $b = \frac{1}{2}$ $a^1 = \frac{1}{2}$ $a = \frac{1}{2}$	✓ answer ✓ answer (2)
6.2	$y = 2^{-x}$ $y = \left(\frac{1}{2}\right)^x$ $f^{-1} : x = \left(\frac{1}{2}\right)^y$ $y = \log_{\frac{1}{2}} x$	✓ substitution ✓ $f^{-1} : x = \left(\frac{1}{2}\right)^y$ ✓ answer (3)
6.3	The inverse is not a function because for example, $g^{-1}\left(\frac{1}{2}\right) = 1 \text{ or } -1$	✓✓ answer (2)
6.4	$x \in [0 ; \infty)$ or $x \in (-\infty ; 0]$	✓ $x \in [0 ; \infty)$ ✓ $x \in (-\infty ; 0]$ (2)
6.5.1	$0 < x < 1$	✓✓ answer (2)
6.5.2	$f(x) - 1 = g(x)$ $f(x) - g(x) = 1$ $x = 0$	✓ statement ✓ answer (2)
[13]		

QUESTION 8

8.1	Period = 360°	✓ answer (1)
8.2	The shift changes the range of g and will become now [-1 ; 3]	✓ - 1 ✓ 3 (2)
8.3	$h(x) = \cos(x + 30^\circ - 30^\circ)$ $h(x) = \cos x$	✓ answer (1) [4]

QUESTION 9

9.1	$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$ $= \lim_{h \rightarrow 0} \frac{\frac{x - (x+h)}{x(x+h)}}{h}$ $= \lim_{h \rightarrow 0} \frac{x - (x+h)}{x(x+h)} \div h$ $= \lim_{h \rightarrow 0} \frac{-h}{x(x+h)} \times \frac{1}{h}$ $= \lim_{h \rightarrow 0} \frac{-1}{x(x+h)}$ $= \frac{-1}{x^2}$	✓ substitution ✓ common denominator ✓ simplification ✓ simplification ✓ answer (5)
9.2.1	$D_x[-5x^2 + 2x]$ $= -10x + 2$	✓✓ answer (2)
9.2.2	$y = \sqrt{x^3} + \frac{1}{3x^3}$ $y = x^{\frac{3}{2}} + \frac{1}{3}x^{-3}$ $\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} - x^{-4}$	✓ $x^{\frac{3}{2}}$ ✓ $\frac{1}{3}x^{-3}$ ✓✓ answer (4) [11]

QUESTION 10

10.1	$f'(x) = 3x^2 - 8x - 11$ $0 = (3x - 11)(x + 1)$ $x = \frac{11}{3}$ or $x = -1$ $A(-1; 36)$ and $B\left(\frac{11}{3}; -14,81\right)$ Or Some candidates may know Horner's' method $f\left(\frac{11}{3}\right) = \left(\left(\left(\frac{11}{3} - 4\right) \times \frac{11}{3}\right) - 11\right) \times \frac{11}{3} + 30$ (this makes calculator work much easier) i.e. $f(x) = ((x - 4)x - 11)x + 30$	✓ derivative ✓ derivative = 0 ✓ factors ✓ x-values ✓ points (5)
10.2	$(1; 36)$ $\left(\frac{17}{3}; -14,81\right)$	✓ $x = 1$ ✓ y-values remain the same (2)
10.3	Average rate of change $= \frac{36 - (-14,8)}{-1 - \frac{11}{3}}$ $= \frac{50,8}{-4,6}$ $= -10,89$	✓ formula ✓ substitution ✓ answer (3)
10.4	$f'(1) = 3x^2 - 8x - 11$ $= 3(1)^2 - 8(1) - 11$ $= -16$ $f(1) = 1^3 - 4(1)^2 - 11(1) + 30$ $f(1) = 16$ $y - 16 = -16(x - 1)$ $y = -16x + 32$	✓ $m = -16$ ✓ point $(1; 16)$ ✓ substitution ✓ answer (4)
10.5	$-16x + 32 = x^3 - 4x^2 - 11x + 30$ $0 = x^3 - 4x^2 + 6x - 2$ $0 = (x - 1)(x^2 - 3x + 2)$ $0 = (x - 1)(x - 1)(x - 2)$ $x = 1$ or $x = 2$ the tangent cuts the graph again at $x = 2$	✓ substitution ✓ standard form ✓ factors ✓ answer (4)
10.6	$k > 36$ or $k < -14,81$	✓✓ answers (2)

10.7	$f'(x) = 3x^2 - 8x - 11$ $f''(x) = 6x - 8$ $0 = 6x - 8$ $x = \frac{4}{3}$ $f''\left(\frac{4}{3}\right) < 0 \qquad f''\left(\frac{4}{3}\right) > 0$  Point of inflection $\left(\frac{4}{3}; \frac{286}{27}\right)$ or (1,33 ; 10,59)	$\checkmark f''(x) = 6x - 8$ $\checkmark f''(x) = 0$ $\checkmark\checkmark$ argument $\checkmark$ x-value $\checkmark$ y-value (6) [26]
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QUESTION 11

11.1	$\text{area} = \frac{\text{volume}}{\text{height}} = \frac{x^3 - 8x^2 + 5x + 50}{5 - x} = -x^2 + 3x + 10$	$\checkmark$ division by 5 - x $\checkmark\checkmark$ answer (3)
11.2	$f(x) = x^3 - 8x^2 + 5x + 50$ $f'(x) = 3x^2 - 16x + 5$ $0 = 3x^2 - 16x + 5$ $0 = (3x - 1)(x - 5)$ $x = \frac{1}{3} \quad \text{or} \quad x = 5$ But $x = 5$ is not valid $\therefore x = \frac{1}{3}$	$\checkmark f'(x)$ $\checkmark = 0$ $\checkmark$ factors $\checkmark$ answers $\checkmark$ reject $x = 5$ $\checkmark$ dimensions (6) [9]

QUESTION 12

12.1	$\frac{1}{2}x + \frac{1}{5}y \leq 480$ $\therefore 5x + 2y \leq 4800$ $\frac{y}{x} \geq \frac{3}{2}$ $\therefore y \geq \frac{3x}{2}$	✓✓ inequality ✓✓ inequality (4)
12.2 and 12.3		✓✓✓ constraints correctly plotted ✓ feasible region (1)
12.4	$P = 12\,000x + 4\,000y$	✓✓ equation (2)
12.5	$x = 600$ and $y = 900$	✓✓ answer (2)
12.6	$P = 12\,000(600) + 4\,000(900)$ $= R\,10\,800\,000$	✓ substitution ✓ answer (2) [14]

Total: 150