PROVINCE OF THE

EASTERN CAPE

EDUCATION

DIRECTORATE:

CURRICULUM FET PROGRAMMES

LESSON PLANS

TERM 3

MATHEMATICS GRADE 12

FOREWORD

The following Grade 10, 11 and 12 Lesson Plans were developed by Subject Advisors during May 2009. Teachers are requested to look at them, modify them where necessary to suit their contexts and resources. It must be remembered that Lesson Plans are working documents, and any comments to improve the lesson plans in this document will be appreciated. Teachers are urged to use this document with the following departmental policy documents: Subject Statement; LPG 2008; SAG 2008; Examination Guidelines 2009 and Provincial CASS Policy / Guidelines.

Lesson planning is the duty of each and every individual teacher but it helps when teachers sometimes plan together as a group. This interaction not only helps teachers to understand how to apply the Learning Outcomes (LOs) and Assessment Standards (ASs) but also builds up the confidence of the teachers in handling the content using new teaching strategies.

It must please be noted that in order to help teachers who teach across grades and subjects, an attempt has been made to **standardise lesson plan templates** and thus the new template might not resemble the templates used in each subject during the NCS training. However, all the essential elements of a lesson plan have been retained. This change has been made to assist teachers and lighten their administrative load.

Please note that these lesson plans are to be used only as a guide to complete the requirements of the Curriculum Statements and the work schedules and teachers are encouraged to develop their own learner activities to supplement and /or substitute some of the activities given here (depending on the school environment, number and type of learners in your class, the resources available to your learners, etc).

Do not forget to build in the tasks for the Programme of Assessment into your Lesson Plans.

Strengthen your efforts by supporting each other in clusters and share ideas. Good Luck with your endeavours to improve Teaching, Learning and Assessment.

SUBJECT: MATHEMATICS **GRADE 12** **LESSON PLAN 1** **TIME : 13 ½ HOURS**

Context: mathematical, architecture and engineering

Link with previous lesson: knowledge of grade 11 trigonometry and 3D shapes

KNOWLEDGE (K): compound angle identities Sin $(\alpha \pm \beta)$; cos $(\alpha \pm \beta)$; sin 2α ; cos2α ; 2D and 3d Shapes

SKILLS (S): use and apply compound angle identities to solve problems; interpret 2D and 3D geometric shapes to solve problems; VALUES: Respect , appreciate

Learning Outcome 1: Number and Number Relationships *When solving problems, the learner is able to recognise, describe, represent and work confidently with numbers and their relationships to estimate, calculate and check solutions.*		Learning Outcome 2: Functions and Algebra *The learner is able to investigate, analyse, describe and represent a wide range of functions and solve related problems.*		Learning Outcome 3: Space, Shape and Measurement *The learner is able to describe, represent, analyse and explain properties of shapes in 2-dimensional and 3-dimensional space with justification.*		Learning Outcome 4: Data Handling and Probability *The learner is able to collect, organise, analyse and interpret data to establish statistical and probability models to solve related problems.*	
12.1.2Demonstrate an understanding of the definition of a logarithm and any laws needed to solve real-life problems		12.2.1(a)Demonstrate the ability to work with various types of functions and relations including the inverses listed in the following Assessment Standard.(b)Demonstrate knowledge of the formal definition of a function		12.3.3Use a two dimensional Cartesian co-ordinate system to derive and apply: the equation of a circle (any centre);the equation of a tangent to a circle given a point on the circle		11.4.1 Calculate and represent measures of central tendency and dispersion	
12.1.3a) Identify and solve problems involving number patterns, including but not limited to arithmetic and geometric sequences and series. (b)Correctly interpret sigma notation.(c)Prove and correctly select the formula for and calculate the sum of series,		12.2.2a)Investigate and generate graphs of the inverse relations of functions, in particular the inverses of: $y = ax + q$ $y = ax^2$ $y = a^x$; $a > 0$(b) Determine which inverses are functions and how the domain of the original function needs to be restricted so that the inverse is also a function.		12.3.4(a)Use the compound angle identities to generalise the effect on the co-ordinates of a point $(x ; y)$ after rotation about the origin through an angle θ.\(b)Demonstrate the knowledge that rigid transformations		Represent bivariate numerical data as a scatter plot and suggest intuitively whether a linear, quadratic or exponential function would best fit the data (problems should include issues related to health	
12.1.4(a)Calculate the value of n in the formula $A = P(1 \pm i)^n$ b)Apply knowledge of geometric series to solving annuity, bond and sinking fund problems, with or without the use of the formulae:		12.2.3 Identify characteristics as listed below and hence use applicable characteristics to sketch graphs of the inverses of the functions listed above:(a)domain and range;(b)intercepts with the axes;(c)turning points, minima and maxima; (d)asymptotes;(e)shape and symmetry;(f)average gradient (average rate of change); intervals on which the function increases/decreases		12.3.5Derive and use the following compound angle identities: sin (α±β) ; cos (α±β) ; sin 2α ; cos2α	√		
12.1.5Critically analyse investment and loan		12.2.4 Factorise third degree polynomials		12.3.6 Solve problems in two and three	√		

options and make informed decisions as to the best option(s) (including pyramid and micro-lenders' schemes).		(including examples which require the factor theorem)		dimensions by constructing and interpreting geometric and trigonometric models			
12.1.6Solve non-routine, unseen problems		(12.2.7 a)Investigate and use instantaneous rate of change of a variable when interpreting models of situations: demonstrating an intuitive understanding of the limit concept in the context of approximating the rate of change or gradient at a point; establishing the derivatives of the following functions from first principles: (c)Determine the equations of tangents to graphs.(d)Generate sketch graphs of cubic functions using differentiation to determine the stationary points (maxima, minima and points of inflection) and the factor theorem and other techniques to determine the intercepts with the *x*-axis.(e) Solve practical problems involving optimisation and rates of change.					
		12.2.8 Solve linear programming problems by optimising a function in two variables, subject to one or more linear constraints, by establishing optima by means of a search line and further comparing the gradients of the objective function and linear constraint boundary lines.					

TEACHING ACTIVITIES	**LEARNERS ACTIVITIES**	**RESOURCES**	**ASSESSMENT**	**DATE COMPLETED**
Activity 1 Compound angle identities Sin (α±β) = sin α cos β±cos α sin β cos (α±β) = cos α cosβ-sin α sinβ Teacher demonstrates the derivation of the	Learners work on the exercises using the compound angle formula Learners use the function values of compound angles to simplify or evaluate trigonometric expressions and solve trigonometric equations	Calculator	Class work, home work Memo Educator , individual	

identities Sin (α±β) = sin α cos β±cos α sin β cos (α±β) = cos α cosβ-sin α sinβ Provides exercises for learners to apply the formula Teaching Methods Demonstration , question and answer				
Activity 2 Double angle identities Teacher demonstrates the derivation of the identities Sin 2α = 2 sin α cos α Cos 2α = $\cos^2\alpha - \sin^2\alpha$ = 1-2 $\sin^2\alpha$ = 2 $\cos^2\alpha$ -1 Provides exercises for learners to apply the formula Teaching Methods Demonstration , question and answer	Learners work on the exercises using the double angle formula Learners use the function values of double angles to simplify or evaluate trigonometric expressions and solve trigonometric equations and determine general solutions.	Calculator	Class work, home work Memo Educator , individual	

Activity 3 Solving problems in 2D and 3D models Teacher brings different 2D and 3D models (prisms, pyramids, cylinders, etc.) and demonstrates the various sides, faces, and corners of these models. Gives learners work sheets of different 2D and 3D diagrams Teaching Methods Demonstration , question and answer, problem solving	Learners solve 2D and 3D problems with regard to finding the lengths, heights and angles using the knowledge of compound angles and double angles	Calculator	Class work, home work Memo Educator , individual	
Homework: Exercises given from selected textbooks and various resource material				
Enrichment/Expanded Opportunities: Different examples and remedial work.				
Teacher Reflections:				

SIGNATURES:

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TEACHER DATE HOD / SMT DATE

SUBJECT: MATHEMATICS **GRADE 12** **LESSON PLAN 2** **TIME : 13 ½ HOURS**

Context: Design

Link with previous lesson: co ordinate geometry and compound angles

KNOWLEDGE (K): effect of rotation on the co ordinates of the point (x; y) ; effect of rigid transformations (translation, rotation and reflection) and enlargement on the size and shape

SKILLS (S): apply formula, calculation skills, drawing , visual analysis of space etc

VALUE :(V); Appreciation in the understanding of transformation in real life situations.

Learning Outcome 1: Number and Number Relationships *When solving problems, the learner is able to recognise, describe, represent and work confidently with numbers and their relationships to estimate, calculate and check solutions.*		Learning Outcome 2: Functions and Algebra *The learner is able to investigate, analyse,describe and represent a wide range of functions and solve related problems.*		Learning Outcome 3: Space, Shape and Measurement *The learner is able to describe, represent, analyse and explain properties of shapes in 2-dimensional and 3-dimensional space with justification.*		Learning Outcome 4: Data Handling and Probability *The learner is able to collect, organise, analyse and interpret data to establish statistical and probability models to solve related problems.*	
12.1.2Demonstrate an understanding of the definition of a logarithm and any laws needed to solve real-life problems		12.2.1(a)Demonstrate the ability to work with various types of functions and relations including the inverses listed in the following Assessment Standard.(b)Demonstrate knowledge of the formal definition of a function		12.3.3Use a two dimensional Cartesian co-ordinate system to derive and apply: the equation of a circle (any centre);the equation of a tangent to a circle given a point on the circle	√	11.4.1 Calculate and represent measures of central tendency and dispersion	
12.1.3a) Identify and solve problems involving number patterns, including but not limited to arithmetic and geometric sequences and series. (b)Correctly interpret sigma notation.(c)Prove and correctly select the formula for and calculate the sum of series,		12.2.2a)Investigate and generate graphs of the inverse relations of functions, in particular the inverses of: $y = ax + q$ $y = ax^2$ $y = a^x$; $a > 0$(b) Determine which inverses are functions and how the domain of the original function needs to be restricted so that the inverse is also a function.		12.3.4(a)Use the compound angle identities to generalise the effect on the co-ordinates of a point (x ; y) after rotation about the origin through an angle θ.\(b)Demonstrate the knowledge that rigid transformations	√	Represent bivariate numerical data as a scatter plot and suggest intuitively whether a linear, quadratic or exponential function would best fit the data (problems should include issues related to health	
12.1.4(a)Calculate the value of n in the formula $A = P(1 \pm i)^n$ b)Apply knowledge of geometric series to solving annuity, bond and sinking fund problems, with or without the use of the formulae:		12.2.3 Identify characteristics as listed below and hence use applicable characteristics to sketch graphs of the inverses of the functions listed above:(a)domain and range;(b)intercepts with the axes;(c)turning points, minima and maxima; (d)asymptotes;(e)shape and symmetry;(f)average gradient (average rate		12.3.5Derive and use the following compound angle identities: sin (α±β) ; cos (α±β) ; sin 2α ; cos2α			

		of change); intervals on which the function increases/decreases					
12.1.5Critically analyse investment and loan options and make informed decisions as to the best option(s) (including pyramid and micro-lenders' schemes).		12.2.4 Factorise third degree polynomials (including examples which require the factor theorem)		12.3.6 Solve problems in two and three dimensions by constructing and interpreting geometric and trigonometric models			
12.1.6Solve non-routine, unseen problems		(12.2.7 a)Investigate and use instantaneous rate of change of a variable when interpreting models of situations: demonstrating an intuitive understanding of the limit concept in the context of approximating the rate of change or gradient at a point; establishing the derivatives of the following functions from first principles: (c)Determine the equations of tangents to graphs.(d)Generate sketch graphs of cubic functions using differentiation to determine the stationary points (maxima, minima and points of inflection) and the factor theorem and other techniques to determine the intercepts with the *x*-axis.(e) Solve practical problems involving optimisation and rates of change.					
		12.2.8 Solve linear programming problems by optimising a function in two variables, subject to one or more linear constraints, by establishing optima by means of a search line and further comparing the gradients of the objective function and linear constraint boundary lines.					
TEACHING ACTIVITIES		**LEARNERS ACTIVITIES**		**RESOURCES**		**ASSESSMENT**	**DATE COMPLETED**
Activity 1 Effect of rotation about the origin on the co ordinates of a point Teacher demonstrates the derivation of the		Learners work on worksheets to find the co ordinates of points after rotation through various angles		Work sheet, calculator		Class work, Memo Educator/peer/individual	

formula $P'(x';y') = P'(x\cos\theta - y\sin\theta; y\cos\theta + x\sin\theta)$ And provides learners with work sheets Teaching Methods Demonstration , discussions				
Activity 2 Effect of rigid transformations (translation, rotation, reflection, glide reflection) Teacher provides a worksheet with shapes/ polygons drawn on a grid paper Teaching Methods Discovery method, discussion, group work	Learners draw the images of the polygons after the rigid transformations. Come up with conclusions with regard to: shape and Size	Work sheet, calculator Graph paper	Class work, short test Memo, Educator /peer	
Activity 3 Effects of enlargement Teacher provides a worksheet with shapes/ polygons drawn on a grid paper Teaching Methods	Learners draw the images of the enlarged polygons using the given scale factor. Come up with conclusions with regard to: shape and Size	Work sheet, calculator Graph paper	Class work, home work Memo Educator/peer **Controlled test**	

Discovery method, discussion, group work				
Homework: Exercises given from selected textbooks and various resource material				
Enrichment/Expanded Opportunities: Different examples where learners are given reflection through x=y, to find the scale factor, lengths of images etc and remedial work.				
Teacher Reflections:				

SIGNATURES:

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TEACHER	DATE	HOD / SMT	DATE

SUBJECT: MATHEMATICS	GRADE 12	LESSON PLAN 3	TIME : 9 HOURS

Context: economics, manufacturing and engineering

Link with previous lesson: linear graphs and inequalities and grade 11 knowledge of linear programming

KNOWLEDGE (K): optimizing a function in two variables; establishing optima by means of a search line and compare the gradient of objective function and linear constraint boundary lines. SKILLS (S): decision making, reasoning etc

VALUES: Appreciation

Learning Outcome 1: Number and Number Relationships *When solving problems, the learner is able to recognise, describe, represent and work confidently with numbers and their relationships to estimate, calculate and check solutions.*		Learning Outcome 2: Functions and Algebra *The learner is able to investigate, analyse, describe and represent a wide range of functions and solve related problems.*		Learning Outcome 3: Space, Shape and Measurement *The learner is able to describe, represent, analyse and explain properties of shapes in 2-dimensional and 3-dimensional space with justification.*		Learning Outcome 4: Data Handling and Probability *The learner is able to collect, organise, analyse and interpret data to establish statistical and probability models to solve related problems.*	
12.1.2Demonstrate an understanding of the definition of a logarithm and any laws needed to solve real-life problems		12.2.1(a)Demonstrate the ability to work with various types of functions and relations including the inverses listed in the following Assessment Standard.(b)Demonstrate knowledge of the formal definition of a function		12.3.3Use a two dimensional Cartesian co-ordinate system to derive and apply: the equation of a circle (any centre);the equation of a tangent to a circle given a point on the circle		11.4.1 Calculate and represent measures of central tendency and dispersion	
12.1.3a) Identify and solve problems involving number patterns, including but not limited to arithmetic and geometric sequences and series. (b)Correctly interpret sigma notation.(c)Prove and correctly select the formula for and calculate the sum of series,		12.2.2a)Investigate and generate graphs of the inverse relations of functions, in particular the inverses of:$y = ax + q$ $y = ax^2$ $y = a^x$; $a > 0$(b) Determine which inverses are functions and how the domain of the original function needs to be restricted so that the inverse is also a function.		12.3.4(a)Use the compound angle identities to generalise the effect on the co-ordinates of a point $(x ; y)$ after rotation about the origin through an angle θ.\(b)Demonstrate the knowledge that rigid transformations		Represent bivariate numerical data as a scatter plot and suggest intuitively whether a linear, quadratic or exponential function would best fit the data (problems should include issues related to health	
12.1.4(a)Calculate the value of n in the formula $A = P(1 \pm i)^n$ b)Apply knowledge of geometric series to solving annuity, bond and sinking fund problems, with or without the use of the formulae:		12.2.3 Identify characteristics as listed below and hence use applicable characteristics to sketch graphs of the inverses of the functions listed above:(a)domain and range;(b)intercepts with the axes;(c)turning points, minima and maxima; (d)asymptotes;(e)shape and symmetry;(f)average gradient (average rate of change); intervals on which the function increases/decreases		12.3.5Derive and use the following compound angle identities: sin (α±β) ; cos (α±β) ; sin 2α ; cos2α			
12.1.5Critically analyse investment and loan options and make informed decisions as to the best option(s) (including pyramid and micro-lenders' schemes).		12.2.4 Factorise third degree polynomials (including examples which require the factor theorem)		12.3.6 Solve problems in two and three dimensions by constructing and interpreting geometric and trigonometric models			
12.1.6Solve non-routine, unseen problems		(12.2.7 a)Investigate and use instantaneous rate of change of a variable when interpreting models of situations: demonstrating an intuitive understanding of					

		the limit concept in the context of approximating the rate of change or gradient at a point; establishing the derivatives of the following functions from first principles: (c)Determine the equations of tangents to graphs.(d)Generate sketch graphs of cubic functions using differentiation to determine the stationary points (maxima, minima and points of inflection) and the factor theorem and other techniques to determine the intercepts with the *x*-axis.(e) Solve practical problems involving optimisation and rates of change.					
		12.2.8 Solve linear programming problems by optimising a function in two variables, subject to one or more linear constraints, by establishing optima by means of a search line and further comparing the gradients of the objective function and linear constraint boundary lines.	√				

TEACHING ACTIVITIES	**LEARNERS ACTIVITIES**	**RESOURCES**	**ASSESSMENT**	**DATE COMPLETED**
Teacher provides word problems from examples of finance, transport, business, etc in the form of work sheets. Teaching Methods Question answer, discussion	Learners work on the worksheet to find the optimum values by -forming constraints -drawing lines -findings feasible region -finding optimal value by inspection	Work sheet calculator, graph paper, pencil	Class work, assignment home work memo educator, individual	
Teacher provides word problems on different contexts (preferably a context from the school, classroom situation)	-Learners write down the objective function of the problem	Graph paper, ruler, pencil	Class work, assignment , short test Memo	

Teaching Methods Question answer, discussion	-Find the gradient of the objective function -draw search lines with the same gradient -find the optimal value		Educator , peer	
Homework: Exercises given from selected textbooks and various resource material				
Enrichment/Expanded Opportunities: more word problems on different contexts are given remedial work for slow learners				
Teacher Reflections:				

SIGNATURES:

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TEACHER	DATE	HOD / SMT	DATE

SUBJECT: MATHEMATICS	GRADE 12	LESSON PLAN 4	TIME : 9 HOURS

Revision work for examinations

Learning Outcome 1: Number and Number Relationships *When solving problems, the learner is able to recognise, describe, represent and work confidently with numbers and their relationships to estimate, calculate and check solutions.*		Learning Outcome 2: Functions and Algebra *The learner is able to investigate, analyse, describe and represent a wide range of functions and solve related problems.*		Learning Outcome 3: Space, Shape and Measurement *The learner is able to describe, represent, analyse and explain properties of shapes in 2-dimensional and 3-dimensional space with justification.*		Learning Outcome 4: Data Handling and Probability *The learner is able to collect, organise, analyse and interpret data to establish statistical and probability models to solve related problems.*	
12.1.2Demonstrate an understanding of the definition of a logarithm and any laws needed to solve real-life problems	√	12.2.1(a)Demonstrate the ability to work with various types of functions and relations including the inverses listed in the following Assessment Standard.(b)Demonstrate knowledge of the formal definition of a function	√	12.3.3Use a two dimensional Cartesian co-ordinate system to derive and apply: the equation of a circle (any centre);the equation of a tangent to a circle given a point on the circle	√	11.4.1 Calculate and represent measures of central tendency and dispersion	√
12.1.3a) Identify and solve problems involving number patterns, including but not limited to arithmetic and geometric sequences and series. (b)Correctly interpret sigma notation.(c)Prove and correctly select the formula for and calculate the sum of series,	√	12.2.2a)Investigate and generate graphs of the inverse relations of functions, in particular the inverses of: $y = ax + q$ $y = ax^2 y = a^x$; $a > 0$(b) Determine which inverses are functions and how the domain of the original function needs to be restricted so that the inverse is also a function.	√	12.3.4(a)Use the compound angle identities to generalise the effect on the co-ordinates of a point $(x ; y)$ after rotation about the origin through an angle θ.\(b)Demonstrate the knowledge that rigid transformations	√	Represent bivariate numerical data as a scatter plot and suggest intuitively whether a linear, quadratic or exponential function would best fit the data (problems should include issues related to health	√
12.1.4(a)Calculate the value of n in the formula $A = P(1 \pm i)^n$ b)Apply knowledge of geometric series to solving annuity, bond and sinking fund problems, with or without the use of the formulae:	√	12.2.3 Identify characteristics as listed below and hence use applicable characteristics to sketch graphs of the inverses of the functions listed above:(a)domain and range;(b)intercepts with the axes;(c)turning points, minima and maxima; (d)asymptotes;(e)shape and symmetry;(f)average gradient (average rate of change); intervals on which the function increases/decreases	√	12.3.5Derive and use the following compound angle identities: sin (α±β) ; cos (α±β) ; sin 2α ; cos2α	√		
12.1.5Critically analyse investment and loan options and make informed decisions as to the best option(s) (including pyramid and micro-lenders' schemes).	√	12.2.4 Factorise third degree polynomials (including examples which require the factor theorem)	√	12.3.6 Solve problems in two and three dimensions by constructing and interpreting geometric and trigonometric models	√		
12.1.6Solve non-routine, unseen problems		(12.2.7 a)Investigate and use instantaneous rate of change of a variable when interpreting models of situations: demonstrating an intuitive understanding of the limit concept in the context of approximating the rate of change or	√				

		gradient at a point; establishing the derivatives of the following functions from first principles: (c)Determine the equations of tangents to graphs.(d)Generate sketch graphs of cubic functions using differentiation to determine the stationary points (maxima, minima and points of inflection) and the factor theorem and other techniques to determine the intercepts with the *x*-axis.(e) Solve practical problems involving optimisation and rates of change.					
		12.2.8 Solve linear programming problems by optimising a function in two variables, subject to one or more linear constraints, by establishing optima by means of a search line and further comparing the gradients of the objective function and linear constraint boundary lines.	√				
TEACHING ACTIVITIES		**LEARNERS ACTIVITIES**		**RESOURCES**		**ASSESSMENT**	**DATE COMPLETED**
Activity 1 REVISION WORK FOR TRIAL EXAMS Teacher prepares questions from past papers for revision		Learners work out past paper questions		Exemplars,		Short tests Home work Class work	
Activity 2 REVISION WORK FOR TRIAL EXAMS Exemplar and past paper revision		Study groups are formed for revision of exemplar and past papers		Exemplars,		TRIAL EXAMS	
Homework: Exercises given from selected textbooks and various resource material							
Enrichment/Expanded Opportunities: Remedial work on trial examination							
Teacher Reflections:							

SIGNATURES:

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TEACHER DATE HOD / SMT DATE